Special Dio3-Tuples for Pronic Number –I

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Abstract: We search for three distinct polynomials with integer coefficients such that the product of any two members of the set added with their sum and increased by a non-zero integer (or polynomial with integer coefficients) is a perfect square.

Keywords: NDio 3-tuples, Pronic numbers, Polynomials.

Notation: Proₙ = Pronic number of rank n.

I. INTRODUCTION

Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer $n$ [1] and also for any linear polynomials $n$. Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14]. In this communication, we present a few special dio 3-tuples for Pronic numbers of different ranks with their corresponding properties.

II. BASIC DEFINITION

A set of three distinct polynomials with integer coefficients $(a_1, a_2, a_3)$ is said to be a special dio 3-tuple with property $D(n)$ if $a_i * a_j + (a_i + a_j) + n$ is a perfect square for all $1 \leq i < j \leq 3$, where $n$ may be non-zero integer or polynomial with integer coefficients.

A. Method of Analysis

1) Case 1: Construction of Dio 3-tuples for Pronic number of rank $n - 1$ and $n$.

Let $a = Pro_{n-1}$, $b = Pro_n$ be Pronic number of rank $n - 1$ and $n$ respectively such that $ab + (a + b) + n^2 + 1$ is a perfect square say $\beta^2$.

Let $c$ be any non-zero integer such that

\[ ac + (a + c) + n^2 + 1 = \beta^2 \tag{1} \]
\[ bc + (b + c) + n^2 + 1 = \gamma^2 \tag{2} \]

On solving equations (1) and (2), we get

\[ (a - b) + (n^2 + 1)(b - a) = (b + 1)\beta^2 - (a + 1)\gamma^2 \tag{3} \]

Assume $\beta = x + (a + 1)y$ and $\gamma = x + (b + 1)y$ and it reduces to

\[ x^2 = (a + 1)(b + 1)y^2 + n^2 \tag{4} \]

The initial solution of the equation (4) is given by

\[ x_0 = n^2 + 1, \quad y_0 = 1 \]

Therefore, $\beta = 2n^2 - n + 2$

On substituting the values of $a$ and $\beta$ in equation (1), we get

\[ c = 4n^2 + 3 = Pro_{2n-2} + 6n + 1 \]

Hence, The triple $(Pro_{n-1}, Pro_n, Pro_{2n-2} + 6n + 1)$ is a Dio 3-tuple with property $D(n^2 + 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.
On substituting the values of $\alpha$ and $\beta$ in equation (5), we get

$$c = 4n^2 - 4n + 5 = \text{Pro}_{2n-2} + 2n + 3$$

Hence, the triple $\left(\text{Pro}_{n-2}, \text{Pro}_n, \text{Pro}_{2n-2} + 2n + 3\right)$ is a Dio 3-tuple with property $D(2n^2 - 2n - 1)$.

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.

We present below the Dio 3-tuple for Pronic number of the rank mentioned above with suitable properties.
We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

<table>
<thead>
<tr>
<th>Table 4</th>
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</thead>
<tbody>
<tr>
<td>a</td>
</tr>
<tr>
<td>Pro_{n-2}</td>
</tr>
<tr>
<td>Pro_{n-2}</td>
</tr>
<tr>
<td>Pro_{n-2}</td>
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<tr>
<td>Pro_{n-2}</td>
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<tr>
<td>Pro_{n-2}</td>
</tr>
</tbody>
</table>

3) Case 3: Construction of Dio 3-tuples for Pronic number of rank \( n - 2 \) and \( n - 1 \).

Let \( a = \text{Pro}_{n-2} \), \( b = \text{Pro}_{n-1} \) be Pronic number of rank \( n - 2 \) and \( n - 1 \) respectively such that

\[ ab + (a + b) + (-n^2 + 2n - 1) \]

is a perfect square say \( \alpha^2 \).

Let \( c \) be any non-zero integer such that

\[ ac + (a + c) + (-n^2 + 2n - 1) = \beta^2 \]  \hspace{1cm} (9)

\[ bc + (b + c) + (-n^2 + 2n - 1) = \gamma^2 \] \hspace{1cm} (10)

On solving equations (9) and (10), we get

\[ (a - b) + (-n^2 + 2n - 1)(b - a) = (b + 1)\beta^2 - (a + 1)\gamma^2 \] \hspace{1cm} (11)

Assume \( \beta = x + (a + 1)y \) and \( \gamma = x + (b + 1)y \) and it reduces to

\[ x^2 = (a + 1)(b + 1)y^2 + (-n^2 + 2n - 2) \] \hspace{1cm} (12)

The initial solution of the equation (12) is given by

\[ x_0 = n^2 - 2n + 1, \quad y_0 = 1 \]

Therefore,

\[ \beta = 2n^2 - 5n + 4 \]

On substituting the values of \( a \) and \( \beta \) in equation (9), we get

\[ c = 4n^2 - 8n + 5 = \text{Pro}_{n-2} - 2n + 3 \]

Hence, The triple \( (\text{Pro}_{n-2}, \text{Pro}_{n-1}, \text{Pro}_{n-2} - 2n + 3) \) is a Dio 3-tuple with property \( D(-n^2 + 2n - 1) \).

A few numerical examples of the Dio 3-tuples satisfying the above property are mentioned below.
Table 5

<table>
<thead>
<tr>
<th>n</th>
<th>(a, b, c)</th>
<th>D(n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0, 0, 1)</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>(0, 2, 5)</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>(2, 6, 17)</td>
<td>-4</td>
</tr>
<tr>
<td>4</td>
<td>(6, 12, 37)</td>
<td>-9</td>
</tr>
<tr>
<td>5</td>
<td>(12, 20, 65)</td>
<td>-16</td>
</tr>
</tbody>
</table>

We present below, a few Dio 3-tuple for Pronic number of rank mentioned above with suitable properties.

Table 6

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(D(n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Pro}_{n-2})</td>
<td>(\text{Pro}_{n-1})</td>
<td>(\text{Pro}_{2n-2} - 2n + 5)</td>
<td>(D(n^2 - 2n + 2))</td>
</tr>
<tr>
<td>(\text{Pro}_{n-2})</td>
<td>(\text{Pro}_{n-1})</td>
<td>(\text{Pro}_{2n-2} - 2n + 7)</td>
<td>(D(3n^2 - 6n + 7))</td>
</tr>
<tr>
<td>(\text{Pro}_{n-2})</td>
<td>(\text{Pro}_{n-1})</td>
<td>(\text{Pro}_{2n-2} - 2n + 9)</td>
<td>(D(5n^2 - 10n + 14))</td>
</tr>
<tr>
<td>(\text{Pro}_{n-2})</td>
<td>(\text{Pro}_{n-1})</td>
<td>(\text{Pro}_{2n-2} - 2n + 11)</td>
<td>(D(7n^2 - 14n + 23))</td>
</tr>
<tr>
<td>(\text{Pro}_{n-2})</td>
<td>(\text{Pro}_{n-1})</td>
<td>(\text{Pro}_{2n-2} - 2n + 13)</td>
<td>(D(9n^2 - 18n + 34))</td>
</tr>
</tbody>
</table>

III. CONCLUSION

In this paper we have presented a few examples of constructing a special Dio 3-tuples for Pronic number of different ranks with suitable properties. To conclude one may search for Dio 3-tuples for higher order Pronic number with their corresponding suitable properties.

REFERENCES