Mitigation of Non-linear Impairments in Optical Fast-OFDM using Wiener-Hammerstein Electrical Equalizer

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Abstract: India is presently witnessing substantial rise in investments especially industrial and infrastructural areas. This growth is further being augmented by the increase in power generation capacity and reliable power distribution and measurement network. In some developed countries like the United States smart meters are already deployed. Although some companies have introduced smart meters, many households have conventional meters. This provides a chance for companies as well as engineers to develop innovative metering solutions. The proposed system can be incorporated to the installed energy meters without much effort. It consists of a light sensor module which detects the pulses given out by the meter and the python program developed computes the energy consumed by the load and sends these values monthly to the Google spreadsheets via Drive API. The users can access the information on a webpage as well as an Android App. In addition to this, they also receive an SMS with the help of a GSM module interfaced to the Raspberry Pi.

Keywords: Fast-OFDM, Equalizers, Wiener-Hammerstein model, Self-Phase Modulation

I. INTRODUCTION

One of the major enabling technologies of next-generation of optical systems or optical communication as a whole is Fast Orthogonal Frequency Division Multiplexing or Fast-OFDM. Fast-OFDM has a half frequency separation between the subcarriers when compared to conventional OFDM. As a result of partially overlapping sub-carriers, we observe very high spectral-efficiency. Inter-symbol interference caused largely due to fiber distortions like polarization mode dispersion (PMD) and Chromatic dispersion (CD) are overcome using Cyclic Prefix code added in the Fast-OFDM system. A major concern regarding Fast-OFDM is its vulnerability to fiber non-linear effects such as Self-Phase Modulation or SPM. Optical Signal Intensity fluctuation is one of the major causes of SPM. In this paper, we concentrate on Mitigation of Non-linear Impairments like SPM in Optical Fast-OFDM using Wiener-Hammerstein Electrical Equalizer.

Equalizers are blocks that combat both linear and nonlinear imparities in any communication system. The first order Equalizer can only work on linear distortions, so we opt for third order Equalizer to work on mitigation of nonlinear distortions. The adaptive Normalized Least Mean Square (NLMS), Recursive Least Square (RLS) and Constant Modulus Algorithm (CMA) are the three popular adaptive signal processing algorithms which enable the Equalizers to adjust their parameters to produce an output which matches the output of an unknown system.

II. FAST OFDM SYSTEM

A. Principle of Fast-OFDM

Fast-OFDM works on a principle similar to conventional OFDM but with an added advantage of having double the bandwidth efficiency in contrast to conventional OFDM. Thus the frequency separation between the subcarriers is 1/(2xT), where ‘T’ is the Subcarrier Time Interval.

Thus, Fast-OFDM will produce the same data rate as that of the conventional OFDM but will use half of the bandwidth only.

Figure 1. Frequency Spectrum of Fast-OFDM signal
The Sequence of symbols generated in OFDM is given by:

\[ S_{\text{tx}} = \sum_{R=-\infty}^{\infty} k = \infty \sum_{n=0}^{N-1} a_n k g_n(t - kT) \]

and

\[ g_n(t) = 1/(T)^{1/2} e^{j2\pi nt/2T} \]

Where, \( S_{\text{total}} \) is the complex envelope of OFDM, \( a_n \) is the complex symbol transmitted on the \( n^{th} \) subcarrier, \( g_n \) is the complex subcarrier, \( T \) is the Time Interval and \( N \) is the total number of subcarriers.

**B. Fast-OFDM System Block Schematic**

Data is first generated and mapped on to separate OFDM subcarriers. The total number of bits we have used for generation of data is of 1024 bits. Serial to Parallel Converter is used to convert Serial input data into parallel transmission of data at the transmitter. Inverse Fast Fourier Transformation is used to convert the symbols at input into time domain. Correctness of data is ensured with the addition of Cyclic Prefix. Parallel to Serial Converter is used to convert Parallel input data into Serial data so as to provide input to DAC. The serial data is then converted to analog signal by DAC (Digital to Analog Converter) for the purpose of analog transmission. The analog signal is transmitted over a Fiber channel, where the effect of nonlinearities occurs. The signal at the end of fiber channel is fed to an Equalizer to combat nonlinear distortions and then given as input to ADC. Analog to Digital Converter (ADC) to converts the received signal from analog to digital, Serial to Parallel Converter is used to convert Serial received data into parallel data and then Cyclic Prefix is removed at receiver. The signal is then sent to FFT (Fast Fourier Transformation) to convert the symbols from time domain back to frequency domain. Parallel to Serial Converter is used to convert the obtained Parallel data (without CP) into Serial data so as to obtain the initial input data. Finally, Bit Error Rate (BER) is calculated based on the received data.

**C. Wiener Hammerstein Equalizer**

Wiener-Hammerstein model is one of the most commonly used nonlinear block-oriented structures because of its simpler structure and requires less calculation. The number of coefficients are significantly reduced in this model when compared to the volterra approach. This model is a cascaded arrangement of subsystems, the first subsystem is the FIR filter, second subsystem is a nonlinear polynomial filter and the third subsystem is also a FIR filter. The block diagram is shown in fig:
After the first FIR filter, the input-output relation of the first subsystem is displayed as below, where $M_1-1$ is the memory length of the first FIR filter.

$$y(n) = \sum_{i=0}^{M_1-1} u(i) x(n-i)$$

. The output signal of the center nonlinear filter is written as:

$$z(n) = v(0)y(n) + v(1)y^2(n)y'(n)$$

The relationship between the input signals $z(n)$ and output signals $o(n)$ of the third subsystem is as:

$$o(n) = \sum_{i=0}^{M_2-1} w(i)z(n-i)$$

Where $M_2-1$ is the memory length of the second FIR filter and $u(\cdot)$, $v(\cdot)$ and $w(\cdot)$ are vector coefficients of respective subsystems.

Adaptive algorithms are the algorithms which enable the Equalizers to adjust their parameters to produce an output which matches the output of an unknown system. This algorithm employs an individual convergence factor that is updated for each equalizer coefficient at each iteration.

1) **Normalized Least Mean square (NLMS) Algorithm**: This algorithm is a class of adaptive filter used to get a desired filter by finding the filter coefficients that intern produces the least mean squares of the error signal which is the difference between the desired signal and the actual one. The algorithm starts by assuming small weights (mostly zero), and after each step, the weights are updated. The input-output relations of an adaptive Wiener-Hammerstein Equalizer consisting of three subsystems is given in the below equations:

$$y(n) = \sum_{i=0}^{M_1-1} u(i) x(n-i)$$

$$z(n) = v(0)y(n) + v(1)y^2(n)y'(n)$$

$$o(n) = \sum_{i=0}^{M_2-1} w(i)z(n-i)$$

The difference between desired signal $d(n)$ and the filter output $o(n)$ is as:

$$e(n) = d(n) - o(n)$$

The individual subsystems coefficients can be represented in vector form as:

$$u(n) = [u_0(n), u_1(n), \ldots, u_{M_1-1}(n)]^T$$

$$v(n) = [v_0(n), v_1(n)]$$

$$w(n) = [w_0(n), w_1(n), \ldots, w_{M_2-1}(n)]^T$$

The output signal after the first, second and third subsystems can be calculated as:

$$y(n) = u^T(n)x(n)$$

$$z(n) = v^T(n)y(n)$$

$$o(n) = w^T(n)z(n)$$

Both linear filters have the memory length as $1$. We use this normalized LMS algorithm to update the linear subsystems and nonlinear subsystem coefficients jointly. This estimation depends on initial conditions and the positive constants chosen.

2) **Recursive Least Square (RLS) Algorithm**: This is an algorithm that recursively finds the filter coefficients which minimize a weight vector coefficient related to input signal. This algorithm is faster than NLMS method as it converges faster. The stability of this algorithm is more when compared to NLMS. RLS comes into limelight because of its rapid rate of convergence and for this reason it is preferred over other algorithms.

![Figure 4. Recursive Least Square (RLS) Algorithm](image)

where, $x(n)$ is the input signal to the Equalizer, $W(n)$ is the Filter tap weight vector, $d(n)$ is the desired signal, $y(n)$ is the output of the equalizer and $e(n)$ is the error estimation of the signal.
The following equations show the procedure of updating the weight vector and coefficients of individual subsystems

\[ W(n) = W(n-1) + k(n) \cdot e(n) \]

\[ k(n) = u(n) / (\lambda + x^T(n) \cdot u(n)) \]

\[ u(n) = W^T(n-1) \cdot x(n) \]

Where \( k(n) \) is the intermediate gain vector used to update the weight vector and \( u(n) \) is also an intermediate equation. \( \lambda \) is a small positive constant very close to, but smaller than 1. \( \lambda \) is called the forgetting factor (0 << \lambda < 1).

The filter output is calculated using the following equations consisting of filter tap weight vector and input signal to the equalizer:

\[ y(n) = W^T(n-1) \cdot x(n) \]

The error estimation the signal is done with respect to the desired signal and the governing equation is:

\[ e(n) = d(n) - y(n) \]

3) **Constant Modulus algorithm (CMA):** Constant Modulus algorithm uses the technique of Blind channel equalization. It is also known as self-recovering equalization. It is easier in designing and has robustness in operation compared to other adaptive algorithms like NLMS and RLS, but it has a slow convergence rate.

\[ J(n) = E[|y(n)|^2 - R_2]^2 \]

Where \( J(n) \) is a cost function, \( p \) is dispersion constant and positive. The cost function of cma undergoes a non-linear behavior. Since LMS and RLS are training based algorithms it leads to a global minimum convergence. The Mean Square Error generated in this method is also high since it uses the technique of blind equalization. There is an inverse relationship between Mean Square Error (MSE) and Convergence rate.

The tap weight vector \( w(n) \) is updated with input signal \( u(n) \) and step size \( \mu \) and is given by,

\[ W(n+1) = W(n) + \mu u(n) \cdot e(n) \]

Error estimation \( e(n) \) is given by,

\[ e(n) = y(n)|y(n)|^p \cdot (R_p - |y(n)|^p) \]

![Figure 5. Constant Modulus Algorithm](image-url)

The output equation of W-H/E based CMA equalizer is given by,

\[ y(n) = W(n) \cdot u(n) \]

where \( y(n) \) is the output of the equalizer, \( W(n) \) is the tap weight vector and \( u(n) \) is the input vector.

Estimation of reference signal:

\[ d(n) = y(n)(R_2 - |y(n)|^2) + y(n) \]

Estimation of error signal:

\[ e(n) = y(n)(R_p - |y(n)|^2) \]

Where \( R_p \) is the property restoral signal and is described as:

\[ R_p = E[|u(n)|^4] / E[|u(n)|^2] \]
III. CONCLUSION

In this project, we present a Wiener-Hammerstein electrical Equalizer based on the 3 popular algorithms. A 1024 bit, 16 QAM Fast-OFDM system is used to evaluate the performance of the proposed W-H/E. As observed in the simulation results, the Wiener-Hammerstein Equalizer is capable of mitigating nonlinearities of Fast-OFDM system. The RLS algorithm outperforms both NLMS, CMA. The results generated in this project could lead to rapid development of the nonlinear electrical equalizer and the next generation Optical communication system

REFERENCES