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Square Sum Difference Product Prime Labeling of Some Path Related Graphs

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Abstract: ***Square sum difference product prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the sum of the squares of the labels of the incident vertices and product of the labels of the incident vertices. The greatest common incidence number of a vertex (gcin) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits square sum difference product prime labeling. In this paper we investigate some path related graphs for square sum difference product prime labeling.***
Keywords: ***Graph labeling, greatest common incidence number, square sum, prime labeling, paths.***

I. INTRODUCTION

All graphs in this paper are simple, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.
A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1[2]. In this paper we investigated square sum difference product prime labeling of some path related graphs.

A. Definition: 1.1

Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor(gcd) of the labels of the incident edges.

II. MAIN RESULTS

A. Definition 2.1

Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,3,---------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping $f^*_{sqsdppl}$: E(G) → set of natural numbers N by $f^*_{sqsdppl}(uv) = |\{f(u)\}^2 + \{f(v)\}^2 - f(u)f(v)|$.The induced function $f^*_{sqsdppl}$ is said to be square sum difference product prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

B. Definition 2.2

A graph which admits square sum difference product prime labeling is called a square sum difference product prime graph.

1) Theorem 2.1 : Path $P_n$admits square sum difference product prime labeling.

Proof: Let G = P_n and let v_1,v_2,----------------,v_n are the vertices of G
Here |V(G)| = n and |E(G)| = n-1
Define a function f : V → {0,1,2,3,----------------,n-1 } by
$f(v_i) = i-1$, i = 1,2,------,n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1}) = i^2-i+1$, i = 1,2,------------,n-1

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{sqsdppl}(v_i v_{i+1}), f^*_{sqsdppl}(v_{i+1} v_{i+2})\}$
= gcd of $\{i^2-i+1, i^2+i+1\}$

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= gcd of $\{2i , i^2\text{-}i+1\}$
= gcd of $\{i , i^2\text{-}i+1\}$
= 1, $\quad$ i = 1,2,------------,n-2

So, gcin of each vertex of degree greater than one is 1.

Hence P_n , admits square sum difference product prime labeling. ■

2) Theorem 2.2: Middle graph of Path P_n admits square sum difference product prime labeling.

Proof: Let G =M(P_n) and let v_1,v_2,----------------,v_{2n-1} are the vertices of G

Here |V(G)| = 2n-1 and |E(G)| = 3n-4

Define a function f : V → {0,1,2,3,----------------,2n-2 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1})$ = i^2-i+1, $\quad$ i = 1,2,------------,2n-3

$f^*_{sqsdppl}(v_{2i-1} v_{2i+1})$ = $4i^2$-4i+4, $\quad$ i = 1,2,------------,n-2

$f^*_{sqsdppl}(v_1 v_{2n-1})$ = $(2n-2)^2$,

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = gcd of $\{f^*_{sqsdppl}(v_1v_2) , f^*_{sqsdppl}(v_1v_3)\}$
= gcd of {1,4}= 1.

gcin of (v_{i+1}) = 1, $\quad$ i = 1,2,------------,2n-4

So, gcin of each vertex of degree greater than one is 1.

Hence $M(P_n)$, admits square sum difference product prime labeling. ■

C. Theorem 2.3

Duplicate graph of path P_n admits square sum difference product prime labeling.

Proof: Let G = $D(P_n)$and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 2n-2

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1})$ = i^2-i+1, $\quad$ i = 1,2,------------,n-1

$f^*_{sqsdppl}(v_{n+i} v_{n+i+1})$ = $(n+i)^2$-(n+i) + 1, $\quad$ i = 1,2,------------,n-1

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_{i+1}) = 1, $\quad$ i = 1,2,------------,n-2

gcin of (v_{n+i+1}) = 1, $\quad$ i = 1,2,------------,n-2

So, *gcin* of each vertex of degree greater than one is 1.

Hence $D(P_n)$, admits square sum difference product prime labeling. ■

D. Theorem 2.4

Strong Duplicate graph of path P_n admits square sum difference product prime labeling., when $(n+4) \not\equiv 0 \pmod{13}$

Proof: Let G = $SD(P_n)$and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n-2

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1})$ = i^2-i+1, $\quad$ i = 1,2,------------,2n-1

$f^*_{sqsdppl}(v_{2i-1}v_{2i+2})$ = $4i^2$-2i+7, i = 1,2,------------,n-1

Clearly f^*_{ssdppl} is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, i = 1,2,------------,2n-2

gcin of (v_{2n}) = gcd of {$f^*_{ssdppl}(v_{2n-1}v_{2n})$, $f^*_{ssdppl}(v_{2n-3}v_{2n})$ }

= gcd of { $4n^2$-6n+3, $4n^2$-10n+13}

= gcd of { 4n-10, $4n^2$-10n+13}

= gcd of { 2n-5, $4n^2$-10n+13}

= gcd of { 2n-5, 13}

= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence SD(P_n), admits square sum difference product prime labeling. ■

1) *Definition 2.1 :*A tortoise graph TOT_n (n ≥ 4) is obtained from a path v_1,v_2,------------,v_n by attaching edge between v_i and v_{n-i+1} for $1 \leq i \leq [\frac{n}{2}]$.

E. Theorem 2.5

Tortoise graph TOT_nadmits square sum difference product prime labeling, when n is even and (n+8) ≢ 0 (mod 26).

Proof: Let G = TOT_n and let v_1,v_2,----------------,v_n are the vertices of G

Here |V(G)| = n and |E(G)| = $\frac{3n-4}{2}$

Define a function f : V → {0,1,2,3,----------------,n-1 } by

f(v_i) = i-1 , i = 1,2,------,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_iv_{i+1})$ = i^2-i+1, i = 1,2,------------,n-1

$f^*_{sqsdppl}(v_{2i-1}v_{2i+2})$ = $4i^2$-2i+7, i = 1,2,------------,$\frac{n-2}{2}$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = gcd of {$f^*_{sqsdppl}(v_1v_2)$, $f^*_{sqsdppl}(v_1v_4)$ }

= gcd of {1,9} =1.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n-2

gcin of (v_n) = gcd of {$f^*_{sqsdppl}(v_nv_{n-1})$, $f^*_{sqsdppl}(v_nv_{n-3})$ }

= gcd of { n^2-3n+3, n^2-5n+13}

= gcd of {2n-10 , n^2-5n+13}

= gcd of {n-5, n^2-5n+13}

= gcd of {n-5, 13}

= 1,

So, *gcin* of each vertex of degree greater than one is 1.

Hence TOT_n , admits square sum difference product prime labeling. ■

F. Theorem 2.6

Tortoise graph TOT_nadmits square sum difference product prime labeling, when n is odd and (n+3) ≢ 0 (mod 14).

Proof: Let G = TOT_n and let v_1,v_2,----------------,v_n are the vertices of G

Here |V(G)| = n and |E(G)| = $\frac{3n-3}{2}$

Define a function f : V → {0,1,2,3,----------------,n-1 } by

f(v_i) = i-1 , i = 1,2,------,n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1}) = i^2-i+1$, $i = 1,2,\text{------------},n-1$

$f^*_{sqsdppl}(v_{2i-1} v_{2i+2}) = 4i^2-2i+7$, $i = 1,2,\text{------------},\frac{n-3}{2}$

$f^*_{sqsdppl}(v_{n-2} v_n) = n^2-4n+7$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, $i = 1,2,\text{------------},n-2$

gcin of (v_n) = gcd of $\{f^*_{sqsdppl}(v_n v_{n-1}), f^*_{sqsdppl}(v_n v_{n-2})\}$

= gcd of $\{n^2-3n+3, n^2-4n+7\}$

= gcd of $\{n-4, n^2-4n+7\}$

= gcd of $\{n-4, 7\}$

= 1,

So, gcin of each vertex of degree greater than one is 1.

Hence TOT_n, admits square sum difference product prime labeling. ■

G. Theorem 2.7

The join of Path P_n and K_1 admits square sum difference product prime labeling.

Proof: Let $G = P_n + K_1$ and let $a, v_1, v_2, \text{----------------}, v_n$ are the vertices of G

Here $|V(G)| = n+1$ and $|E(G)| = 2n-1$

Define a function $f : V \to \{0,1,2,3,\text{----------------},n\}$ by

$f(v_i) = i$, $i = 1,2,\text{------},n$

$f(a) = 0$.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1}) = i^2+i+1$, $i = 1,2,\text{------------},n-1$

$f^*_{sqsdppl}(a v_i) = i^2$, $i = 1,2,\text{------------},n$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (a) = 1.

gcin of (v_1) = 1.

gcin of (v_{i+1}) = 1, $i = 1,2,\text{------------},n-2$

gcin of (v_n) = gcd of $\{f^*_{sqsdppl}(a v_n), f^*_{sqsdppl}(v_n v_{n-1})\}$

= gcd of $\{n^2, n^2-n+1\}$

= gcd of $\{n, n^2-n+1\}$

= gcd of $\{1, n\}$

= 1.

So,*gcin* of each vertex of degree greater than one is 1.

Hence $P_n + K_1$, admits square sum difference product prime labeling. ■

H. Theorem 2.8

Total graph of Path P_n admits square sum difference product prime labeling, when $(n-3) \not\equiv 0 \pmod 7$.

Proof: Let $G = T(P_n)$ and let $v_1, v_2, \text{----------------}, v_{2n-1}$ are the vertices of G

Here $|V(G)| = 2n-1$ and $|E(G)| = 4n-5$

Define a function $f : V \to \{0,1,2,3,\text{----------------},2n-2\}$ by

$f(v_i) = i-1$, $i = 1,2,\text{------},2n-1$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{sqsdppl}$ is defined as follows

$f^*_{sqsdppl}(v_i v_{i+1}) = i^2-i+1$, $i = 1,2,\text{------------},2n-3$

$f^*_{sqsdppl}(v_{2i+1}v_{2i+3}) = 4i^2+4i+4, \quad i = 1,2,\text{------------},n-3$

$f^*_{sqsdppl}(v_{2i+2}v_{2i+4}) = 4i^2+8i+7, \quad i = 1,2,\text{------------},n-3$

$f^*_{sqsdppl}(v_1v_{2n-1}) = (2n-2)^2$

$f^*_{sqsdppl}(v_2v_{2n-1}) = 4n^2-10n+7.$

$f^*_{sqsdppl}(v_1v_3) = 4$

$f^*_{sqsdppl}(v_1v_4) = 9$

Clearly $f^*_{sqsdppl}$ is an injection.

gcin of (v_1) = 1.

gcin of (v_{i+1}) = 1, $\quad i = 1,2,\text{------------},2n-4$

gcin of (v_{2n-2}) = gcd of $\{f^*_{sqsdppl}(v_{2n-3}v_{2n-2}), f^*_{sqsdppl}(v_{2n-4}v_{2n-2})\}$

= gcd of $\{4n^2-14n+13, 4n^2-16n+19\}$

= gcd of $\{2n-6, 4n^2-16n+19\}$

= gcd of $\{n-3, (n-3)(4n-4)+7\}$

= gcd of $\{n-3, 7\}$

= 1.

gcin of (v_{2n-1}) = gcd of $\{f^*_{sqsdppl}(v_1v_{2n-1}), f^*_{sqsdppl}(v_2v_{2n-1})\}$

= gcd of $\{(2n-2)^2, 4n^2-10n+7\}$

= gcd of $\{2n-2, 4n^2-10n+7\}$

= gcd of $\{2n-2, (2n-2)(2n-3)+1\}$

= 1,

So, gcin of each vertex of degree greater than one is 1.

Hence $T(P_n)$, admits square sum difference product prime labeling. ■

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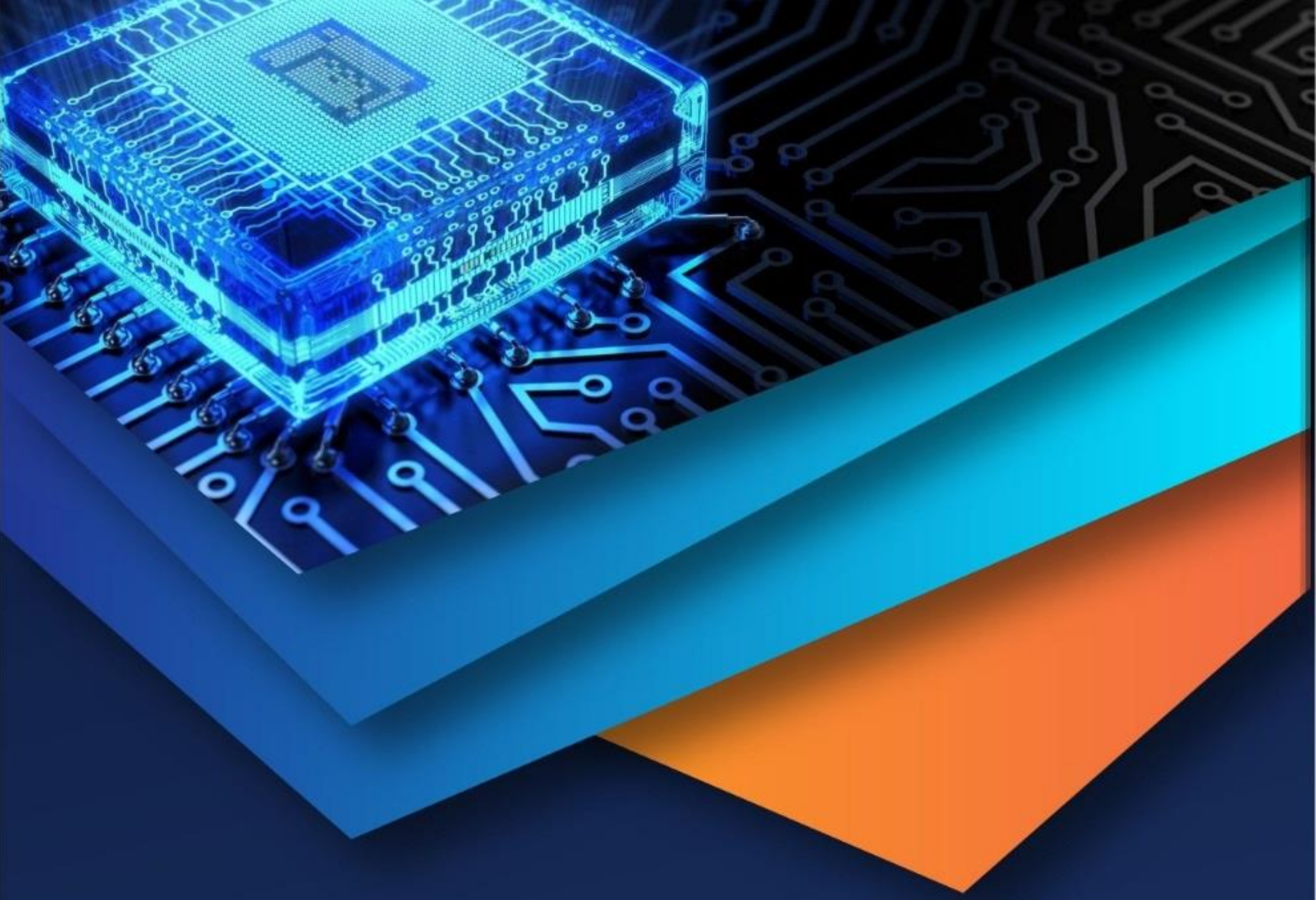

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