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Product Cordial Labelling of Some Connected Graphs

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Abstract: Product cordial labelling is binary labelling of vertices such that edge labelling is defined by difference of labelling of its end vertices followed by difference of number of vertices with 0 labelling and 1 labelling is less than or equal to 1. Here we discuss product cordial labelling of some connected and undirected graphs. We have proved that graph obtained by joining even number of copies of finite, connected and undirected graph by a path admits product cordial labelling.
Keywords: Cordial graph, Product cordial graph, Path, Copies of graph

I. INTRODUCTION

Graph labelling is an immense growing research area, which has many application to the science and technology. We focus on product cordial labelling. We consider finite, connected and undirected graph. We consider graph $G = (V(G), E(G))$ having set of vertices $V(G)$ and set of edges $E(G)$ respectively. We refer Gross and Yellen [2] for all kind of definitions and notations. We refer "A dynamic survey by Gallian [1]" for detailed survey on labelling.

A. Definition 1.1

If the vertices of graph are assigned by some values or numbers subject to certain rules is known as graph labelling.

B. Definition 1.2

Labelling of vertices of graph by binary numbers, 0's and 1's under certain condition is called binary vertex labelling.

Notations:

$v_f(0)$ = Number of vertices with label 0.

$v_f(1)$ = Number of vertices with label 1.

$e_f(0)$ = Number of edges with label 0.

$e_f(1)$ = Number of edges with label 1.

C. Definition 1.3

A binary vertex labelling of graph G with induced labelling $f^*: E \rightarrow \{0,1\}$ defined by $f^*(e = uv) = f(u)f(v)$ is called a product cordial labelling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits product cordial labelling is called product cordial graph.

II. OBSERVATIONS

Present work is motivated by some results of Vaidya and Kanani [6] and Nikhil and Dharamvir [3] in product cordial labelling which are given below:

A. Observation 2.1

The graph obtained by joining two copies of Petersen graph by a path of arbitrary length is a product cordial graph.

B. Observation 2.2

The graph obtained by joining two copies of $C_n(C_n)$ by a path of arbitrary length is a product cordial graph.

C. Observation 2.3

The path union of k copies of $C_n(C_n)$ is a product cordial graph except for odd k.

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D. Observation 2.4

The path union of k copies of Petersen graph is a product cordial graph except for odd k.

E. Observation 2.5

k Copies of $\boldsymbol{C_n(C_n)}$ joined by a path where each consecutive copies joined by a path of same length is product cordial, where k is even.

In the last result [observation 2.5] we have replaced different graphs and try to check whether they admits product cordial labelling or not. It can be seen that in the product cordial labelling of such kind of graphs, it is important to check product cordial labelling of paths which joins copies of graphs. In even number of copies we can easily label half number of copies by 1 and remaining half copies by 0, which is shown as below.

III. RESULTS

Theorem 3.1 Let $G_1, G_2, G_3, \dots, G_n$ be n copies of any graph G'. where G' is finite and connected as well as undirected graph. Each G_i and G_{i+1} connected by a path P_l for $1 \le i \le n-1$.

Then the resultant graph G admits product cordial labelling when n is even.

Proof: Let $G_1, G_2, G_3, \dots, G_n$ be n copies of any finite, connected and undirected graph G'.

Let us assume $|V(G')| = p$ and $|E(G')| = k$

Hence we get $|V(G_i)| = p$ and $|E(G_i)| = k$; for $1 \le i \le n$

Let $u_{i1}, u_{i2}, u_{i3}, \dots, u_{ip}$ be the vertices of graph G_i for $1 \le i \le n$

Now each G_i and G_{i+1} is connected by a path P_l of length $l-1$ for $1 \le i \le n$.

Let $w_{(i)(i+1)1}, w_{(i)(i+1)2}, \dots, w_{(i)(i+1)l}$ be the vertices of path P_l between G_i and G_{i+1} for $1 \le i \le n$.

Here we consider

$u_{11} = w_{121}$

$u_{21} = w_{12l} = w_{231}$

$u_{31} = w_{23l} = w_{341}$

……

$u_{(n-1)1} = w_{(n-2)(n-1)l} = w_{(n-1)n1}$

$u_{n1} = w_{(n-1)nl}$

Note that $|V(G)| = np + (n-1)(l-2)$

$|E(G)| = nk + (n-1)(l-1)$.

To define binary vertex labelling $f: V(G) \to \{0,1\}$.

We consider following:

For $1 \le i \le {}^{n}/_{2}$

$f(u_{i1}) = 1$

$f(u_{i2}) = 1$

……

$f(u_{ip}) = 1$

And

For ${}^{n}/_{2} < i \le n$

$f(u_{i1}) = 0$

$f(u_{i2}) = 0$

……

$f(u_{ip}) = 0$

In this manner vertices of $G_1, G_2, G_3, \dots, G_{n/2}$ are labeled by 1.

So from 1.4 all the edges of the corresponding graph are labeled by 1.

Similarly $G_{n/2+1}, \dots, G_n$ are labeled by 0.

So from 1.4 all the edges of the corresponding graph are labeled by 0.

Now for path P_l we consider following:

Case 1: l is even

$f(w_{(i)(i+1)j}) = 1$ For $1 \le i \le {}^{n}/_{2} - 1$ and $1 \le j \le l$

$f(w_{(i)(i+1)j}) = 0$ For ${}^{n}/_{2} + 1 \le i \le n - 1$ and $1 \le j \le l$

For $i = {}^{n}/_{2}$

$f(w_{(i)(i+1)j}) = 1$ For $1 \le j \le {}^{l}/_{2}$

$f(w_{(i)(i+1)j}) = 0$ For ${}^{l}/_{2} < j \le l$

Case 2: l is odd

$f(w_{(i)(i+1)j}) = 1$ For $1 \le i \le {}^{n}/_{2} - 1$ and $1 \le j \le l$

$f(w_{(i)(i+1)j}) = 0$ For ${}^{n}/_{2} + 1 \le i \le n - 1$ and $1 \le j \le l$

For $i = {}^{n}/_{2}$

$f(w_{(i)(i+1)j}) = 1$ For $1 \le j \le {}^{l+1}/_{2}$

$f(w_{(i)(i+1)j}) = 0$ For ${}^{l+1}/_{2} < j \le l$

In Case 1 $v_f(0) = v_f(1)$ and $e_f(0) = e_f(1) + 1$

In Case 2 $v_f(0) + 1 = v_f(1)$ and $e_f(0) = e_f(1)$

In both the cases $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

Hence graph G admits product cordial labelling.

IV. CONCLUSIONS

Vaidya and Kanani [6] investigated product cordial labelling for path union of various graph while Nikhil and Dharamvir [3] have proved that even copies of $C_n(C_n)$ admits product cordial labelling when they join by path. We have extended their result to any finite, connected and undirected graph. Whether similar type of result can be true for odd number of copies of any finite, connected and undirected graph is an open research problem.

V. ILLUSTRATIONS

Figure 5.1: Product cordial labelling of 4 copies of Fan graph $f_4 = P_4 + K_1$, each one joined by P_3 (l is odd).

Figure 5.2: Product cordial labelling of 4 copies of Fan graph $f_4 = P_4 + K_1$, each one joined by P_4 (l is even).

Here we can replace Fan graph by any finite, connected and undirected graph. And also replace 4 copies by any even number.

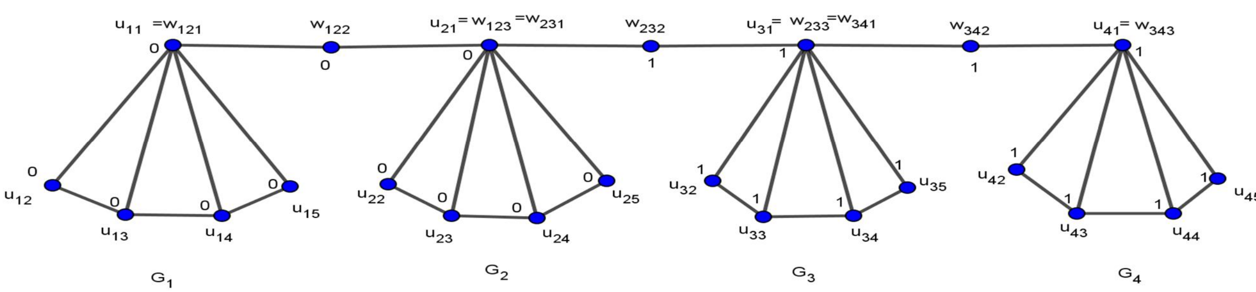

Figure 5.1: Product cordial labelling of 4 copies of Fan graph $f_4 = P_4 + K_1$, each one joined by P_3 (l is odd).

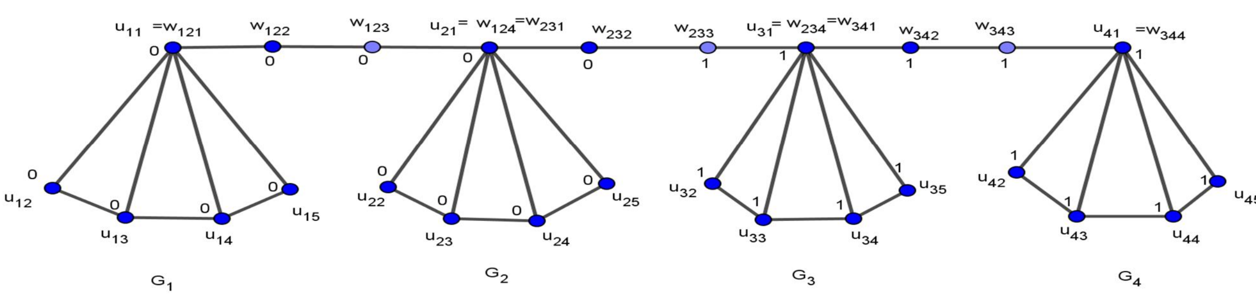

Figure 5.2: Product cordial labelling of 4 copies of Fan graph $f_4 = P_4 + K_1$, each one joined by P_4 (l is even).

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