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The Comparison of Bondage Number and Domination Number with Minimum and Maximum Degrees of an Interval Graph G Using an Algorithm

Dr. A. Sudhakaraiah (Asst. Prof.)[1], K.Narayana[2], T.Venkateswarlu[3]
[1]Department of Mathematics, S.V.University, Tirupati,Andhra Pradesh, India-517502

Abstract: The theory of domination and the distance, the most often discussed is a communication network. This network consists of communication links all distance between affixed set of sites. Interval graphs are rich in combinatorial structures and have found applications in several disciplines such as Biology, Ecology, Psychology, Traffic control, Genetics, Computer sciences and particularly useful in cyclic scheduling and computer storage allocation problems etc. The problem is to select the smallest set of sites at which transmitters are placed so that every site in the network that does not have a transmitters is joined by a direct communication link to the site, which has a transmitter then this problem reduces to that of finding a minimum dominating set in the graph corresponding to this network. Then the problem is what is the fewest number of communication links such that at least one additional transmitter would be required in order that communication with all sites as possible. This leads to the introducing of the concept of the domination number, bondage number and the degree of vertex of graph G. In this paper we explored "the comparison of bondage number and domination Number with minimum and maximum degrees of an interval graph G using an algorithm ".
Keywords: Interval family, Interval graph, dominating set, Bondage Number, Maximum degree, Minimum degree

I. INTRODUCTION

Few subjects in mathematics have as specified an origin as graph theory. Graph theory originated with the Konigsberg Bridge Problem, which Leonhard Euler solved in 1736. Over the past sixty years, there has been a great deal of exploration in the area of graph theory. Its popularity has increased due to its many modern day applications in Interval graphs corresponding to Interval family and it has become the source of interest to many researchers. Among the various applications of the theory of domination that have been considered, the one that is perhaps most often discussed concerns a communications network. This network consists of existing communications links between a fixed set of sites. The problem at hand is to select a smallest set of sites at which to place transmitters so that every site in the network that does not have a transmitter is joined by a direct communication link to one that does have transmitter. This problem reduces to finding a minimum dominating set in the graph, corresponding to this network , that has a vertex corresponding to each site, and an edge between two vertices if and only if the corresponding sites have direct communication link joining them.

We now carry the foregoing example further and examine a question concerning the vulnerability of the communications network under link failure. In particular, suppose that someone does not know which sites in the network act as transmitters, but does know that the set of such sites corresponds to a minimum dominating set in the related graph. With this in mind, we introduce the bondage number of a graph. Interval graphs have drawn the attention of many researchers for over 25years.They have extensively been studied and revealed their practical relevance for modelling problems arising in the real world. Interval graphs found applications in archaeology, genetics, ecology, psychology, traffic control, computer scheduling, storage information retrieval and electronic circuit design and a wide variety of algorithms have been developed.

II. PRELIMINARIES

Let I = { $I_1, I_2, I_3 \ldots. I_n$} be an interval family where each I_i is an interval on the real line and I_i= [a_i, b_i] for i=1,2,3,…,n. Here a_i is called the left end point and b_i is called the right end point of I_i. Without loss of generality, we assume that all end points of the intervals in I are distinct numbers between 1 and 2n. Two intervals i and j are said to intersecteach other if they have non-empty intersection. Two intervals are said to overlap if they have non- empty intersection and neither one of them contains the other.Let G

= (V, E) be a graph. G is called an interval graph if there is a one to one correspondence between V and I such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I intersect. We denote this interval graph by G [I].

A set D of vertices in a graph G is a dominating set [1,3] if each vertex of G that is not in D is adjacent to at least one vertex of D. A dominating set of minimum cardinality in G is called a minimum dominating set, and its cardinality is termed the domination number of G and denoted by $\gamma(G)$. The bondage number b(G) [2,4,5,6,8] of a non-empty graph G is the minimum cardinality among all sets of edges E for which $\gamma(G-E) > \gamma(G)$. Thus, the bondage number of G is the smallest number of edges whose removal will render every minimum dominating set in G a "non dominating'' set in the resultant spanning sub graph. Since the domination number of every spanning sub graph of a non empty graph G is at least a great as $\gamma(G)$. The bondage number of a non empty graph is well defined. In that follows, we investigate the value of the bondage number in progressively more general settings.The degree $d_G(v)$ of a vertex v in G is the number of edges of G incident with v, each loop counting as two edges. Minimum degree of a vertex v in G is the minimum number of edges of G incident with v and Maximum degree of a vertex v in G is the maximum number of edges of G incident with v. We denote by $\delta(G)$ and $\Delta(G)$[7] the minimum and maximum degrees, respectively, of vertices of G.

III. MAIN THEOREMS

A. theorem 1: if g is an interval graph corresponding to an interval family i then the bondage number

b(G) ≤ $\Delta(G) + \delta(G) - 1$.

1) *Proof:*let I= { $I_1, I_2, \ldots, I_n$} be an interval family and G be an interval graph corresponding to an interval family I. our aim to show that b(G) ≤ $\Delta(G) + \delta(G) - 1$,where b(G) is the bondage number of G corresponding to I, $\gamma(G-E) > \gamma(G)$ *Here* $\Delta(G), \delta(G)$ are maximum and minimum degrees of G. first we have to prove that $\gamma(G-E) > \gamma(G)$. it will arise three cases.
2) *Case (i):*in this case $\gamma(G)$ is a minimum domination number of G.

we will find $\gamma(G)$ towards using an algorithm as follows.

B. Procedure for finding a minimum dominating set (d) of an interval graph using an algorithm.

1) *Input:*Interval family I= { $I_1, I_2, \ldots, I_n$} .
2) *Output:*minimum dominating set of an interval graph of a given familyI.

Step1: take i= 1, D= ∅.
Step2: S_1= nbd [i]
Step3: LHDI= largest interval of the highest degree intervals in S_1.
Step4: D= D∪ $\{LHDI\}$
Step5: find NI (LHDI)
Step6: if i= NI(LHDI) exists.
Then go to step2.
Else.
Step7: write minimum dominating set D.
Step8: End.

Therefore $\gamma(G)$ is hold in this case.

In the above algorithm, D is minimum dominating set and $\gamma(G)$ is cardinality of the dominating set D. Again we will find $\gamma(G-E)$ of G corresponding to an interval family I = { $I_1, I_2, \ldots, I_n$} .

3) *Case(ii):*NowG[I] be the interval graph corresponding to an interval family I. let v, u be any two vertices of graph G[I].suppose v∈ $D[G]$ because there is no other vertex in G other than v that dominates u. we consider the set of E= {e= (v, u)} in G corresponding to an interval family I. If we remove this edge from an interval graph G, then u becomes an isolated vertex in G-E. Hence the minimum dominating set D(G-E) = D(G)∪ {u} becomes a domination number of G-E and since $\gamma(G)$ is a minimum domination number of G it follows that $\gamma(G-E)$ is also a minimum domination number of G-E.

Hence the minimum dominating set D(G-E) = D(G)∪ {u}. And hence

$$\gamma(G-E) = |D(G)| + |\{u\}|$$

$$\gamma(G-E) = \gamma(G)+1 \quad > \gamma(G)$$

$$\gamma(G-E) > \gamma(G)$$

Bondage number b(G) = $|E|$ = 1

There fore b(G) = 1

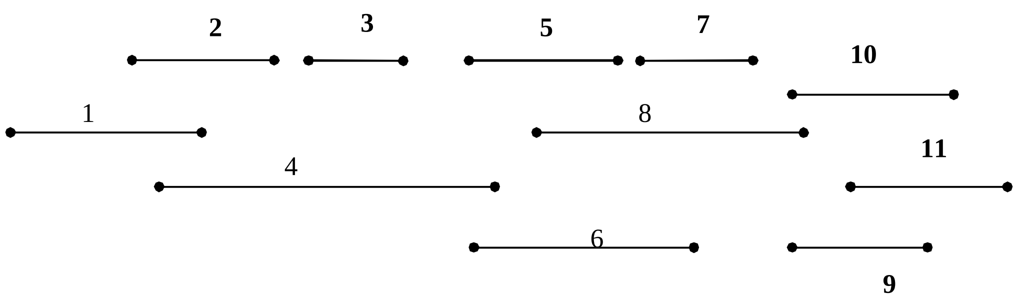
.

4) *Case(iii):* let i be a vertex of minimum degree $\delta(G)$ in G and let j be any vertex adjacent to j of G corresponding to an interval family I= $\{ I_1, I_2, \ldots, I_n\}$ .In this we can easily verify $\Delta(G) + \delta(G) - 1$.from case (i), case(ii) and case(iii) we get

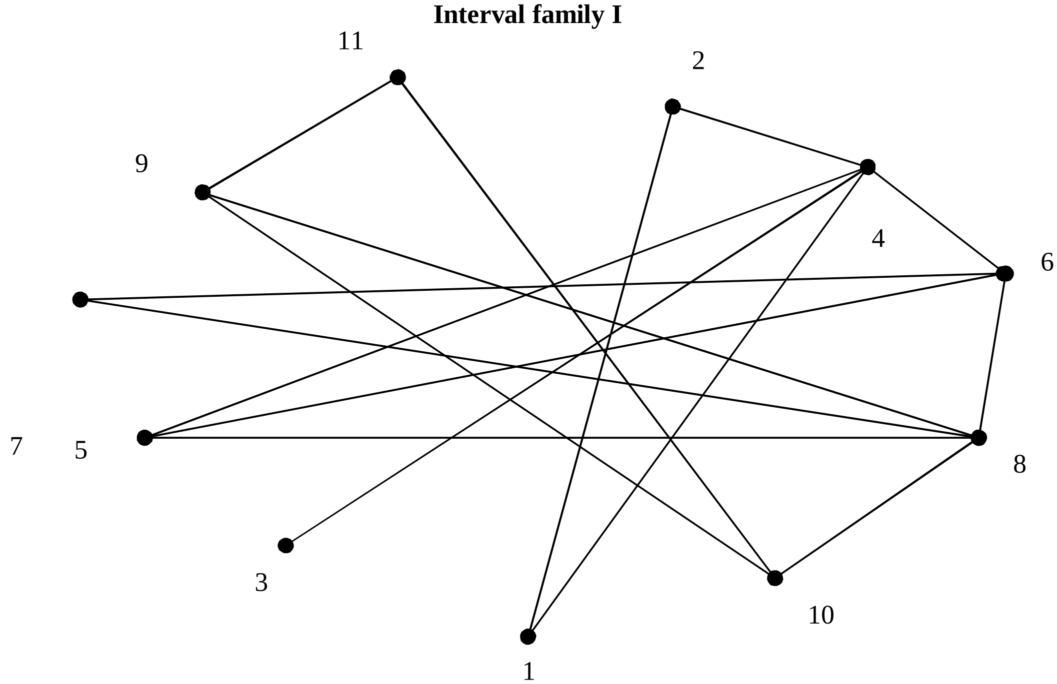
b(G) $\leq \Delta(G) + \delta(G) - 1$.

As follow the practical problem of theorem (1) of G.

C. Experimental problem 1:



Interval family I



Interval graph G

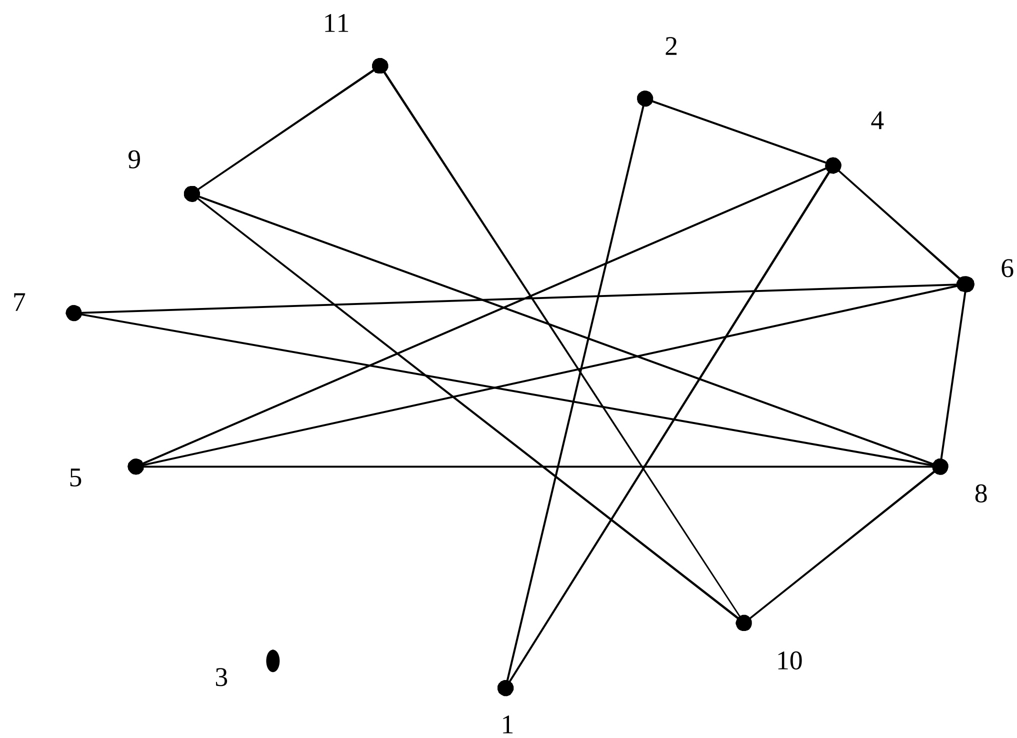
Dominating set D= {4,8,11} and $\gamma(G)$ = 3.Remove the edge = (3,4) from G, then the corresponding interval graph G-E is as follows.

A. Degrees of vertices:

deg(1) =2	deg(2) = 2	deg(3) = 1
deg(4) = 5	deg(5) = 3	deg(6) = 4
deg(7) = 2	deg(8) = 5	deg(9) = 3
deg(10) = 3	deg(11) = 2	

B. Neighbourhood of vertices

nbd[1] = {1,2,4}	nbd[2] = {1,2,4}
nbd[3] = {3,4}	nbd[4] = {1,2,3,4,5,6}
nbd[5] = {4,5,6,8}	nbd[6] = {4,5,6,7,8}
nbd[7] = {6,7,8}	nbd[8] = {5,6,7,8,9,10}
nbd[9] = {8,9,10,11}	nbd[10] = {8,9,10,11}
nbd[11] = {9,10,11}	



Interval graph G – E

Dominating set of G-E= {3,4,8,11} and $\gamma\ (G-E)$= 4. Therefore $\gamma\ (G-E) > \gamma\ (G)$ and hence b(G)= 1.

And also we proved b(G) $\leq\ \Delta(G)\ +\ \delta(G) -\ 1$.i.e, 1 ≤ 5+1-1. Which implies 1≤ 5

Therefore the result is true.

C. Non intersecting vertex of vertices

NI(1) = 3	NI(2) = 3	NI(3) = 5	NI(4) = 7
NI(5) = 7	NI(6) = 9	NI(7) = 9	NI(8) = 11
NI(9)= null	NI(10)= nullNI(11)=null		

D. To finding a minimum dominating set d of an interval graph using an algorithm:

1) *Input:* interval family I= { 1,2,3, ... ,11} .

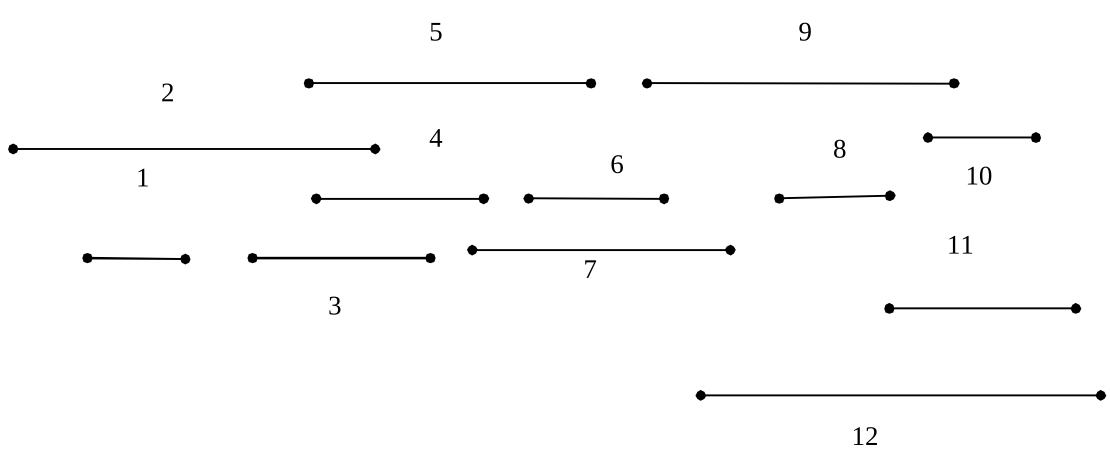
Step1: take i= 1,D= ∅
Step2: S_1= nbd[1] = {1,2,4}
Step3: LHDI = 4
Step4: D= D∪{LHDI}= ∅ ∪**{4}= {4}**
Step5: find NI(LHDI)= NI(4)= 7
Step6: if i= NI(LHDI)= 7 exists.Then go to step 2
Step2: S_1= nbd[7] = {6,7,8}
Step3: LHDI = 8
Step4: D= D∪{LHDI}= {4}∪ {8}= {4,8}
Step5: find NI(LHDI)= NI(8)= 11
Step6: if i= NI(LHDI)= 11 exists.Then go to step 2
Step2: S_1= nbd[11] = {9,10,11}
Step3: LHDI = 11
Step4: D= D∪{LHDI}= {4,8}∪ {11}= = {4,8,11}
Step5: find NI(LHDI)= NI(11)= null
ELSE
Step7: minimum dominating set D= {4,8,11}
Step8: End

2) *Out Put*: minimum dominating set D= {4,8,11}
3) *THEOREM 2: if G is a non-empty interval graph corresponding to an interval family I with domination number*

γ (G) ≥ 2 then the bondage number b(G) ≤ [γ (G) – 1] $\Delta(G)$ + 1.

4) *PROOF*: we consider I= { $I_1, I_2, \ldots, I_n$} be an interval family and Let I_i,$I_j$be any two intervals such that I_j is contained in $I_i$and there is no interval other than $I_i$that intersect I_j then we have to take two vertices V_1,V_2 of G[I] are joined by an edge in E, if their corresponding interval I_i and I_j in I intersect such that V_1 is dominatedby only one vertex v_2 and suppose e= (V_1,V_2) and γ (G) be a minimum domination number of G. Already we have to find the domination number γ (G) of G[I]by using an algorithm in THEOREM 1.The vertex V_1 becomes an isolated vertex in G-e. Hence $\gamma(G-e) = \gamma(G)+1$ becomes a minimum domination number of G-e so that $\gamma(G-e) > \gamma(G)$ and therefore the bondage number b(G)=1.And we will find b(G)of G[I]. In this connection we get b(G) ≤ [γ (G) – 1] $\Delta(G)$ + 1. As follows the practical problem.

E. Experimental problem 2:



Interval family I

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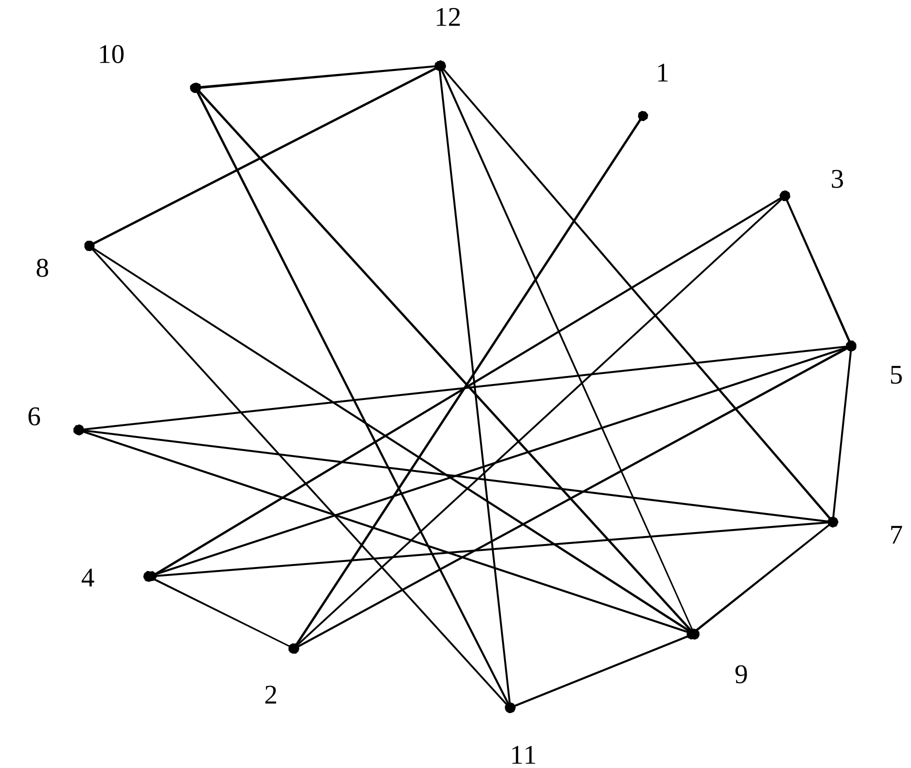
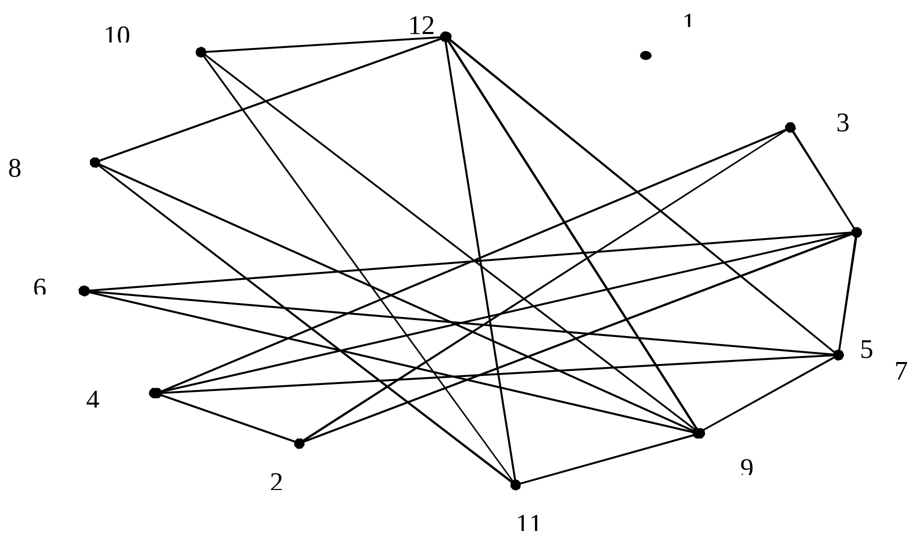


Interval graph G

Dominating set D= {2,9} and $\gamma(G)$ = 2.

Remove the edge = (1,2) from G, then the corresponding interval graph G-e is as follows.



Interval graph G-e

Dominating set of G-e= {1,2,9} and $\gamma\ (G-e)_{=\ 3}$

Therefore $\gamma\ (G-e) > \gamma\ (G)$ and hence b(G)= 1.

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And also we proved b(G) ≤ [γ (G) – 1] $\Delta(G)$ + 1

i.e, 1 ≤ (2-1) 6+ 1

1≤ 7

Therefore the result is true.

1) Degrees of vertices:

deg(1) =1	deg(2) = 4	deg(3) = 3
deg(4) = 4	deg(5) = 5	deg(6) = 3
deg(7) = 5	deg(8) = 3	deg(9) = 6
deg(10) = 3	deg(11) = 4	deg(12) = 5

2) Neighbourhood of vertices:

nbd[1] = {1,2}	nbd[2] = {1,2,3,4,5}
nbd[3] = {2,3,4,5}	nbd[4] = {2,3,4,5,7}
nbd[5] = {2,3,4,5,6,7}	nbd[6] = {5,6,7,9}
nbd[7] = {4,5,6,7,9,12}	nbd[8] = {8,9,11,12}
nbd[9] = {6,7,8,9,10,11,12}	nbd[10] = {9,10,11,12}
nbd[11] = {8,9,10,11,12}	nbd[12] = {7,8,9,10,11,12}

3) Non intersecting vertex of vertices:

NI(1) = 3	NI(2) = 6	NI(3) = 6	NI(4) = 6
NI(5) = 8	NI(6) = 8	NI(7) = 8	NI(8) = 8
NI(9)= null	NI(10)= null	NI(11)=null	NI(12)=null

F. To find a minimum dominating set d of an interval graph using an algorithm:

4) Input: interval family I= { 1,2,3, … ,12} .

Step1: take i= 1,D= ∅
Step2: S_1= nbd[1] = {1,2}
Step3: LHDI = 2
Step4: D= D∪{LHDI}= ∅ ∪{2}= {2}
Step5: find NI(LHDI)= NI(2)= 6
Step6: if i= NI(LHDI)= 6 exists.
Then go to step 2
Step2: S_1= nbd[6] = {5,6,7,9}
Step3: LHDI = 9
Step4: D= D∪{LHDI}= {2}∪ {9}= {2,9}
Step5: find NI(LHDI)= NI(9)= null
ELSE
Step7: minimum dominating set D= {2,9}
Step8: End

5) Out Put: minimum dominating set D= {2,9}

6) THEOREM 3: if G is a connected interval graph of order n≥ 2, thenb(G)≤ $n - \gamma$ (G)+ 1.

7) Proof: let the dominating set D of G consists of two vertices only, say x and y. suppose x dominates the vertex set S_1= {1,…,i} and y dominates the vertex set S_2= {i+1,…,n}. we consider an interval family I= { $I_1, I_2, \dots, I_n$}. Suppose there is no vertex in S, other than x that dominates S_1, other than x that dominates S_1 and no vertex in S_2 other than y that dominates S_2 than the bondage number b(G)=1.Now we will prove that b(G)≤ $n - \gamma$ (G)+ 1. First we will prove that γ (G), already proved in THEOREM1 towards an algorithm. Now we will prove that b(G)=1.

Let the domination number γ (G)= {x,y}. Suppose x and y satisfy the hypothesis of the THEOREM. Since x alone dominates S_1, there is no vertex in S_3= {1,…, i| x} that can dominates, let j be any any vertex in S_3 and e= {x, j}. consider an interval graph G-e

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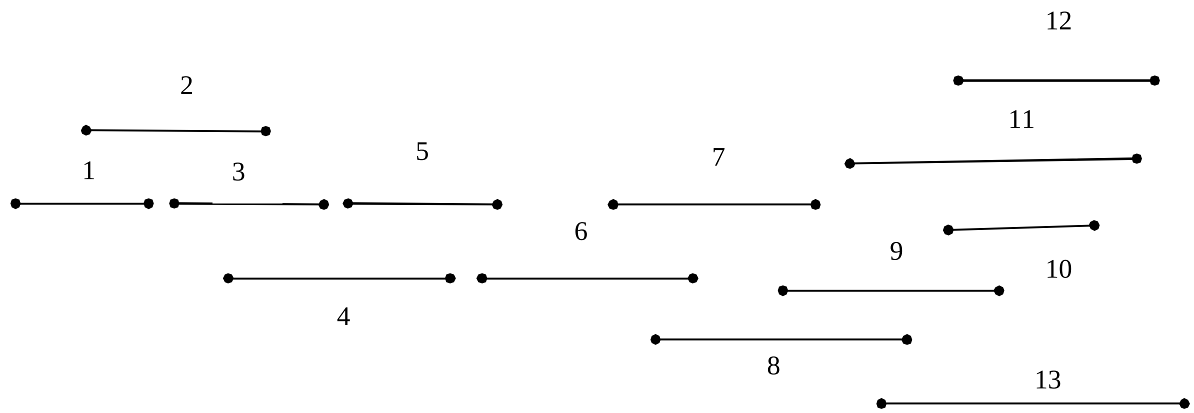
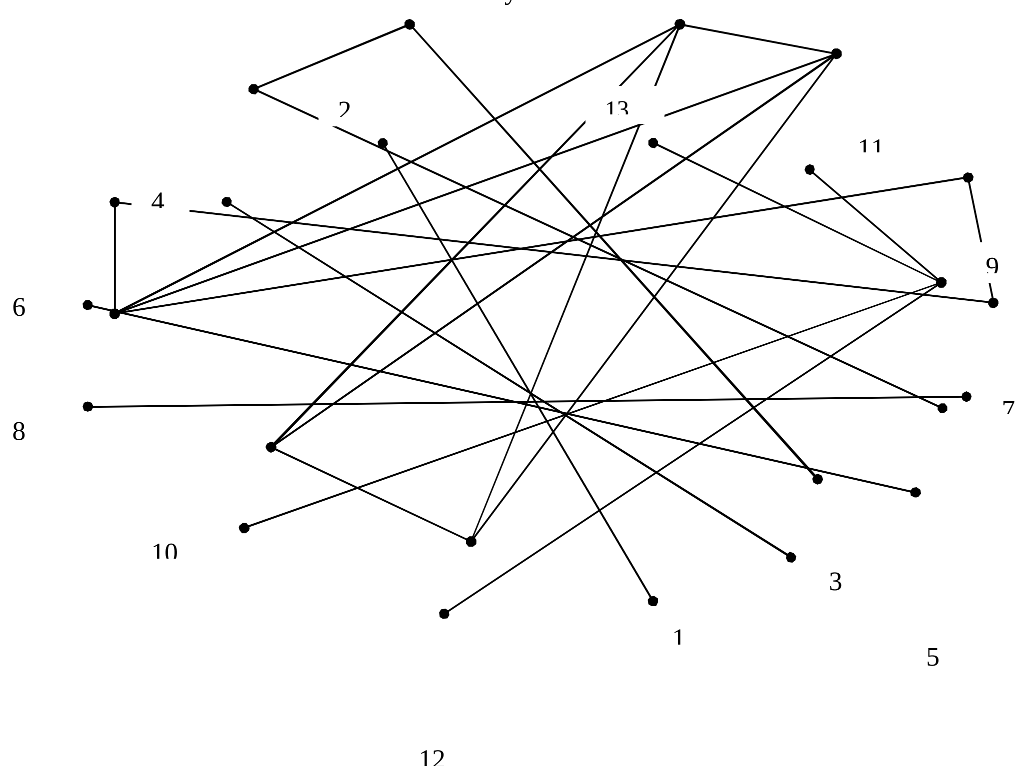
corresponding to I. in this graph, x dominates every vertex in S_1, except j. Now consider a vertex in S_1 which is adjacent with j, say k. then clearly the set {x.k} dominates the set S_1 in G-e. If there is a vertex in s_1 that is adjacent with j, then clearly the graph G becomes disconnected. So there is at least one vertex in S_1 that is adjacent with j. Let us assume that there is a single vertex say z, z= x such hat z dominates the set S_1 in G-e. This implies that z also dominates the set S_1in G, a contradiction, because by hypothesis x is the only vertex that dominates the set S_1 in G. Hence a single vertex cannot dominate S1 in G-e.

Hence γ (G-e) = γ (G)∪ |{k} | becomes a domination number of G-e. since γ (G) is minimum domination in G, γ (G-e) is also minimum domination in G-e, so that $\gamma\,(G-e) > \gamma\,(G)$.Hence the bondage number b(G) = 1, this proves the THEOREM As follows the practical problem.

G. Experimental problem 3:



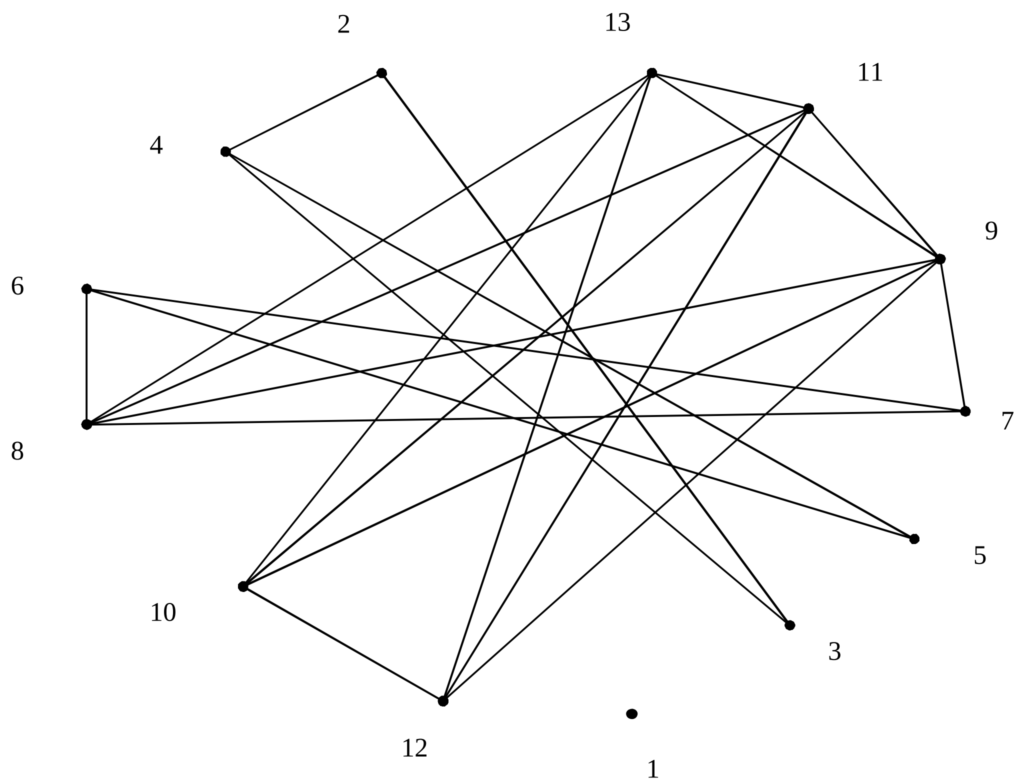
Interval family I



Interval graph G

Dominating set D= {2,6,13} and $\gamma(G)$ = 3.

Remove the edge = (1,2) from G, then the corresponding interval graph G-e is as follows



Interval graph G-e

Dominating set of G-e= {1,2,6,13} and $\gamma\ (G-e)_{= 4}$

Therefore $\gamma\ (G-e)\ >\ \gamma\ (G)$ and hence b(G)= 1.

And also we proved b(G)≤ $n-\gamma$ (G)+ 1

i. e., 1 ≤ 13-3+ 1

1≤ 11

Therefore the result is true.

1) Degrees of vertices:

deg(1) =1	deg(2) = 3	deg(3) = 2
deg(4) = 3	deg(5) = 2	deg(6) = 3
deg(7) = 3	deg(8) = 5	deg(9) = 6
deg(10) = 4	deg(11) = 5	deg(12) = 4
deg(13) = 5		

2) Neighbourhood of vertices:

nbd[1] = {1,2}	nbd[2] = {1,2,3,4}
nbd[3] = {2,3,4}	nbd[4] = {2,3,4,5}
nbd[5] = {4,5,6}	nbd[6] = {5,6,7,8}

nbd[7] = {6,7,8,9}   nbd[8] = {6,7,8,9,11,13}
nbd[9] = {7,8,9,10,11,12,13}   nbd[10] = {9,10,11,12,13}
nbd[11] = {8,9,10,11,12,13}   nbd[12] = {9,10,11,12,13}
nbd[13] = {8,9,10,11,12,13}

3) *Non intersecting vertex of vertices:*

NI(1) = 3	NI(2) = 5	NI(3) = 5	NI(4) = 6
NI(5) = 7	NI(6) = 9	NI(7) = 10	NI(8) = 10
NI(9)= null	NI(10)= null	NI(11)=null	NI(12)=null
NI(13)=null			

H. To find a minimum dominating set d of an interval graph using an algorithm:

4) *Input:* Interval family I= { 1,2,3, … ,13} .

Step1: take i= 1,D= ∅
Step2: S_1= nbd[1] = {1,2}
Step3: LHDI = 2
Step4: D= D∪{LHDI}= ∅ ∪{2}= {2}
Step5: find NI(LHDI)= NI(2)= 5
Step6: If i= NI(LHDI)= 5exists.
Then go to step 2
Step2: S_1= nbd[5] = {4,5,6}
Step3: LHDI = 6
Step4: D= D∪{LHDI}= {2}∪ {6}= {2,6}
Step5: find NI(LHDI)= NI(6)= 9
Step6: If i= NI(LHDI)= 9exists.
Then go to step 2
Step2: S_1= nbd[9] = {7,8,9,10,11,12,13}
Step3: LHDI = 13
Step4: D= D∪{LHDI}= {2,6}∪ {13}
Step5: find NI(LHDI)= NI(13)= null
Step6: If i= NI(LHDI)= null
ELSE
Step7: minimum dominating set D= {2, 6, 13}
Step8: End

5) *Out Put:*minimum dominating set D= {2, 6, 13}.

IV. CONCLUSIONS

Resolving the Domination number of some special classes of Interval graphs has been the main focus of the paper. Nature of the Intervals of the graph and the relation between the Domination number and the Bondage number paved the way for the present progressive revelations. Especially, the nature of the Interval family played a major role in determining the Domination number, Bondage number, Maximum degree and Minimum degree of the Interval graphs with amazing ease. Some categorized graphs have been chosen in the process of exploration. In future, efforts will be put to identify the Interval graphs with the comparison of bondage number and domination Number with minimum and maximum degrees of an interval graph G using an algorithm.

V. ACKNOWLEDGMENT

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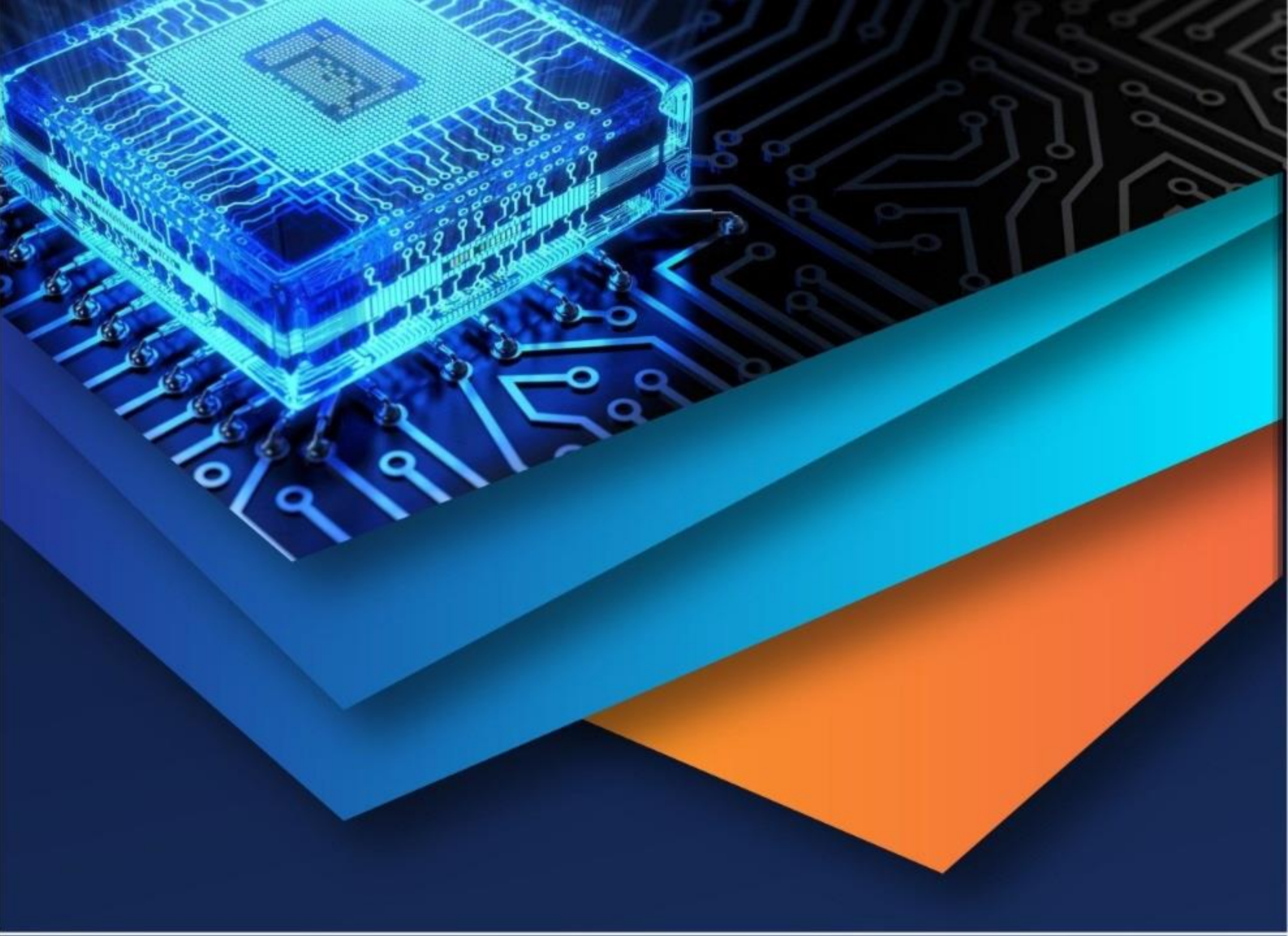

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