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Methodology of Optimal Link Shape Synthesis of Planar Mechanisms

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Abstract: The optimization problem formulation for link shape synthesis for the optimally balanced simple and multiloop planar mechanisms is presented in this paper. The closed parametric curve is used to represent the link shape and its geometric and inertial properties are calculated using well known Green's theorem. The proposed optimization problem includes the equality constraints to keep the resulting inertial properties same as the inertial properties of the optimally balanced mechanisms.

I. INTRODUCTION

In this paper, the link shapes are synthesized for optimally balanced mechanism for the given motion. The link shapes satisfying kinematic and dynamic requirements are very crucial for the design of a mechanism and its performance. The shape synthesis using parametric curves like Hermit, Bezier and B-spline curves leads to computer-aided design (CAD) and manufacturing of the mechanism links. Through CAD modeling of the links using these curves; the design, production and functional details can beeasily transmitted between engineering and manufacturing operations. The CAD modeling of the links is also useful in analyzing the static and dynamic response of the designed mechanism. The real-time behavior of the mechanism is evaluated through computer simulation and thus it eliminates the need of the experimental tests for the actual mechanism. Therefore, the cost and time are saved to a great extent and any possible error is realized before manufacturing of the mechanism links.

II. LINK SHAPE

The link shape is represented by the parametric curve, i.e., closed cubic B-spline curve as shown in Fig. 1. If the curve interpolates or approximates a set of n+1 control points, P_0, P_1,…,P_n(Zeid and Sivasubramanian, 2009; Mortenson, 2006) then the position of any point on the curve is defined as:

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(u), \quad 0 \le u \le u_{\max} \tag{1}$$

For a curve of degree (k-1), the B-spline function $N_{i,k}(u)$ is computed iteratively as:

$$N_{i,k}(u) = (u - u_i)\frac{N_{i,k\text{-}1}(u)}{u_{i+k\text{-}1} - u_i} + (u_{i+k} - u)\frac{N_{i+1,k\text{-}1}(u)}{u_{i+k} - u_{i+1}} \tag{2}$$

where

$$N_{i,1} = \begin{cases} 1, & u_i \le u \le u_{i+1} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

In Eq. (3), $N_{i,1}$ is a unit step function and u_i are known as parametric knots or knot values. These values form a sequence of nondecreasing integers called the knot vector. The parametric equation of ith curve segment of a cubic B-spline curve having control points $P_{i\text{-}1}$, P_i, P_{i+1} and P_{i+2} for $u \in [u_{i\text{-}1}, u_i]$ is given as:

$$P_i(u) = \frac{\alpha_1 P_{i\text{-}1} + \alpha_2 P_i + \alpha_3 P_{i+1} + \alpha_4 P_{i+2}}{6} \tag{4}$$

where

$$\alpha_1 = -u^3 + 3u^2 i - 3ui^2 + i^3 \tag{5}$$

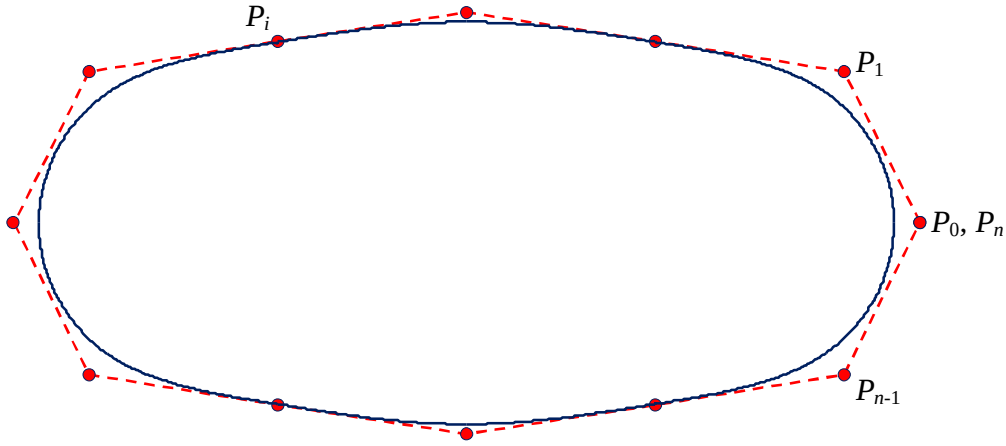


Fig. 1 Closed cubic B-spline curve and its control points

$$\alpha_2 = 3u^3 + u^2(3-9i) + u(-3+9i^2-6i) - 3i^3 + 3i^2 + 3i + 1 \tag{6}$$

$$\alpha_3 = -3u^3 + u^2(-6+9i) + u(-9i^2+12i) + 3i^3 - 6i^2 + 4 \tag{7}$$

$$\alpha_4 = u^3 + u^2(3-3i) + u(3+3i^2-6i) - i^3 + 3i^2 - 3i + 4 \tag{8}$$

The control points form the vertices of the characteristic polygon of the B-spline curve as shown in Fig. 1. Note that the cubic B-spline curve is a composite sequence of curve segments connected with C^2 continuity which blends two curve segments with same curvature. The coordinates of any point on the *i*th segment of the curve are given by Eq. (4) as:

$$x_i(u) = \frac{\alpha_1 x_{i-1} + \alpha_2 x_i + \alpha_3 x_{i+1} + \alpha_4 x_{i+2}}{6} \tag{9}$$

$$y_i(u) = \frac{\alpha_1 y_{i-1} + \alpha_2 y_i + \alpha_3 y_{i+1} + \alpha_4 y_{i+2}}{6} \tag{10}$$

where the terms α_1, α_2, α_3 and $\alpha_4$are defined in Eqs. (5) – (8),and (x_{i-1}, y_{i-1}), (x_i, y_i), etc. are the coordinates of points P_{i-1}, P_i, etc. respectively. The mass and inertia of the link that is synthesized using closed cubic B-spline curve can be calculated using Green's theorem (Crisco et al., 1998; Brlek et al., 2005).For two functions $P(x, y)$and $Q(x, y)$ over a closed region D in the plane with boundary ∂D , Green's theorem presents:

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy = \oint_{\partial D} \left(Pdx + Qdy \right) \tag{11}$$

The area of closed region D is calculated as:

$$A = \iint_D dxdy \tag{12}$$

This area is calculated using Green's theorem by taking $P(x,y) = 0$ and $Q(x,y) = x$ that gives:

$$A = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy \tag{13}$$

$$A = \oint_{\partial D} xdy \tag{14}$$

For a plane curve specified parametrically as $(x(u), y(u))$ for $u \in [u_0, u_1]$, Eq. (14) becomes:

$$A = \int_{u_0}^{u_1} xy'du \tag{15}$$

Similarly, the moment about *x*-axis and *y*-axis of plane are computed as:
using $P = -y^2/2$ and $Q = 0$

$$M_x = \iint ydxdy = -\frac{1}{2}\oint y^2 dx = -\frac{1}{2}\int_{u_0}^{u_1} y^2 x'du \tag{16}$$

using $P = 0$ and $Q = x^2/2$

$$M_y = \iint xdxdy = \frac{1}{2}\oint x^2 dy = \frac{1}{2}\int_{u_0}^{u_1} x^2 y'du \tag{17}$$

The geometric centroid $(\bar{x}, \bar{y})$ of plane curve is given by $\bar{x} = M_y/A$ and $\bar{y} = M_x/A$. Finally, the area moments of inertia can be computed as:
using $P = -y^3/3$ and $Q = 0$

$$I_{xx} = \iint y^2 dxdy = -\frac{1}{3}\oint y^3 dx = -\frac{1}{3}\int_{u_0}^{u_1} y^3 x'du \tag{18}$$

using $P = 0$ and $Q = x^3/3$

$$I_{yy} = \iint x^2 dxdy = \frac{1}{3}\oint x^3 dy = \frac{1}{3}\int_{u_0}^{u_1} x^3 y'du \tag{19}$$

Hence,the area *A*, centroid $(\bar{x}, \bar{y})$ and area moment of inertia about centroidal axes [I_{xx}, I_{yy}, I_{zz}] of the closed curve made of *n* cubic B-spline segments are calculated as:

$$A = \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} x_i(u) y_i'(u)\, du \tag{20}$$

$$\bar{x} = \frac{1}{2A_i} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} x_i^2(u) y_i'(u)\, du \tag{21}$$

$$\bar{y} = -\frac{1}{2A_i} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} y_i^2(u) x_i'(u)\, du \tag{22}$$

$$I_{xx} = -\frac{1}{3}\sum_{i=1}^{n}\int_{u_{i-1}}^{u_i} y_i^3(u)x_i^{'}(u)\,du \tag{23}$$

$$I_{yy} = \frac{1}{3}\sum_{i=1}^{n}\int_{u_{i-1}}^{u_i} x_i^3(u)y_i^{'}(u)\,du \tag{24}$$

$$I_{zz} = I_{xx} + I_{yy} \tag{25}$$

The first derivatives $x_i^{'}(u)$ and $y_i^{'}(u)$ of $x_i(u)$ and $y_i(u)$ with respect to u, respectively, in Eqs. (20) – (24) are given by:

$$x_i^{'}(u) = \frac{\beta_1 x_{i-1} + \beta_2 x_i + \beta_3 x_{i+1} + \beta_4 x_{i+2}}{6} \tag{26}$$

$$y_i^{'}(u) = \frac{\beta_1 y_{i-1} + \beta_2 y_i + \beta_3 y_{i+1} + \beta_4 y_{i+2}}{6} \tag{27}$$

where

$$\beta_1 = -3u^2 + 6ui - 3i^2 \tag{28}$$

$$\beta_2 = 9u^2 + 2u(3 - 9i) - 3 + 9i^2 - 6i \tag{29}$$

$$\beta_3 = -9u^2 + 2u(-6 + 9i) - 9i^2 + 12i \tag{30}$$

$$\beta_4 = 3u^2 + 2u(3 - 3i) + 3 + 3i^2 - 6i \tag{31}$$

For geometric properties defined in Eqs. (20) – (25), the mass and mass moment of inertia of a link with shape represented by the closed curve are calculated as:

$$m = At\rho \tag{32}$$

$$I = I_{zz}t\rho \tag{33}$$

where t and ρ represent thickness and material density for the link, respectively.

III. OPTIMIZATION PROBLEM FORMULATION

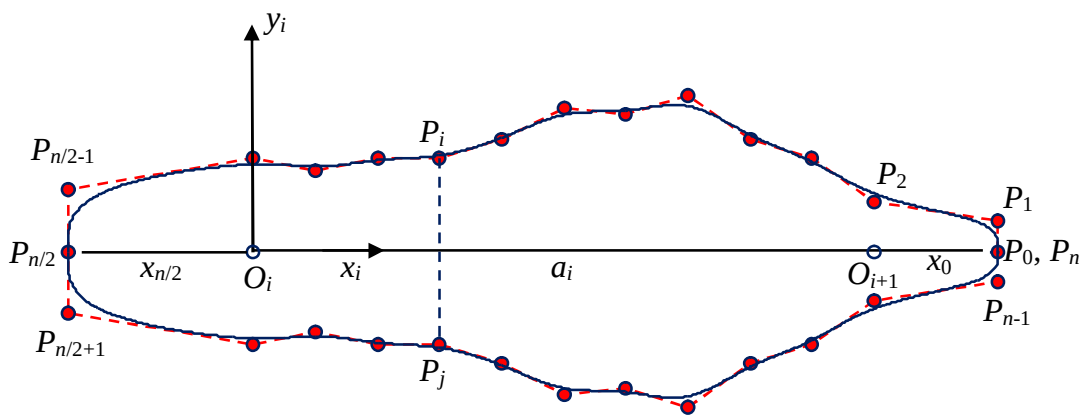
In this section, an optimization problem is formulated to find the optimum link shapes corresponding to the inertial parameters of the optimally balanced mechanisms. To formulate the optimization problem, the Cartesian coordinates of control points of cubic B-spline curve are taken as design variables as shown in Fig. 2.

For a binary link, Fig. 2 (a), the link length, a_i, between joint origins O_i to O_{i+1} is divided into equal parts. Hence, the x-coordinates of the control points lying between O_i and O_{i+1} are fixed according to the link length. Now, the y-coordinates are taken as the design variables. Furthermore, the extension of link beyond O_i and O_{i+1} is controlled by points P_0, P_1, P_{n-1} at one end and by points $P_{n/2-1}$, $P_{n/2}$, $P_{n/2+1}$ at other end. Hence, x-coordinate of P_0, y-coordinates of P_1 and P_{n-1} are chosen as the design variables at the right end and same is done at left end. Finally, the design vector is proposed as:

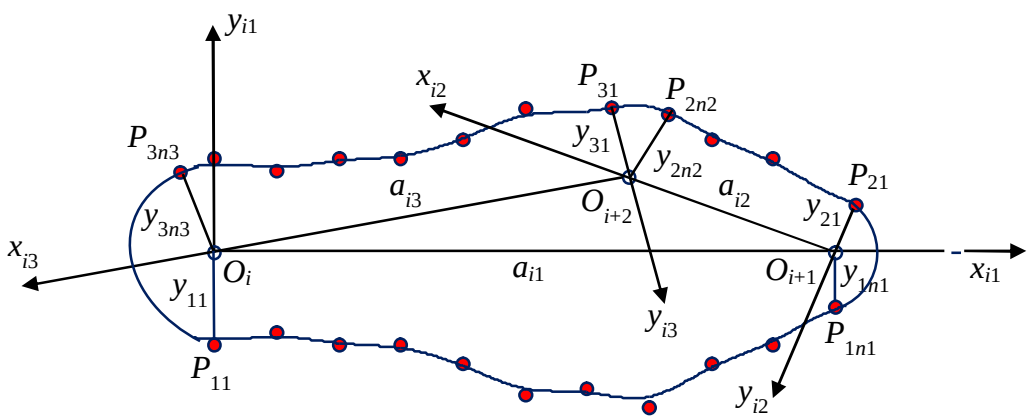
$$\mathbf{x} = [x_0\ y_1 \ldots y_{n/2\text{-}1}\ x_{n/2}\ y_{n/2+1} \ldots y_{n\text{-}1}]^{\mathrm{T}} \tag{34}$$

The conditions for symmetrical and non-symmetrical shapes are imposed by controlling coordinates of the opposite points as $y_j = -y_i$ and $y_j \neq y_i$, respectively. In addition to manufacturing benefits, the symmetrical shapes have zero products of inertia.

For a ternary link having joint origins as O_i,O_{i+1} and O_{i+2} shown in Fig. 2(b), the link length, a_i, can be defined as summation of the distances between joints, i.e., $a_i = a_{i1} + a_{i2} + a_{i3}$. The number of control points between two joints can be decided according to the distance between them.



(a) Binary link



(b) Ternary link

Fig. 2 Closed cubic B-spline curve representing link shape and its control points where P_i and P_j are two opposite points about x-axis

If n_1, n_2, and n_3 are number of control points for lengths a_{i1}, a_{i2}, and a_{i3}, respectively, then total number of control points is the sum of n_1, n_2, and n_3. At each joint, two points coincide and their y-coordinate can be determined by considering the local coordinate frame in the link as shown in Fig. 2 (b). The design vector in this case can be defined as:

$$\mathbf{x} = [y_{11} \ldots y_{1n_1}\ y_{21} \ldots y_{2n_2}\ y_{31} \ldots y_{3n_3}]^{\mathrm{T}} \tag{35}$$

Note that for the link having three or more joints, the shapes can be synthesized by selecting y-coordinates for each segment of the length between joints.The inertial properties of resulting shapes are constrained by the optimal properties. These constraints ensure that the links with optimum shapes have the same inertial properties as that of the optimally balanced mechanism links. The objective function is formulated to minimize the percentage error in resulting links inertia values as:

$$\text{Minimize Z} = \frac{(I_i^* - I_i)}{I_i^*} \times 100 \tag{36}$$

$$\text{Subject to } m_i = m_i^*;\ \bar{x}_i = \bar{x}_i^*;\ \bar{y}_i = \bar{y}_i^* \text{ for } i = 1, 2, \ldots, n \tag{37}$$

$y_j = -y_i$ (for symmetrical binary link); $y_j \neq y_i$ (for non-symmetrical binary link)

here parameters with superscript '*' represent parameters obtained for the optimally balanced mechanism and subscript '*i*' is used for *i*th link of mechanism. The teaching-learning-based optimization (TLBO) algorithm is used to solve this optimization problem. It is advantageous to use TLBO as compared to the other evolutionary optimization algorithms, as (1)it doesn't requireany algorithm specific parameters to be defined to start the optimization procedure and (2) it converges to the optimum solution faster than other evolutionary optimization algorithms. Also, the initial values of the design variables are not required to start searching the optimum solution and hence no initial shape is required.The thickness of mechanism links is taken as 10 percent of the driving link length and the link material is chosen as the mild steel (density = 7850 kg/m^3) for deciding the density and maximum permissible stress. Furthermore, the thickness of the link is taken uniform normal to the plane of motion and can be different for different link in the mechanism considered. The stress at the weakest section in each link is calculated for the maximum joint force occurred during the complete cycle of operation. Moreover, the von mises stresses for the peak load is considered to determine minimum cross-section of each link. The inertial properties of links are calculated using Eqs. (32) – (33) and verified by CAD models developed using Autodesk Inventor software. The flow chart shown in Fig. 4 illustrates the two-stage optimization method proposed for the optimum design of the planar mechanisms.

IV. PLANAR MECHANISMS

In this section, the effectiveness of the proposed optimization method for link shape synthesis is shown for a optimally balanced planar mechanism. Based on the design variables defined in Fig. 2, total 28 design variables, namely, x_0, x_{13}, $y_1 \ldots y_{26}$ are now considered for the optimum link shape synthesis for the planar mechanisms (Fig. 3).

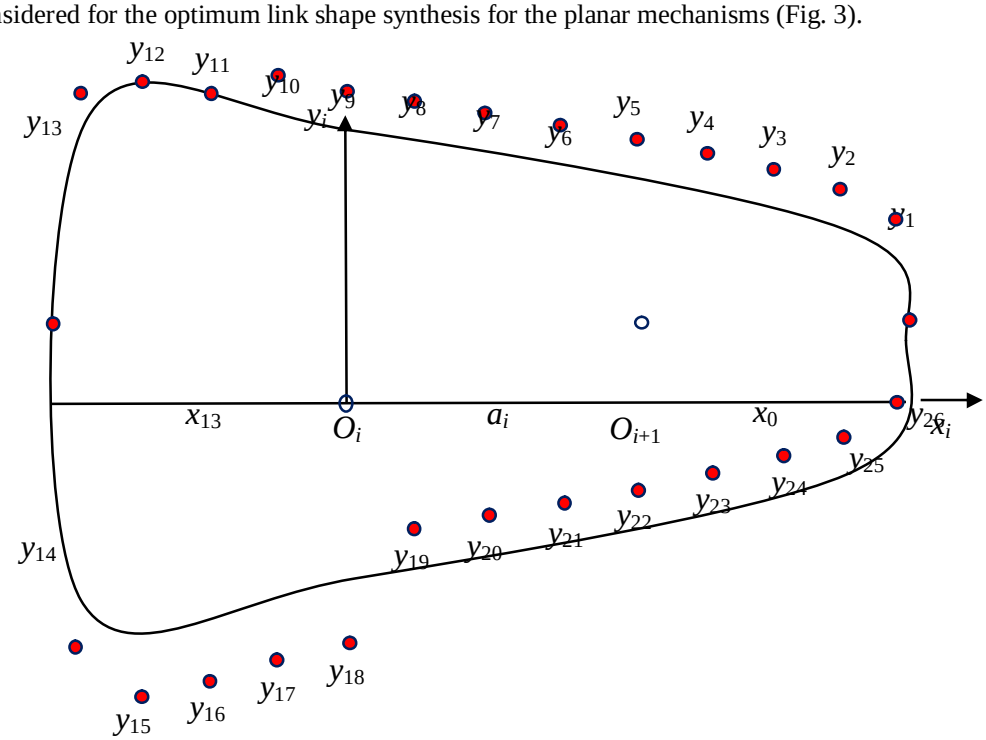


Fig. 3 Design variables to find optimum link shape of planar mechanisms

Here, a_i represents the link length between joints O_i and O_{i+1}. The design variables x_0 and x_{13} are representing link lengths beyond the joints O_{i+1} and O_i, respectively.

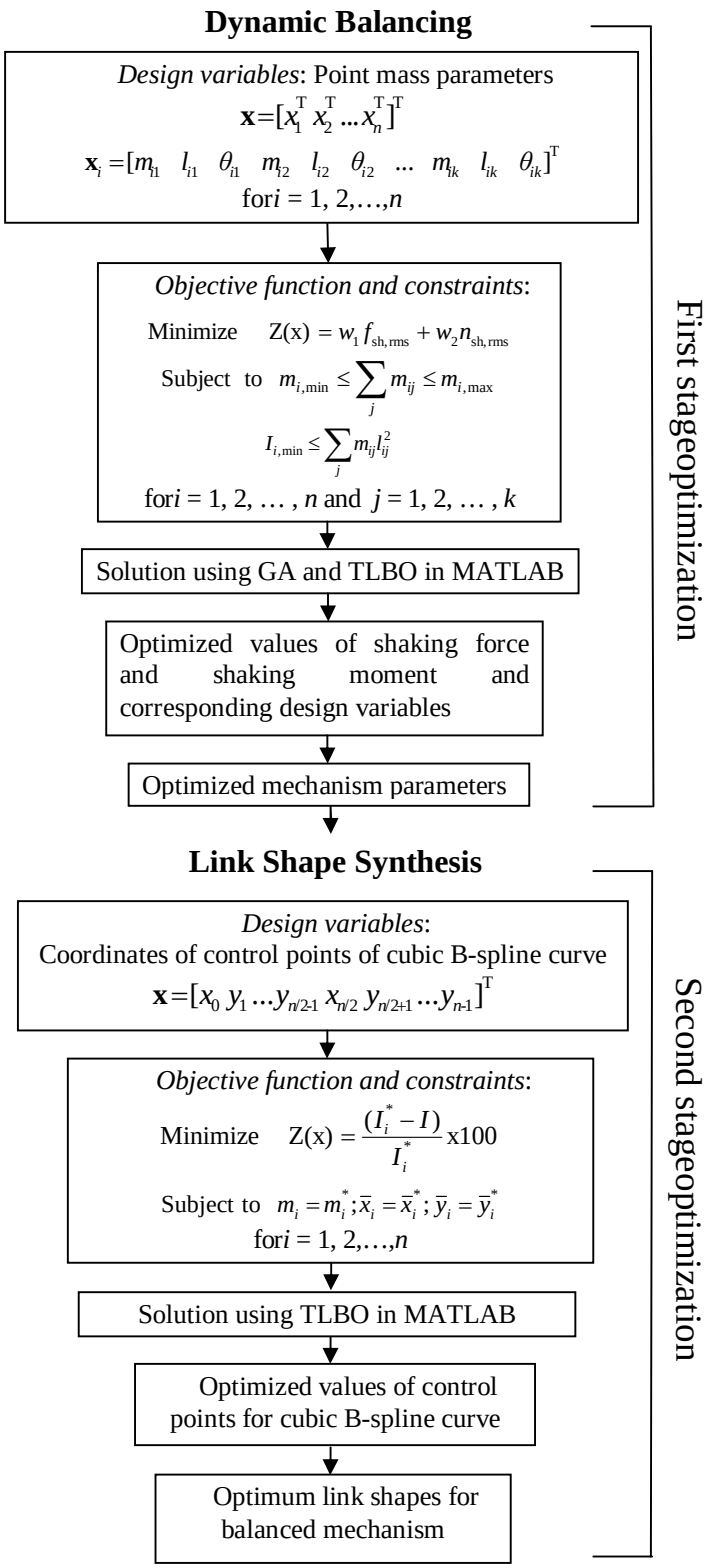


Fig. 4 Two stage optimization scheme to balance mechanism and shape synthesis

The lengths a_i, x_0 and x_{13} are divided each into equal parts which decide the x-coordinates of control points. So, these x-coordinates are given as follows:

$x_1 = a_i + x_0$; $x_2 = a_i + 0.75x_0$; $x_3 = a_i + 0.50x_0$; $x_4 = a_i + 0.25x_0$; $x_5 = a_i$;
$x_6 = 0.75a_i$; $x_7 = 0.50a_i$; $x_8 = 0.25a_i$; $x_9 = 0$;
$x_{10} = -0.25x_{13}$; $x_{11} = -0.50x_{13}$; $x_{12} = -0.75x_{13}$; $x_{13} = -x_{13}$;
$x_{14} = -x_{13}$; $x_{15} = -0.75x_{13}$; $x_{16} = -0.50x_{13}$; $x_{17} = -0.25x_{13}$;
$x_{18} = 0$; $x_{19} = 0.25a_i$; $x_{20} = 0.50a_i$; $x_{21} = 0.75a_i$;
$x_{22} = a_i$; $x_{23} = a_i + 0.25x_0$; $x_{24} = a_i + 0.50x_0$; $x_{25} = a_i + 0.75x_0$; $x_{26} = a_i + x_0$.

Moreover, the symmetrical link shapes can be obtained by controlling the y-coordinates as:

$$y_{14} = -y_{13} \qquad y_{21} = -y_6$$
$$y_{15} = -y_{12} \qquad y_{22} = -y_5$$
$$y_{16} = -y_{11} \qquad y_{23} = -y_4$$
$$y_{17} = -y_{10} \qquad y_{24} = -y_3$$
$$y_{18} = -y_9 \qquad y_{25} = -y_2$$
$$y_{19} = -y_8 \qquad y_{26} = -y_1$$
$$y_{20} = -y_7$$

Note that lengths x_0 and x_{13} are variables while a_i is the length of the ith link.

V. CONCLUSIONS

The physically possible shapes are constructed for the optimal inertial parameters of the mechanism links and the given kinematic structure. The percentage error of resulting link inertia values defined as the objective function was found within ± 5 percent. Thus, the two-stage optimization formulation including the dynamic balancing and the dynamics of mechanism has been brought in shape synthesis of links.The benefit associated with the proposed method is that the links of balanced mechanism are of the uniform thickness while the force and inertia counterweights added to the original mechanisms in traditional methods (Berkof, 1973; Farmani et al., 2011; Berkof and Lowen, 1969)are of large thickness and radius compared to the original link parameters. Also, the proposed method doesn't require any pre-defined shapes or design domain to start withas suggested in (Farmani et al., 2011; Verschuure et al., 2007).The resulting stresses for links of the balanced mechanism can be calculated at the weakest sections under external loads.

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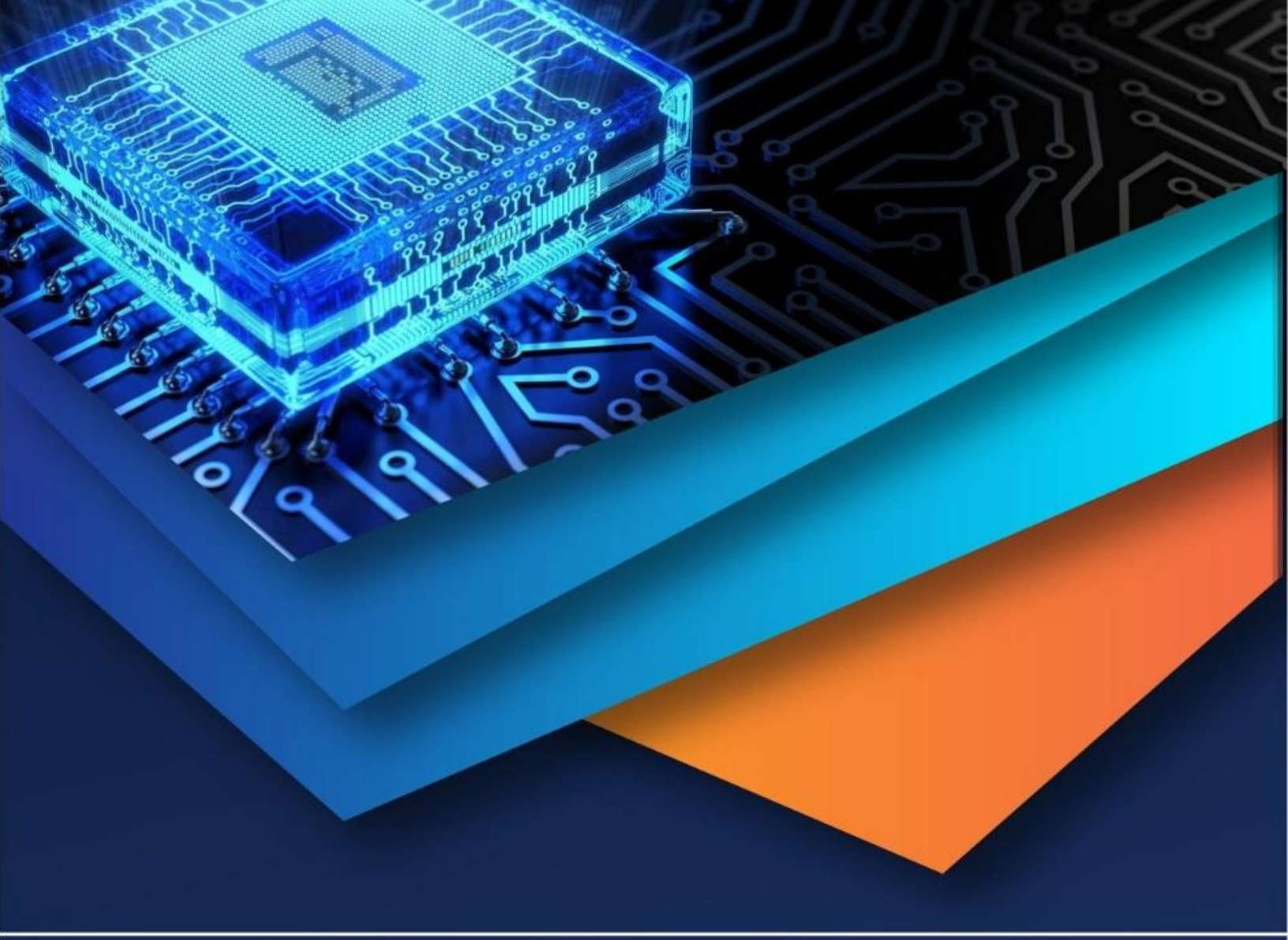

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