iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 5 **Issue:** IX **Month of publication:** September 2017

DOI:

www.ijraset.com

Call: 08813907089 | E-mail ID: ijraset@gmail.com

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue IX, September 2017- Available at www.ijraset.com

Some Modified Exponential Ratio Type Estimators of Finite Population Mean in Survey Sampling

Archana Panigrahi[1], Gopabandhu Mishra[2]
[1,2] Department of Statistics, Utkal University, Bhubaneswar-751004, Odisha, India

***Abstract:* Bahl and Tuteja (1991) have suggested an exponential ratio type estimator to estimate finite population mean. In this paper some modified exponential ratio type estimators of finite population mean under simple random sampling without replacement have been proposed following Searls (1964), Srivastava (1974) and Upadhyaya and Srivastava (1976a and 1976b). The efficiencies of these estimators are compared with exponential ratio type estimator as regard to bias and mean square error both theoretically and empirically.**

***Keywords :* Simple random sampling, Exponential ratio type estimators, Bias, Mean square error, Efficiency.**

I. INTRODUCTION

In survey sampling, the utilisation of auxiliary information for improving precision of the estimate is well recognised. The classical ratio estimator of Cochran (1940) is one of such estimators which make use of population mean of auxiliary variable to increase the efficiency of the estimator. Searls (1964) utilised the known coefficient of variation of study variable y for estimation of population mean ($\overline{Y}$). Srivastava (1974) developed an estimator of population mean ($\overline{Y}$) using estimated coefficient of variation of y. Upadhyaya and Srivastava (1976a and 1976b) suggested an improved estimator of $\overline{Y}$ in a symmetrical population using estimated coefficient of variation of study variable y. Following Searls (1964), Srivastava (1974) and Upadhyaya and Srivastava (1976a and 1976b), some modified exponential ratio type estimators of population mean ($\overline{Y}$) under SRSWOR have been proposed.

Let there be a finite population U consisting of N units U_1, U_2, U_3 ,..., U_N. The i^{th} unit is indexed by a pair of real value (y_i, x_i). It is assumed that the study variable y is positively correlated with the auxiliary variable x and is denoted by ρ.

II. PROPOSED ESTIMATORS

A sample size 'n' is selected from U with simple random sampling without replacement (SRSWOR), denoting the sample mean of study variable and auxiliary variable $\overline{y}$ and $\overline{x}$ respectively.

Searls (1964) proposed an estimator to estimate finite population mean $\overline{Y}$ using known population coefficient of variation, i.e. C_y ,where $C_y = \frac{S_y}{\overline{Y}}$ and $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \overline{Y})^2$, is given by

$$\widehat{\overline{Y}}_s = \frac{\overline{y}}{1+\theta_1 C_y^2} \qquad (2.1)$$

where, $\theta_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$

An exponential ratio estimator for estimating $\overline{Y}$ suggested by Bahl and Tuteja (1991), which is more efficient than the conventional ratio estimator ($\widehat{\overline{Y}}_R = \frac{\overline{y}}{\overline{x}}\overline{X}$) when there exist a low correlation between y and x, is given by

$$t_{ER1} = \overline{y}\,\mathrm{Exp}\left[\frac{\overline{X}-\overline{x}}{\overline{X}+\overline{x}}\right] \qquad (2.2)$$

Now, we suggest a modified exponential ratio type estimator of population mean when population coefficient of variation of y, i.e. C_y is known in advance

$$t_{ER2} = \frac{\overline{y}}{1+\theta_1 C_y^2}\,\mathrm{Exp}\left[\frac{\overline{X}-\overline{x}}{\overline{X}+\overline{x}}\right] \qquad (2.3)$$

Further in absence of known C_y, considering estimated coefficient of variation i.e. $\hat{C}_y$ from sample data, we suggest another estimator for $\overline{Y}$

$$t_{ER3} = \frac{\overline{y}}{1+\theta_1 \hat{C}_y^2}\,\mathrm{Exp}\left[\frac{\overline{X}-\overline{x}}{\overline{X}+\overline{x}}\right] \qquad (2.4)$$

where, $\hat{C}_y^2 = \frac{s_y^2}{\bar{y}^2}$ and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$

Following Upadhyaya and Srivastava (1976a and 1976b), we suggest another modified exponential ratio type estimators for $\bar{Y}$ using estimated $\hat{C}_y^2$ is given by

$$t_{ER4} = \bar{y}(1 + \theta_1 \hat{C}_y^2)\text{Exp}\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \tag{2.5}$$

III. BIAS AND MSE OF DIFFERENT ESTIMATORS

Assuming the validity of Taylor's series expansion of t_{ER1}, t_{ER2}, t_{ER3} and t_{ER4} and considering the expected value to $O(\frac{1}{n})$, the bias of the different estimators are

$$B(t_{ER1}) = E(t_{ER1}) - \bar{Y} = \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} - \frac{1}{2}C_{11}\right] \tag{3.1}$$

$$B(t_{ER2}) = E(t_{ER2}) - \bar{Y} = \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} - C_{02} - \frac{1}{2}C_{11}\right] \tag{3.2}$$

$$B(t_{ER3}) = E(t_{ER3}) - \bar{Y} = \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} - C_{02} - \frac{1}{2}C_{11}\right] \tag{3.3}$$

$$B(t_{ER4}) = E(t_{ER4}) - \bar{Y} = \theta_1 \bar{Y}\left[\frac{3}{8}C_{20} + C_{02} - \frac{1}{2}C_{11}\right] \tag{3.4}$$

where, $C_{rs} = \frac{K_{rs}(x,y)}{\bar{X}^r\,\bar{Y}^s}$

$K_{rs}(x,y)$ being the $(r,s)^{th}$ cumulant of x and y.

The mean square errors (MSEs) of different estimators to $O(\frac{1}{n^2})$ are derived as

$$MSE(t_{ER1}) = \bar{Y}^2[\theta_1(C_{02} + \frac{1}{4}C_{20} - C_{11}) + (\theta_2 - \frac{3\theta_1}{N})(\frac{5}{4}C_{21} - \frac{3}{8}C_{30} - C_{12}) + \theta_1^2(C_{20}C_{02} + 2C_{11}^2 - \frac{31}{8}C_{11}C_{20} + \frac{79}{64}C_{20}^2)] \tag{3.5}$$

where, $\theta_2 = (\frac{1}{n^2} - \frac{1}{N^2})$

$$MSE(t_{ER2}) = MSE(t_{ER1}) + \theta_1\bar{Y}^2\left(3C_{11}C_{02} - \frac{5}{4}C_{02}C_{20} - C_{02}^2\right) \tag{3.6}$$

$$MSE(t_{ER3}) = MSE(t_{ER1}) + \theta_1\bar{Y}^2\left(C_{11}C_{02} - \frac{5}{4}C_{02}C_{20} + 3C_{02}^2 - 2C_{03} + C_{12}\right) \tag{3.7}$$

$$MSE(t_{ER3}) = MSE(t_{ER1}) + \theta_1\bar{Y}^2\left(-C_{11}C_{02} + \frac{5}{4}C_{02}C_{20} - C_{02}^2 + 2C_{03} - C_{12}\right) \tag{3.8}$$

IV. COMPARISON OF BIASES AND MEAN SQUARE ERRORS

The biases of t_{ER1}, t_{ER2}, t_{ER3} and t_{ER4} are of order $O(\frac{1}{n})$ and hence, are negligible when sample size is large. From (3.2) and (3.3), the biases of modified estimators t_{ER2} and t_{ER4} are same i.e.

$$B(t_{ER2}) = B(t_{ER4}) \tag{4.1}$$

However the estimators t_{ER2}, t_{ER3} and t_{ER4} are more biased than t_{ER1}.

The mean square errors of t_{ER1}, t_{ER2}, t_{ER3} and t_{ER4} to $O(\frac{1}{n})$ are same. Thus for the purpose of comparison of efficiencies, the mean square error of the estimators are considered up to $O(\frac{1}{n^2})$.

The comparisons of efficiencies of different estimators are made (a) under general conditions and (b) under bivariate symmetrical distribution.

1) t_{ER2} is more efficient than t_{ER1} if

Case (a) $C_{11} < \frac{1}{3}\left(\frac{5}{4}C_{20} + C_{02}\right)$ (4.2)

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor:6.887
Volume 5 Issue IX, September 2017- Available at www.ijraset.com

i.e. $\rho < \frac{1}{12Z}(5Z^2 + 4)$ (4.3)

Case (b) same as above condition

where, $Z = \left(\frac{C_{20}}{C_{02}}\right)^{\frac{1}{2}}$

2) t_{ER3} is more efficient than t_{ER1} if

Case (a) $C_{11} < \frac{1}{C_{02}}\left(\frac{5}{4}C_{20}C_{02} - 3C_{02}^2 + 2C_{03} - C_{12}\right)$ (4.4)

Case (b) $\rho < \frac{1}{4Z}(5Z^2 - 12)$ (4.5)

3) t_{ER4} is more efficient than t_{ER1} if

Case (a) $C_{11} > \frac{1}{C_{02}}\left(\frac{5}{4}C_{20}C_{02} - C_{02}^2 + 2C_{03} - C_{12}\right)$ (4.6)

Case (b)) $\rho > \frac{1}{4Z}(5Z^2 - 4)$ (4.7)

4) t_{ER3} is more efficient than t_{ER2} if

Case (a) $C_{11} > \frac{1}{2C_{02}}(4C_{02}^2 - 2C_{03} + C_{12})$ (4.8)

Case (b) $\rho > \frac{2}{Z}$ (4.9)

5) t_{ER4} is more efficient than t_{ER2} if

Case (a) $C_{11} > \frac{1}{4C_{02}}\left(\frac{5}{2}C_{20}C_{02} + 2C_{03} - C_{12}\right)$ (4.10)

Case (b) $\rho > \frac{5}{8}Z$ (4.11)

6) t_{ER4} is more efficient than t_{ER3} if

Case (a) $C_{11} > \frac{1}{2C_{02}}\left(\frac{5}{2}C_{20}C_{02} - 4C_{02}^2 + 4C_{03} - 2C_{12}\right)$ (4.12)

Case (b) $\rho > \frac{1}{4Z}(5Z^2 - 8)$ (4.13)

V. EMPIRICAL STUDY

To study the efficiency of different estimators we have considered eight natural populations from different textbooks. The comparison is based on exact mean square errors. We have drawn all possible $({}^{N}C_{n})$ samples of size four without replacement from given populations and the exact mean square errors are calculated. Table 1 gives the descriptions of population with Correlation Coefficient ρ and the Coefficient of Variation C_x and C_y. Table 2 gives the exact MSE of different estimators i.e. mean per unit estimator $t_0(=\bar{y})$, t_{ER1}, t_{ER2}, t_{ER3} and t_{ER4}.

TABLE 1: DESCRIPTION OF POPULATION

Population No.	Description	N	Y	X	ρ	C_x	C_y
1	Cochran(1977) p.325	10	Persons	Rooms	0.651	0.135	0.153
2	Cochran(1977) p.34	17	Food Cost	Family Size	0.466	0.393	0.319
3	Drapper & Smith (1966) p.352	25	Response Vector	Operating days per month	0.536	0.149	0.173
4	Drapper & Smith (1966) p.352	25	Response Vector	Average wind velocity	0.474	0.276	0.173
5	Drapper & Smith	25	Response	Pounds of	0.305	0.181	0.173

	(1966) p.352		Vector	crude glycerine			
6	Panse and Sukhatme (1967) p.108	25	Progeny mean (mm)	Parental plant value (mm)	0.678	0.071	0.054
7	Gujrati (1978) p.228	23	Per capita consumption of chicken (in lbs)	Real retail price of chicken (in lbs)	0.839	0.231	0.185
8	Swain (2003) p.224	20	Defence budget outlay in different years	US military sales in different years	0.724	1.14	1.117

TABLE 2: MSE OF DIFFERENT ESTIMATORS

Population No.	$t_0 = \bar{y}$	t_{ER1}	t_{ER2}	t_{ER3}	t_{ER4}
1	35.953	22.325	22.170	22.169	22.461
2	13.844	10.914	10.500	10.465	11.625
3	0.558	0.402	0.396	0.388	0.417
4	0.558	0.474	0.468	0.465	0.486
5	0.558	0.531	0.525	0.517	0.594
6	0.337	0.1819	0.1817	0.1803	0.1837
7	11.228	3.916	3.860	4.084	3.800
8	511.017	254.651	163.896	154.842	481.965

VI. CONCLUSION

A. For populations 1, 2, 3, 4, 6, 7 and 8, the estimators, t_{ER1}, t_{ER2}, t_{ER3} and t_{ER4} are more efficient than the mean per unit estimator $t_0 = \bar{y}$.

B. For populations 1, 2, 3, 4, 5, 6 and 8, the estimator t_{ER3} is most efficient.

C. For population 7, the estimator t_{ER4} is most efficient.

As the estimator t_{ER3} perform better than other estimators in most of populations considered here, so it may be used as an alternative estimator of $t_{ER1.}$

REFERENCES

[1] W. G. Cochran, Sampling Techniques, 3rd ed., New York, US: Wiley Eastern Limited, 1977.
[2] N. R. Draper and H. Smith, Applied Regression Analysis, New York, US: John Wiley & Sons, 1966.
[3] D. Gujrati, Basic Econometrics, New York, US: McGraw Hill Book Co., 1978.
[4] V. G. Panse and P. V. Sukhatme, Statistical Method for Agricultural Workers, New Delhi, India: Indian Council of Agricultural Research, 1967.
[5] D.T. Searls, "The utilization of known Coefficient of Variation in estimation procedure," Jour. Amer. Stat. Assoc., vol. 59, no, pp. 1225-26, 1964.
[6] V. K. Srivastava, "On use of Coefficient of Variation in estimating mean," Jour. Ind. Soc. Agrl. Stat., vol. 26(2), pp. 33-36, 1974.
[7] L. N. Upadhyaya and S. R. Srivastava, "A note on use of Coefficient of Variation in estimating mean," Jour. Ind. Soc. Agrl. Stat., vol. 28(2), pp. 97-98, 1976a.
[8] L. N. Upadhyaya and S. R. Srivastava, "An efficient estimator of mean when population variance is known," Jour. Ind. Soc. Agrl. Stat., vol. 28(1),, pp. 9-10, 1976b.