A Computational Geometry Based Facility Location Model for Healthcare Management System

Parthajit Roy

1Department of Computer ScienceThe University of Burdwan West Bengal, India-713104

Abstract: Facility location problem is an important problem in the field of optimization. Identification of existing locations, allocation of resources is long standing topics for research communities. This paper proposes a novel facility location identification and resource allocation model based on computational geometric tools namely Delaunay triangulation and Voronoi diagram. The proposed model has been applied in the field of Healthcare management system for suggesting locations of new hospitals as well as locations for deployment of mobile hospitals. The model is fully automated and computationally efficient and performs well in the existing setup.

Keywords: Facility Location; Voronoi Diagram; Delaunay Triangulation; Push-Pull Model; Computational Geometry.

I. INTRODUCTION

The usage of Computer for the sake of mankind is the main objective of computer science and applications since its inception. The applications of computer ranges from scientific computation to data science, from rocket science to bioinformatics. There is another area where the computers have been used for the past few decades. This is Geographical Information Systems or GIS. GIS is an interdisciplinary branch of study where Geography and Information technology merges for giving better services to the mankind. The present work is the proposal of a GIS system for healthcare management systems. The proposed model is a generic model which suggests the best possible locations for setting up the different medical facilities to minimize the allocation cost as well as to maximize the utilization. In that sense it is an optimization model also. The proposal of the model comes from an economical model, namely called Pull-Push model. The pool-push model works as follows. In a society, where there are peoples with average and less than average literacy, the people do not know which one is good for their health. For example, consider the usage of mosquito net, or spraying of DDT around their house. Poor people generally do not spent money for these. This is because that the effect of mosquito net and DDT spraying is not directly visible to them. The disease like Dengue spreads through mosquito but not that every single mosquito bite spreads it and in a village of thousand people with almost the same setup, a few gets attacked by Dengue. Rest all somehow bypasses it. So, rural people do not care much for it. The observation is, rural people do not spend much money for prevention purposes[1]. On the other hand, whenever a disease occurs, people becomes active and in such cases they travel a long distance to a good hospital for better services. Sometimes they even move to private healthcare service sectors and spend a lot of money. The curtail observation is that the rural poor people spends more money in cure and recovery process because that has a clear visible outcome. The role of government always remain crucial in such situations. They uses a model called push and pull model. Whenever there is a policy of prevention, the governments generally supply it forcefully. Either free of cost or in a subsidized form. i.e. they send their workers to different villages or different slums and distributes them to the people. This is called push model. For curing process, government uses a pull model. Government sets up different healthcare centres of different levels and people goes there for availing the facilities. This is because it is uneconomical for governments to set up hospitals at every locality. So, government carefully selects some locations and sets up facility centres. In this way government gives the facilities by push/pull model. A good discussion of such model can be found in the work of Banerjee and Duflo [1]. In our present work, we have suggested a three tier model of facility location in the context of Indian healthcare systems. Apart from push and pull tier, there is a middle tier that acts as both push and pull facilities with a moderate degree. Middle tier increases overall performance at reduced cost. For this, a fully automated model has been proposed. The model is a computational geometry based model where Delaunay Triangulation and Voronoi diagram has been used for suggestion of facility locations. The proposed model takes the population of the village or locality and the spatial distance among the villages as input. The model also considers the existing facility locations like existing health centres or Hospitals as the input parameters. The model then forms a graph model and computes on that and suggests the places for facility locations. The proposed model suggests two different types of locations. Locations where the static facility centres (Hospital or other healthcare setup of the pull tier) can be set up, and the locations where the dynamic facility centres (Mobile healthcare van or similar facility centre of push tier) can be set up. The proposed model is also very efficient computationally as
well as capable of suggesting the best location considering the existing healthcare setup. The rest of the paper is organized as follows. Section 2 gives the literature review, section 3 presents the details of the geometrical phenomena used model. The actual model is proposed in section 4. Section 5 discusses some of the problems and drawbacks of the proposed model. Conclusion comes in section 6 and the references come at the end.

II. LITERATURE REVIEW

Facility location is a long standing problem and it has a fascinating history. Some early proposal of facility location problem in the context of network and communication is due to McGregor et al [2]. Some early proposal of facility location was made by Calvo et al [3]. The model was mainly concentrated on integer programming models. Some refinements of Calvo’s model was done by Tien et al [4]. A good survey on facility locations has been done by ReVelle et al [5]. A survey on multi criteria facility locations can be found in the work of Farahani et al [6]. An up-to-date survey on facility location in the field of healthcare is due to Ahmadi-Javid et al [7]. The application of Voronoi in the field of optimization is not new. A Voronoi based model for optimum location for goods has been proposed by Riol et al [8] whereas a nearest hospital suggestion model has been proposed by Roy et al [9]. A case study on rail station identification is due to Rosa Mota et al [10]. An electrical distribution boundary using Voronoi diagram has been proposed by Held et al [11]. Some mixing of GA and Voronoi diagram for system expansions have been proposed by Jimenez-Estvez et al [12]. Geographical Information System in the field of healthcare is a fascinating area of research. Some examples of GIS in primary medical care accessibility has been proposed by Parker et al [13] and its impact on health has been considered by Nguyen et al [14].

Delaunay Triangulation (DT) is a planar decomposition of spatial points. It has some interesting properties like its planarity. A DT is a planar decomposition of a complete graph such that the minimum spanning tree of the DT agrees with the minimum spanning tree of the complete graph given all edge weights are metric Berg et al. [15]. Voronoi diagram is the planar dual of the Delaunay Triangulation. A good survey of Delaunay Triangulation and Voronoi diagram can be found in the book of O’Rourke [16]. Some application of DT for spectral clustering is done by Roy et al [17].

III. ESSENTIAL GEOMETRY

We now turn to mathematics needed to explain the proposed model. First we shall discuss Voronoi diagram. A Voronoi diagram is a decomposition of a spatial domain around some spatial points in such a way that the spatial point is the closest from any point in the cell. Let us consider the sample points shows in Figure 1(a). These are the sample points. The Voronoi diagram of the sample points are shown in Figure 2. There are several important properties of Voronoi diagram These are as follows.

A. A Voronoi diagram is a planar decomposition.
B. The closed area formed by the Voronoi lines is called a Voronoi cell and each Voronoi cell contains ona single nodal point.
C. From every location inside a Voronoi cell, the inside nodal point is the closest nodal point.
D. From every Voronoi line, the neighboring nodal points are equidistant. It is shown in Figure 3. In Figure 3 location A and B are equidistant from their separating Voronoi line. The red arrows show this. Similarly, the blue arrows indicate that the points A and C are at same distance from their separating Voronoi lines.
E. The meeting point of three or more Voronoi lines is called Voronoi junction and the nodal points of the corresponding Voronoi cells are all equidistant.

In our present model we are going to use Voronoi diagram in push based model.

![Figure 1](image)
Now we shall discuss the Delaunay Triangulation. Given a set of points in an Euclidean metric space, a Delaunay decomposition is a planar decomposition of the plane with the following properties.

_F._ The decomposition is a planar triangulation where each the points resides at each corner of the triangles.

_G._ The circumcircle of the triangle is empty. i.e. circumcircle of any triangle does not contain any other points from the point set.

_H._ The Decomposition alwayes form a convex hull.

As an illustration, consider the same set of points as shown earlier in Figure 2(a). The black points are the nodes. The Delaunay triangulation of the points is shown in Figure 2(b). Clearly the Delaunay Triangulation is a convex hull. Further the Delaunay triangulation is a planar decomposition of the set of points. Figure 2(b) shows the empty circumcircle property. The circle passing through the points A, B and C is shown in red colour in Figure 3. Clearly, there is no other node inside the circle. i.e the circle is empty. Fra a Delaunay triangulation, all such circles are empty. This completes the geometrical discussions. We are now ready to propose the model.

![Sample points in the spatial domain. (b) Delaunay Triangulation of the spatial points. (c) The empty circumcircle property of Delaunay Triangulation.](image)

**IV. PROPOSED MODEL**

Let us now propose the model. The model is a Delaunay-Voronoi diagram based model. Let us consider that in a large area (district of a state), there are several villages or localities.

![The distribution of village or localities. (b) The Distribution of Hospitals and Villages, The hospitals are shown in black circles. (c) The hospitals are shown in spatial domain. These points are used for Voronoi and Delaunay computations.](image)

In Figure 3(a), the red squares are the villages. Figure 3(b) shows the distribution of the villages and the distribution of the hospitals. Our target is to give maximum facility to the villagers considering the existing facility locations. The model also suggests the new places for building up the new set up. For this, the proposed model considers the existing facility locations. In the present case, the facility locations are the Hospitals or the health centres. The spatial location as shown using black circles in Figure 3(b). We shall compute the Voronoi diagram of these nodal points.
A. Push and Intermediary Tier

First we shall propose the model for push based location centres. To compute such suggestions with maximum facility in the existing setup, the model considers only the hospitals in a spatial domain. These are considered as the nodes of a graph (Figure 2(a)). The Voronoi diagram of the nodes is then computed. The Voronoi diagram is shown in Figure 4(a). The Voronoi diagram has a property that the Voronoi node is the closest node from all of the points in the concern cell. This means, the people of a particular Voronoi cell will know which one is the nearest hospital to them (Figure 4(a)). Further, the Voronoi junction points are the very good locations for Mobile Hospital location. It can be noted that keeping the mobile hospital van at hospital is not an optimum idea. Some mobile van/mobile hospitals may be placed at certain Voronoi junctions. This will reduce the total traveling time. i.e. if the Mobile van is kept at hospital, then van has to go to patient location and has to come back to hospital. Instead if we place the mobile van at the junction points (Shown in arrow in Figure 4(b)), then it can give better support to the surrounding villages to three different hospitals. So, the suggestion is that the mobile hospital vans have to be kept at Voronoi corners. Obviously, depending upon the resources, the allocation should be done. Our proposal is, do not deploy a mobile vans should be deployed in such a way that no cell gets the access of two vans. As in the Figure 4(b), as the arrow centred location covers hospital no. 8, 11 and 13, No van should be placed in a way that any one of 8, 11 or 13 becomes accessible. In that way we can minimize the number of vans without compromising the service quality. In Figure 4(c) the red marked location are the good junction points for such push based facility locations.

B. Pull Tier Facility Location Model

The second task of the model is to propose the new locations for building new health centres. The new locations should be as far as possible from the existing locations. This will increase the utilization. The present model proposes a Delaunay triangulation based suggestion for this purpose. In the proposed pull model, we are computing Delaunay triangulation of the Hospitals (Figure 5 (a) and (b)). Now consider Figure 5(c). In this figure, two circles are shown in red colour. One is passing through facility location A,B,C and the other is passing through D,E,F. We have already seen in Section 3, that one property of Delaunay triangle is that all such circumcircles are empty i.e. no other facility point will be inside these circles. So, the centre of the circle is the farthest point from all other facility locations in the periphery. Further, if all the circles are considered, then the centre of the largest circle, i.e. location S in Figure 5(c), is the best location for such facility. If there are funds for n facility locations, then the n largest circles can be chosen and the centres of them may be suggested as the place for facility locations.

Figure 4(a) Voronoi decomposition of the points shown in Figure 3(c). (b) The population distribution is superimposed in the Voronoi distribution for better understanding of the nearest hospitals. (c) According to Voronoi decomposition, some possible locations of mobile hospitals. The Red plus sign shows these.

Figure 5 (a) Delaunay triangulation of the facility locations with the distribution of the villages. (b) The Delaunay triangulation of the facility locations. (c) The pull tier suggestion made by the Delaunay triangulation. The point S is the best location for new Hospital.
C. Effectiveness of the Proposed Model

The proposed model is a Delaunay-Voronoi based model for deploying mobile health facilities as well as for deploying new static facility locations. If there are \( n \) facility location in a given area, the model computes Delaunay triangles for those \( n \) locations. The time complexity of Delaunay triangulation is \( O(n\sqrt{n}) \). The polynomial time shows that the computation is very efficient. The Voronoi computation also takes \( O(n \log n) \) time complexity. So computation of Voronoi diagram is also very efficient. Secondly, there is another striking feature of both Delaunay triangulation and Voronoi diagram. The constructions of both Delaunay triangulation and Voronoi diagram are incremental. Means, we can add new points and recomputed the DT or Voronoi diagrams. The Voronoi diagram or the DT will be affected locally surrounding the newly added point. Not the entire structure will be affected. To understand this in case of Voronoi diagram, consider Figure 6(a) and Figure 6(b). In Figure 6(a), the red point is the location of new health centre. Once, the new location has been added, the Voronoi cells are recomputed. Recomputation is shown in red dotted lines. Most of the lines are superposed with the existing lines. Only few are different. This proves that the addition of a new facility location doesn’t alter all the positions of the Mobile facility centres. This is an essential requirement. Final outcome is shown in Figure 6(b).

![Figure 6(a) Re-computation of Voronoi diagram after the addition of a new hospital. The red dotted lines are the extra lines. (b) The final Voronoi diagram. It is clear that most part of diagram is same. Only some portion near newly added point has been deviated.](image)

Secondly, DT also follows incremental construction. Addition of a new point alters only some of the proximities of the point. Figure 7(b) shows this in dotted red lines. The final output is shown in figure 7(b). The main advantage of this property is, that the administration can take a decision about more than one facility locations at a time.

![Figure 7(a)The Delaunay triangulation after the addition of new point. The red dotted lines are the new triangles. All other triangle are same as before. (b) The final triangulation after removing the old triangles.](image)

V. CONCLUSION

This paper proposed a facility location model for healthcare management. The model proposeda pull push model for maximum outcome. Proposed model also considers the existing facilities facility setups and suggests the up-gradation of the system. It is also evident that the proposal of the new health centres do not alter existing setup. The model is also computationally efficient. However there are scopes of improvements. First of all, only largest circumcircle for new facility location has been considered. But population density may not be high in that place. So, the population may be considered along with the Delaunay triangulation. Secondly, the placing of mobile locations can be done using better algorithms for better distribution. Finally, some fuzzy modelling can be incorporated to give the model a more realistic look.
REFERENCES


