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# Detection of Isomorphism among Kinematic Chains 

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#### Abstract

This paper presents a new of method determining the isomorphic kinematic chains during the design stage of the mechanisms. A new invariant SAEV is introduced for representing the kinematic chains uniquely. The invariant SAEV is the sum of absolute values of the eigenvectors of the chain identification matrix (CIM) obtained from the kinematic graph of the chain; isomorphic chains have the same values of SAEV. The method is applied to every known case of isomorphic chains some of them are shown in detail in this paper to prove the worth of this method.


Keywords: Isomorphism, kinematic chains, chain identification matrix, eigenvectors, kinematic graph

## I. INTRODUCTION

The literature pertaining to representation of kinematic chains in matrix form and determination of isomorphism using the properties of these matrices is abundantly available like Wilson (1972) [1] gives a weighted adjacency matrix by numerically assigning the weights for different types of the joints in the kinematic chains, Uicker and Raicu (1975) [2] used the adjacency matrix representing the pattern of connection among the links of the kinematic chains, Mruthyunjaya and Raghavan (1979) [3] introduced a new matrix based on the degrees of the links of the kinematic chains and the characteristics coefficients of the polynomial of this matrix serves as an indicator of isomorphism, Cubillo and Wan (2005) [4] uses Eigenvalues and eigenvectors of the adjacency matrices for the detection of isomorphism, for relating the input and output of a mechanism Zhang and Li (1999) [5] combines the motion property to the adjacency matrix. Twang, Chao and Chu (1988) [6] introduced a weighted graph's matrix known as combinatorial matrix for representing the chains with gears and lower pairs . various other matrices like hamming distance matrix by Rao A.C.(1988) [7], joint-joint matrix by Khan (2007) [8], canonical adjacency matrix by Ding and Huang(2007) [9], flow matrix by Rao and Rao (1996) [10] ,stratified adjacency matrix by Butcher (2005) [11] physical connectivity matrix by Ali Hasan (2007) [12] incident matrix Yang (2012) [13]. all the above matrices are used to represent kinematic chains and their spectral properties are used to eliminate the possible isomer among, but they fail in some cases, some are difficult to grasp, some require large calculations and some are working only up to ten link chains.
The author also introduced three different matrices in the previous work i.e. inversion adjacency matrix(IAM)[14],chain identification matrix(CIM)[15],link identity matrix(LI)[16] all these matrices are capable of identifying isomorphism among kinematic chains with no restriction on number of links and also their derived distinct mechanisms. Here in this paper the author uses the chain identification matrix (CIM) for introducing a new invariant for isomorphism detection among kinematic chains.

## II. CHAIN IDENTIFICATION MATRIX (CIM)

A chain identification matrix (CIM) of a kinematic chain is a unique representation that defines a chain completely, but before obtaining the matrix the following definitions should be kept in mind.

## A. Link

Each component of the kinematic chain is a single link provided that it has a relative motion with the other, but if two components are rigidly fixed they are not considered as two different links they are treated as a single link. The type of the link depends upon the connections they have with the other links if a link is connected with two other links then it is a binary link, if it has three connections then it is termed as a ternary link and so on.

## B. Degree of a Link

The degree of a binary link is 2 , ternary link is 3 and the quaternary link is 4 and so on
CI matrix of the selected kinematic chain is obtained as explained below

1) $\operatorname{CIM}(\mathrm{i}, \mathrm{j})=0$ if $\mathrm{i}^{\text {th }}$ link is connected to $\mathrm{j}^{\text {th }}$ link
2) $\operatorname{CIM}(\mathrm{i}, \mathrm{j})=\mathrm{n}$, where ' n ' is the number of links commonly connected to both $\mathrm{i}^{\text {th }}$ and $\mathrm{j}^{\text {th }}$ links
3) $\operatorname{CIM}(\mathrm{i}, \mathrm{j})=$ degree of that link, if $\mathrm{i}=\mathrm{j}$,
4) $\mathrm{CIM}(\mathrm{i}, \mathrm{j})=0$ if it ${ }^{\text {th }}$ link and $\mathrm{j}^{\text {th }}$ link are not commonly connected to any other link of the chain.

## III.ISOMORPHISM IDENTIFICATION

The two kinematic chains are isomorphic if the sum of absolute values of eigenvectors (SAEV) of chain identification matrices (CIM) of the two chains. The invariant SAEV serves as an indicator for isomorphism for the chains under consideration. The following procedure is used to check the isomorphism.

1) Write the CIM matrices for the chains to be checked for isomorphism as explained in the section II.
2) Now use MATLAB for finding the eigenvectors for the CIM matrices of the chains.
3) Obtain the structural invariant SAEV by summing the absolute values of eigenvectors.
4) If the values of SAEV comes out to be same the chains are isomorphic or otherwise

The above method is now being tested for the following known cases available in the literature to show the worth of the new method developed for this paper

## A. Example-1

A pair six link non-isomorphic chains (Watt's and Stephenson's chain) shown in figure -1 are tested for isomorphism.


Fig -1 kinematic graphs of Watt's and Stephenson's chain
The CIM matrices for both the chains shown in Fig-1 are written as

$$
=\mathrm{CIM}_{1(\mathrm{a})}\left|\begin{array}{cccccc}
3 & 0 & 2 & 0 & 2 & 0 \\
0 & 2 & 0 & 2 & 0 & 1 \\
2 & 0 & 2 & 0 & 1 & 0 \\
0 & 2 & 0 & 3 & 0 & 2 \\
2 & 0 & 1 & 0 & 2 & 0 \\
0 & 1 & 0 & 2 & 0 & 2
\end{array}\right| \text { and } \mathrm{CIM}_{1(\mathrm{~b})}=\left|\begin{array}{cccccc}
3 & 0 & 2 & 0 & 1 & 0 \\
0 & 2 & 0 & 2 & 1 & 1 \\
2 & 0 & 3 & 0 & 0 & 1 \\
0 & 2 & 0 & 2 & 1 & 1 \\
1 & 1 & 0 & 1 & 2 & 0 \\
0 & 1 & 1 & 1 & 0 & 2
\end{array}\right|
$$

The absolute eigenvector matrices of $\mathrm{CIM}_{1(\mathrm{a})}$ and $\mathrm{CIM}_{2(\mathrm{~b})}$ are $\mathrm{EV}_{1(\mathrm{a})}$ and $\mathrm{EV}_{1(\mathrm{~b})}$ obtained by using MATLAB

$$
\begin{gathered}
\mathrm{EV}_{1(\mathrm{a})}=\left|\begin{array}{cccccc}
0.6992 & 0.1055 & 0.0000 & 0.0000 & 0.7071 & 0.0000 \\
0.0746 & 0.4944 & 0.0207 & 0.7068 & 0.0000 & 0.5000 \\
0.4944 & 0.0746 & 0.7068 & 0.0207 & 0.5000 & 0.0000 \\
0.1055 & 0.6992 & 0.0000 & 0.0000 & 0.0000 & 0.7071 \\
0.4944 & 0.0746 & 0.7068 & 0.0207 & 0.5000 & 0.0000 \\
0.0746 & 0.4944 & 0.0207 & 0.7068 & 0.0000 & 0.5000
\end{array}\right| \\
\mathrm{EV}_{1(\mathrm{~b})}=\left|\begin{array}{llllll} 
& & & & & \\
0.0000 & 0.6015 & 0.1359 & 0.3717 & 0.5059 & 0.4750 \\
0.7071 & 0.0000 & 0.3516 & 0.0000 & 0.4675 & 0.3973 \\
0.0000 & 0.6015 & 0.1359 & 0.3717 & 0.5059 & 0.4750 \\
0.7071 & 0.0000 & 0.3516 & 0.0000 & 0.4675 & 0.3973 \\
0.0000 & 0.3717 & 0.5982 & 0.6015 & 0.1599 & 0.3414 \\
0.0000 & 0.3717 & 0.5982 & 0.6015 & 0.1599 & 0.3414
\end{array}\right|
\end{gathered}
$$

The invariant $\mathrm{SAEV}_{1(\mathrm{a})}=10.2098$ and $\mathrm{SAEV}_{(\mathrm{b})}=12.1725$ are obtained by summing all the elements of matrices $\mathrm{EV}_{1(\mathrm{a})}$ and $\mathrm{EV}_{(\mathrm{b})}$. As the condition for isomorphism states that the invariant SAEV should be same for the chains to be tested it concludes that the
chains shown in fig-1 are non-isomorphic as the values of the SAEV for both the chains are different. The available literature [17] also supports the fact that the Watt's and Stephenson's chain with same link assortment are non-isomorphic.

## B. Example - 2

The two kinematic chains with 10 links, single degree of freedom as shown in Fig-2. The task is to examine whether these two chains are isomorphic.


Fig-2 kinematic graphs of 10 link single degree of freedom isomorphic chains

The CIM matrices and corresponding eigenvector matrices for both the chains shown in Fig-2 are written below

$$
\begin{aligned}
& \mathrm{CIM}_{2(a)}=\left|\begin{array}{llllllllll}
4 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 \\
0 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 2 & 0 \\
2 & 0 & 3 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\
0 & 1 & 0 & 3 & 0 & 1 & 0 & 0 & 2 & 0 \\
0 & 0 & 1 & 0 & 2 & 0 & 1 & 1 & 0 & 1 \\
2 & 0 & 0 & 1 & 0 & 3 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 \\
0 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 3 & 0 \\
1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 2
\end{array}\right| \\
& \mathrm{CIM}_{2(b)}=\left|\begin{array}{llllllllll}
4 & 1 & 1 & 0 & 0 & 2 & 1 & 1 & 1 & 2 \\
1 & 3 & 1 & 2 & 0 & 0 & 1 & 1 & 2 & 1 \\
1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 2 \\
0 & 2 & 1 & 3 & 1 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 1 \\
2 & 0 & 0 & 1 & 1 & 3 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 0 \\
1 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 2 & 1 \\
2 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 1 & 3
\end{array}\right| \\
& \mathrm{EV}_{2(\mathrm{a})}=\left|\begin{array}{llllllllll}
0.0000 & 0.3816 & 0.2544 & 0.0823 & 0.2780 & 0.3927 & 0.2492 & 0.2241 & 0.4878 & 0.4484 \\
0.0000 & 0.2635 & 0.0528 & 0.5468 & 0.5612 & 0.3274 & 0.0654 & 0.0184 & 0.3354 & 0.2993 \\
0.0000 & 0.4764 & 0.2847 & 0.3978 & 0.1790 & 0.0806 & 0.3683 & 0.2645 & 0.3936 & 0.3670 \\
0.0000 & 0.3576 & 0.2786 & 0.0894 & 0.3500 & 0.4373 & 0.3571 & 0.4265 & 0.2626 & 0.3071 \\
0.0000 & 0.0913 & 0.6368 & 0.2039 & 0.0462 & 0.5672 & 0.0505 & 0.4148 & 0.0305 & 0.2126 \\
0.0000 & 0.3904 & 0.0862 & 0.0282 & 0.3876 & 0.3827 & 0.4491 & 0.4833 & 0.1825 & 0.2721 \\
0.7071 & 0.0246 & 0.2937 & 0.0938 & 0.1347 & 0.0069 & 0.3835 & 0.3226 & 0.2550 & 0.2645
\end{array}\right|
\end{aligned}
$$

$$
\mathrm{EV}_{2(\mathrm{~b})}=\left|\begin{array}{llllllllll|}
0.7071 & 0.0246 & 0.2937 & 0.0938 & 0.1347 & 0.0069 & 0.3835 & 0.3226 & 0.2550 & 0.2645 \\
0.0000 & 0.4499 & 0.3889 & 0.4067 & 0.0654 & 0.2265 & 0.2480 & 0.1351 & 0.4366 & 0.3937 \\
0.0000 & 0.2565 & 0.1925 & 0.5501 & 0.5102 & 0.1458 & 0.3324 & 0.2561 & 0.2660 & 0.2572
\end{array}\right|
$$

The invariant $\mathrm{SAEV}_{2(a)}=26.3534$ and $\mathrm{SAEV}_{2(\mathrm{~b})}=26.3534$ are obtained by summing all the elements of matrices $\mathrm{EV}_{2(\mathrm{a})}$ and $\mathrm{EV}_{2(\mathrm{~b})}$. The values of SAEV for the chains shown in fig- 2 are same which indicates that the chains are isomorphic as already proved isomorphic by the author [18] and other researchers also[19]

## C. Example 2

A pair eight link isomorphic chains shown in fig-3 are tested for isomorphism.

(a)

(b)

Fig-3 kinematic graphs of eight link isomorphic chains
The CIM matrices and corresponding eigenvector matrices for both the chains shown in Fig-3 are written below

$$
\begin{aligned}
\mathrm{CIM}_{3(a)} & =\left|\begin{array}{llllllll}
3 & 0 & 1 & 0 & 1 & 0 & 0 & 2 \\
0 & 2 & 0 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 3 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 2 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 2 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 3 & 2 & 0 \\
0 & 1 & 1 & 0 & 0 & 2 & 2 & 0 \\
2 & 1 & 0 & 1 & 1 & 0 & 0 & 3
\end{array}\right| \\
\mathrm{CIM}_{3(b)} & =\left|\begin{array}{llllllll}
3 & 0 & 2 & 0 & 0 & 1 & 1 & 1 \\
0 & 3 & 0 & 2 & 1 & 1 & 0 & 1 \\
2 & 0 & 2 & 0 & 0 & 0 & 1 & 1 \\
0 & 2 & 0 & 3 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 2 & 0 & 1 & 0
\end{array}\right|
\end{aligned}
$$

$$
\left.\begin{array}{llllllllll|ll|} 
& & \left|\begin{array}{cccccccc}
1 & 1 & 0 & 0 & 0 & 2 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 0 & 3 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 2
\end{array}\right| & \\
\mathrm{EV}_{3(\mathrm{a})}=\left|\begin{array}{llllllll}
0.1757 & 0.4664 & 0.0000 & 0.3889 & 0.4129 & 0.1644 & 0.4887 & 0.4051 \\
0.1975 & 0.4734 & 0.5000 & 0.4408 & 0.0414 & 0.4094 & 0.1780 & 0.3026 \\
0.1560 & 0.1830 & 0.5000 & 0.1139 & 0.3601 & 0.6316 & 0.0825 & 0.3793 \\
0.4532 & 0.0472 & 0.0000 & 0.4084 & 0.5986 & 0.4476 & 0.0805 & 0.2459 \\
0.1107 & 0.2112 & 0.5000 & 0.5797 & 0.4032 & 0.2387 & 0.2662 & 0.2583 \\
0.4642 & 0.0793 & 0.5000 & 0.2528 & 0.0845 & 0.0165 & 0.5268 & 0.4236 \\
0.6442 & 0.2735 & 0.0000 & 0.2555 & 0.4006 & 0.1063 & 0.4101 & 0.3239 \\
0.2401 & 0.6301 & 0.0000 & 0.0975 & 0.0980 & 0.3710 & 0.4468 & 0.4347
\end{array}\right| \\
& & & & & & & \\
0.4642 & 0.0793 & 0.5000 & 0.2528 & 0.0845 & 0.0165 & 0.5268 & 0.4236 \\
0.2401 & 0.6301 & 0.0000 & 0.0975 & 0.0980 & 0.3710 & 0.4468 & 0.4347 \\
0.6442 & 0.2735 & 0.0000 & 0.2555 & 0.4006 & 0.1063 & 0.4101 & 0.3239 \\
0.1757 & 0.4664 & 0.0000 & 0.3889 & 0.4129 & 0.1644 & 0.4887 & 0.4051 \\
0.1107 & 0.2112 & 0.5000 & 0.5797 & 0.4032 & 0.2387 & 0.2662 & 0.2583 \\
0.4532 & 0.0472 & 0.0000 & 0.4084 & 0.5986 & 0.4476 & 0.0805 & 0.2459 \\
0.1560 & 0.1830 & 0.5000 & 0.1139 & 0.3601 & 0.6316 & 0.0825 & 0.3793 \\
0.1975 & 0.4734 & 0.5000 & 0.4408 & 0.0414 & 0.4094 & 0.1780 & 0.3026
\end{array} \right\rvert\,
$$

The invariant $\operatorname{SAEV}_{3(\mathrm{a})}=19.3806$ and $\mathrm{SAEV}_{3(\mathrm{~b})}=19.3806$ are obtained by summing all the elements of matrices $\mathrm{EV}_{3(\mathrm{a})}$ and $\mathrm{EV}_{3(\mathrm{~b})}$. The values of SAEV for the chains shown in fig-3 are same which indicates that the chains are isomorphic as already proved isomorphic in the literature [20]

## IV.RESULT

The method is applied for developing the 16 kinematic chains of 8-links shown in Appendix-I and the results is depicted in the table below.

TABLE I: INVARIANT SAEV FOR ALL SINGLE DEGREE OF FREEDOM 8-LINK CHAINS

| Chain No. | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SAEV | 19.8651 | 15.1557 | 18.8982 | 19.3621 | 16.2342 | 20.1904 | 19.3806 | 18.7792 |
| Chain No. | E9 | E10 | E11 | E12 | E13 | E14 | E15 | E16 |
| SAEV | 16.8957 | 20.2107 | 15.8447 | 16.1110 | 18.5167 | 19.1823 | 17.8143 | 17.1204 |

## V. CONCLUSION

The method discussed in this paper is proved worthy with the help of the three different examples which are available in literature, the structural invariant SAEV introduced in this paper not only checks isomorphism but gives a unique identity to every nonisomorphic chain and upon application it is found capable of developing all known 16 kinematic chain of 8 -links, 230 kinematic chain of 10 -links having 1-F, 98 kinematic chains of 10 -links having 3-F. The advantage of this method is that it can be easily converted into MATLAB code or any other programming language.

## REFERENCES

[1] Wilson R., Introduction to graph theory. 3rd edition, , Longman, 1972.
[2] Uicker J. J, Raicu A., "A method for the identification and recognition of equivalence of kinematic chains." Journal of Mechanism and Machine Theory, Vol. 10 (5), pp.375-383. 1975.
[3] Mruthyunjaya T.S, Raghavan M.R., "Structural analysis of kinematic chains and mechanisms based on matrix representation." Transaction of the ASME, Journal of Mechanical Design, Vol.101, pp. 488 - 518, 1979.
[4] Cubillo J.P. and Wan. J, "Comments on mechanism kinematic chain isomorphism identification using adjacent matrices." Journal of Mechanism and Machine Theory, Vol. 40(2), pp. 131-139, 2005.
[5] Zhang W. J, Li Q., "On a New approach to mechanism topology identification.." Journal of Mechanical Design ASME, Vol. 121(1), pp.57-64, 1999.
[6] Li TJ, Cao WQ, Chu JK., "The topological representation and detection of isomorphism among geared linkage kinematic chains." Proceedings of ASME Design Engineering Technical Conference, 1988.
[7] Rao AC., "Kinematic chains, isomorphism, inversions, and type of freedom using the concept of hamming distances." Indian J of Tech., Vol.26,pp.105109,1988.
[8] Khan R.A., Ali Hasan and Aas M., "Structural synthesis of planar kinematic chains using matrix." Journal of Intelligent System Research, Vol. 1(1) ,pp.11-23, 2007.
[9] Ding H, Huang Z., "The establishment of the canonical perimeter topological graph of kinematic chains and isomorphism identification.., Journal of Mechanical Design ASME, Vol. 129(9) pp. 915-923,2007.
[10] Rao C.N. and Rao A.C., "Selection of best frame, input and output links for function generators modeled a probabilistic system." Journal of Mechanism and Machine Theory, Vol. 31(7),pp. 973 -983, 1996.
[11] Butcher EA, Hartman C., "Efficient enumeration and hierarchical classification of planar simple -jointed kinematic chains: Application to12- and 14-bar single degree-of-freedom chains." Journal of Mechanism and Machine Theory, Vol. 40,pp.1030-1050, 2005.
[12] Hasan A., "Isomorphism and inversions of kinematic chains up to 10-links." 13th National Conference on Mechanisms and Machines IISc, Bangalore, India, December 12-13, 2007.
[13] Yang F., Deng Z., Tao J. and Li L., "A new method for isomorphism identification in topological graphs using incident matrices." Journal of Mechanism and Machine Theory, Vol. 49,pp. 298-307. 2012.
[14] Rizvi, Syed Shane Haider, Ali Hasan, and R. A. Khan., "An efficient algorithm for distinct inversions and isomorphism detection in kinematic chains." Elsever Journal Perspectives in Science Vol.8,pp.251-253, 2016.
[15] Rizvi, Syed Shane Haider, Ali Hasan, and R. A. Khan., "New Matrix Representation of Kinematic Chains and Determination of Isomorphism." Sch. J. Eng. Tech., Vol.2(4B),pp 561-565, 2014
[16] Rizvi, Syed Shane Haider, Ali Hasan, and R. A. Khan. "A new method for distinct inversions and isomorphism detection in kinematic chains", Int. J. Mechanisms and Robotic Systems, Vol. 3(1), pp.48-59, 2016.
[17] Rizvi, Syed Shane Haider, Ali Hasan, and R. A. Khan., "A New Concept to Detect Isomorphism in Kinematic Chains using Fuzzy Similarity Index." , Vol.86(12),pp 30-33, 2014.
[18] Rizvi, Syed Shane Haider. "A New Method Based on the Comparison of the Connection Strings of the Kinematic Chains to detect Isomorphism." Journal of Pure and Applied Science \& Technology. Vol. 3(1), pp. 94-100,2013.
[19] Kong FG, Li Q, Zhang WJ; An artificial neural network approach to mechanism kinematic chain isomorphism identification". Journal of Mechanism and Machine Theory,Vol.34(2), pp.271-283. 1999
[20] Rizvi, Syed Shane Haider, Ali Hasan, and R. A. Khan. "A New Method Based On the Comparison of the Unique Chain Code to Detect Isomorphism among Kinematic Chains." International Journal of Modern Engineering Research,. Vol 1(4): pp 16-21, 2014
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