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Optimization of Complex Function Variable

Kamlesh Kumar Naik[1], Saurabh Kumar[2]
[1]M Tech, Thermal engineering scholar, Mechanical engineering department, Raipur institute of technology, Raipur, India)
[2]Associate Professor, Mechanical engineering department, Raipur institute of technology, Raipur, India)

Abstract: In this paper Non differentiable minimax fractional programming and particle swarm optimization is used optimized the complex function variable. Where the non differentialminimax fractional programming is used for minimization and particle swarm optimization is used for maximized the complex function. Problem on those equations which is not too optimized by conventional minima and maxima method can be easy to formulate and find the maxima and minima.
Keyword: Complex function, particle swarm optimization, non differentialminimax fractional programming

I. INTRODUCTION

The aim of this paper is to optimized a complex function variable by using this method

A. Non differentiable minimax fractional programming

In non differentialminimax fractional programming change fractional complex programming into a non fractional programming problem, and have showed that the optimal solution of complex fractional programming problem can be reduced to an optimal solution of the equivalent non fractional complex programming problem

B. Particle swarm optimization

This paper uses particle swarm optimization (PSO) to solve complex variable fractional programming problems (CVFPP) where the objective function includes the two parts (real and imaginary), the input is complex while the output is always real. Particle swarm optimization can be denoted as an effective technique for solving linear or nonlinear, nonanalytic complex fractional objective functions

II. MINIMAX FRACTIONAL PROGRAMMING PROBLEM WITH COMPLEX VARIABLES

A. Introduction

Weconsider the following minimax fractional complex programming problem [Lai et al. 2010] as the following:

$$\text{P } \min_{\zeta\in X}\max_{\eta\in Y}\frac{Re\left[f(\zeta,\eta)+(z^H Az)^{\frac{1}{2}}\right]}{Re\left[g(\zeta,\eta)-(z^H Bz)^{\frac{1}{2}}\right]}$$

$$\text{S.t.} X = \{\zeta = (z, \bar{z}) \in C^{2n} \mid -h(\zeta) \in S\} \subset C^{2n}$$

Where Y is a compact subset of $\{\eta = (w, \bar{w}) \mid w \in C^m\} \subset C^{2m}$; A and B $\in C^{n\times n}$are positive

Semi definite Hermitian matrices; S is a polyhedral cone in C^p; f (·, ·) and g(·, ·) are continuous functions, and for each $\eta \in Y$, f (·,η) and g(·,η) : $C^{2n} \to C$ are analytic, we assume further that $h(\cdot) : C^{2n} \to C^p$is an analytic map defined on $\zeta = (z, \bar{z}) \in Q \subset C^{2n}$.

This set $Q = \{(z, \bar{z}) \mid z \in C^n\}$is a linear manifold over real field. Without loss of generality, it is assumed that $Re\,[\,f(\zeta,\eta) + (z^H A z)^{1/2}] \geq 0$ and $Re\,[g(\zeta,\eta) - (z^H B z)^{1/2}] > 0$ for each $(\zeta,\eta) \in X \times Y$. This problem will be non smooth if there is a point $\zeta_0 = (z_0, z_0)$ such that$z^H{}_0 Az_0 = 0$ or $z^H{}_0 Bz_0 = 0$.

In complex programming problem, the analytic function f (z, $\bar{z}$) is defined on the set Q

Since a nonlinear analytic function cannot have a convex real part in our requirement by this reason in our programming problem (P), the complex variables are taken

As the form $\zeta = (z, \bar{z}) \in C^{2n}$.

In order to understand some problems studied as before in different viewpoints that are

The special cases of problem (P), we recall these special forms as the following:

In problem (P), if Y vanishes and rewrite $\zeta = (z, \bar{z})$, then (P) is reduced to the following minimization problem

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(P_0) min $_{\zeta=(z,\bar{z})\in X}\frac{Re\left[f(z,\bar{z})+(z^H Az)^{\frac{1}{2}}\right]}{Re\left[g(z,\bar{z})-(z^H Bz)^{\frac{1}{2}}\right]}$

1) If A = 0 and B = 0 are zero matrices in problem (P), then (P) is reduced to (P1).

(P_1) $\min_{\zeta\in X}\max_{\eta\in Y}\frac{Re[f(\zeta,\eta)]}{Re[g(\zeta,\eta)]}$

S.t.X $=\{=(z,\bar{z})\in C^{2n}|-h(\zeta)\in S\}\subset C^{2n}$

Where Y is a specified compact subset in C^{2m}, and for each η∈Y ,f (·,η) and g(·,η) are analytic functions

2) If B = 0, g(·, ·) ≡ 1, then problem (P) is reduced to (P_2).

$P_2\min_{\zeta\in X}\max_{\eta\in Y}Re\left[f(\zeta,\eta)+(z^H Az)^{\frac{1}{2}}\right]$

Subjected to $\zeta\in X=\{\zeta=(z,\bar{z})\in C^{2n}|-h(\zeta)\in S\}\subset C^{2n}$

Where Yis a specified compact subset in C^{2m}

3) If A= 0, B = 0 and g(·, ·) ≡ 1, then problem (P) is reduced to (P_3).

$P_3\min_{\zeta\in X}\max_{\eta\in Y}Re[f(\zeta,\eta)]$

Subjected to $\zeta\in X=\{\zeta=(z,\bar{z})\in C^{2n}|-h(\zeta)\in S\}$

4) If ζ= x $\in R^n$ and η = y $\in Y\subset R^m$, then problem (P) is reduced to the real variable problem.

B. Notation

Let c" denote the n-dimensional complex vector space. The inner product (ζ, η) of two vectors ζ, η $\in C^n$ is defined by

$(\zeta,\eta)=\sum_{k=1}^{n}\zeta_k\eta_k^*$

Where η_k^* denotes the conjugate of η_k If we set

$\zeta_k=x_k+ix_{n+k}$, k =1, 2, 3 ... n, $x_r\in R$, r =1, 2, 3....n

We have

$\zeta=(x_1,x_2\ldots\ldots\ldots x_n)+i(x_{n+1},x_{n+2}\ldots\ldots x_{2n})$

That is

ζ = Re ζ + *i* Im ζ

The one-to-one correspondence ω: C^n+R^{2n} is defined, which associates with every ζ of c^n the vector *x* of R^{2n}, whose first n components are given by Re ζ and the following n by Im ζ

And similarly $\eta=(y_1\ldots.y_n)+i(y_{n+1}\ldots.y_{2n})$,

A $\in C^{nxn}$ is a positive semi definite Hermitian matrix, S is a polyhedral cone in C^p;

f (·, ·)is continuous, and for each η∈ Y;

f(·, η) :$C^{2n}\rightarrow C$ and h(·):$\rightarrow C^p$ are analytic in ζ =(z , $\bar{z}$) ∈ Q

$Q=\{(z,\bar{z})\ |z\in c^n\}\subset C^{2n}$ is a linear manifold over a real field.

C. Formulation

$f(\zeta,\eta)=\{a_1\zeta^3+a_2\zeta^2+a_3\zeta+a_4+\emptyset(\eta)\}-(z^H Az)^{\frac{1}{2}}$

Where $(z^H Az)^{\frac{1}{2}}$ depands on ζ

$$f(\zeta,\eta)+-(z^H Az)^{\frac{1}{2}}=\{a_1\zeta^3+a_2\zeta^2+a_3\zeta+a_4+\emptyset(\eta)\}$$

Similarly

$g(\zeta,\eta)=\{c_1\zeta^3+c_2\zeta^2+c_3\zeta+c_4+\psi(\eta)\}-(z^H Bz)^{\frac{1}{2}}$

Where $(z^H Az)^{\frac{1}{2}}$ depands on ζ

$$g(\zeta,\eta)+-(z^H Bz)^{\frac{1}{2}}=\{c_1\zeta^3+c_2\zeta^2+c_3\zeta+c_4+\psi(\eta)\}$$

Hence from the formulation of minimax complex optimization with condition solve the above formulation equation here we used the condition of 1.1, 1.3,

(P_0) min $_{\zeta=(z,\bar{z})\in X}\frac{Re[f(z,\bar{z})]}{Re[g(z,\bar{z})]}$

$\min_{\zeta=(z,\bar{z})\in X}\frac{\{a_1\zeta^3+a_2\zeta^2+a_3\zeta+a_4\}}{\{c_1\zeta^3+c_2\zeta^2+c_3\zeta+c_4\}}$

$\min_{\zeta=}(z, \bar{z})\in X \frac{\zeta^3\left(a_1+\frac{a_2}{\zeta}+\frac{a_3}{\zeta^2}+\frac{a_4}{\zeta^3}\right)}{\zeta^3\left(c_1+\frac{c_2}{\zeta}+\frac{c_3}{\zeta^2}+\frac{c_4}{\zeta^3}\right)}$

$\min \zeta \to \infty \frac{\left(a_1+\frac{a_2}{\zeta}+\frac{a_3}{\zeta^2}+\frac{a_4}{\zeta^3}\right)}{\left(c_1+\frac{c_2}{\zeta}+\frac{c_3}{\zeta^2}+\frac{c_4}{\zeta^3}\right)}$

$\min \frac{a_1}{c_1}$

Where C_1 is not equal to zero so, $C_1>0$

min a_1

We have to found the third order polynomial equation which roots are complex so the equation is a complex function the equation is written below:

$y = 1.653x^3 - 15.29x^2 + 50.29x + 571.4$[3]

This equation is compare with the above formulate equation which is as follows

$a_1\zeta^3 + a_2\zeta^2 + a_3\zeta + a_4$

and the minimum value of the above equation is a_1

so the minimum value of the equation is 1.653.

III. PARTICLE SWARM OPTIMIZATION (PSO)

A. Introduction

Particle swarm optimization (PSO) to solve complex variable fractional programming problems (CVFPP) where the objective function includes the two parts (real and imaginary), the input is complex while the output is always real. Particle swarm optimization can be denoted as an effective technique for solving linear or nonlinear, non analytic complex fractional objective functions. Problems with an optimal solution at a finite point and an unbounded constraint set can also be solved using the proposed technique [Ibrahim et al.2013]

Particle swarm optimization is a population based stochastic optimization technique developed by Eberhart and Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling

Formulation:

The functions related to the difference examples by PSO is

f_1 : max f (z) = Re | z^2 +1|

Subject to $\{z \in C\, |z| \le 1\}$

This function is just for clear illustration to how PSO can reach almost the same optimized solution. However the function does not represent a fractional programming problem but still a good evidence for the dominancy of PSO in solving complex nonlinear programming optimization problems [Ihrahim et al. 2013] the simple solution. Since

$|z^2 + 1| \le |z|^2 + 1 \le 2 \quad for\ |z| \le 1$

Particle swarm optimization technique can ever solve complex function programming problem easily. (PSO) is effective in nonlinear and nonanalytic fractional objective functions. (PSO) can be efficiently used for large datasets and a multi-processor environment. It also does not require the problem defined function to be continuous. It can find optimal or near-optimal solutions, and may be suitable for discrete and combinatorial problems.

B. Calculation

Applying the PSO algorithm to solve the above problem in maximum case,

The equation is

$y = 1.653x^3 - 15.29x^2 + 50.29x + 571.4$ [3]

And the above equation will be written as

$y = 1.653z^3 - 15.29z^2 + 50.29z + 571.4$

In the form of the complex function because of the x roots are in the form of complex form

To find its root one real and two complexes are to be found is follow:

$z = 3.083 \pm 0.7965\,i$

So

$f(z) \quad = \quad [z - (3.083 + 0.965i)][z - (3.083 - 0.7965i)]$

$$max f(z) \leq |z-(3.083+0.965i)||z-(3.083-0.965i)|$$
$$max f(z) \leq |z-3.083|+|-0.965i| \times |z-3.083|+|-0.7965i|$$
$$max f(z) \leq |3.083-0.7965i-3.083|+|0.7965i||3.083-0.7965i-3.083|+|0.7965i|$$
$$max f(z) \leq |0.7965i|+|0.7965i| \times |0.7965i|+|0.7965i|$$
$$max f(z) \leq 4 \times 0.7965$$
$$max f(z) \leq 3.186$$

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