Two Layered Model of Blood Flow Through a Stenosed Artery with Variable Viscosity in Porous Medium: Effects of External Magnetic Field

M. Chitra[1], D. Karthikeyan[2]

[1]*Associate Professor,* [2]*Research Scholar, Department of Mathematics, Thiruvalluvar University, Vellore-632 115, Tamilnadu, India*

Abstract: ***This paper deals with a two-layered mathematical model for blood flow through stenosed artery in porous medium under the effect of an applied magnetic field. The model consists of a core surrounded by a peripheral layer. It is assumed that fluids of both the region (core and peripheral) are Newtonian having different variable viscosities. For such models, in literature, the peripheral layer thickness and slip are assumed a priori based on experimental observations. The governing equation for laminar incompressible fully developed and Newtonian fluid by assuming slip boundary conditions is solved by using the Frobenius method. It is assumed that the surface roughness is cosine shaped and the maximum height of roughness is very small compared with radius of the unconstructed tube. The effect of hematocrit and Harmann number these parameters on velocity (U), volumetric flow rate (Q) and wall shear stress (τ) are obtained for both core region and peripheral region of the stenosed artery are computed graphically. The analysis developed here could be used to determine the more accurate values of the apparent variable viscosity of blood, agreeability, rigidity and deformability of red cells. This information of blood could be useful in the development of new diagnosis tools for many diseases.***

Key words : ***Stenosed artery, Two layer, Variable viscosity, Slip velocity, Blood flow, Magnetic field, Frobenius method***.

I. INTRODUCTION

Many cardiovascular diseases such as due to the arterial occlusion is one of the leading cause of death world wide. It is known that stenosis is a dangerous disease and is caused due to the abnormal growth in the lumen of the arterial wall. The partial occlusion of the arteries due to stenotic obstruction not only restricts the regular blood flow but also characterizes the hardening and thickening of the arterial wall. However, the main cause of the formation of stenosis is still unknown but it is well established that the fluid dynamical factors play an important role as to further development of stenosis. The presence of a constriction (medically called stenosis) in the lumen of an artery disturbs the normal blood flow and causes arterial diseases (myocardial infarction and cerebral strokes). It is known that hydrodynamic factors (e.g. wall shear stress) play a pivotal role in the development and progression of arterial stenosis. Hematocrit is the most important determinant of whole blood viscosity. Therefore, blood viscosity and vascular resistance affect total peripheral resistance to blood flow, which is abnormally high in the primary stage of hypertension. Again hematocrit is a blood test that measures the percentage of red blood cells present in the whole blood of the body. The percentage of red blood cells blood cells may as in adult human body is approximately 40-45%. Red blood cells may affect the viscosity of whole blood and thus the velocity distribution depends on the hematocrit. So blood can not be considered as homogeneous fluid. Due to the high shear rate near the arterial wall, the viscosity of blood is low and the concentration of red blood cells is high in the central core region.

There are many evidences that non-Newtonian behavior of fluids and the flow type (laminar and turbulence) are responsible for bringing a rapid change in hydrodynamic factors [4,5]. It is further understood that the hydrodynamic factors are influenced by the presence of plasma layer near the arterial wall, ratio between viscosities of blood and plasma and slip velocity at the arterial wall and hence that the mathematical modeling of blood flow through a stenosed artery is very important in view of developing the analytic formulas for computing plasma layer thickness, core viscosity and slip velocity at the arterial wall. In these models, the flow of blood is represented by one-layered model. Bugliarello and Sevilla [7] and Bugliarello and Hayden. Lee and Fung [6] have obtained the numerical results for the streamlines and distribution of velocity, pressure, vorticity and the shear stress for different Reynolds number in blood flow through locally constricted tubes. Many authors [10, 11]have taken two-layered models and analyzed the influence of peripheral plasma viscosity on flow characteristics. Srivastava and Srivastava [1], R. Ponalagusamy, R. Tamil Selvi [12] Srivastava et al. [2] and Srivastava and Rastogi [3] have considered the flow of blood represented by a two-layered model. Srivastava and Saxena [9] have considered a two layered model (Casson–Newtonian) and presented the analytic expressions for velocity profiles, flow rates, wall shear stress and resistance to flow which are the same derived by Chaturani and

©IJRASET (UGC Approved Journal): All Rights are Reserved

Ponalagusamy [6]. In all these models, the peripheral layer thickness is assumed a priori. It is, therefore, of interest to obtain an analytic expression for the calculation of peripheral layer thickness. The study pertains to a situation in which the variable viscosity of blood depending upon hematacrit is taken into consideration. It is assumed that the arterial segment is cylindrical tube and governing equations are solved by using Frobenius method. The focus of this investigation is to obtain analytical expression for peripheral layer thickness and core viscosity in terms of measurable flow variables. The effect of Hartmann number, magnetic field and maximum Hematocrit at the center of the arterial segment on velocity profile, volumetric flow rate and wall shear stress are computed graphically and results are discussed through graphs.

II. FORMULATION OF THE PROBLEM

Consider an axially symmetric, steady imcompressible laminar and fully developed flow of blood through an arterial stenosis as shown in (Fig. 1). Here the flow of blood is represented by a two-layered model (a core of red blood cell suspension surrounded by a peripheral layer of plasma with variable viscosity (Fig. 1)). It is assumed that the rheology of blood in the core region and the peripheral layer has been characterized as a Newtonian fluid. We shall take the cylindrical coordinate system $(\bar{z}, \bar{r}, \bar{\theta})$ whose origin is located on the vessel (stenosed artery) axis.

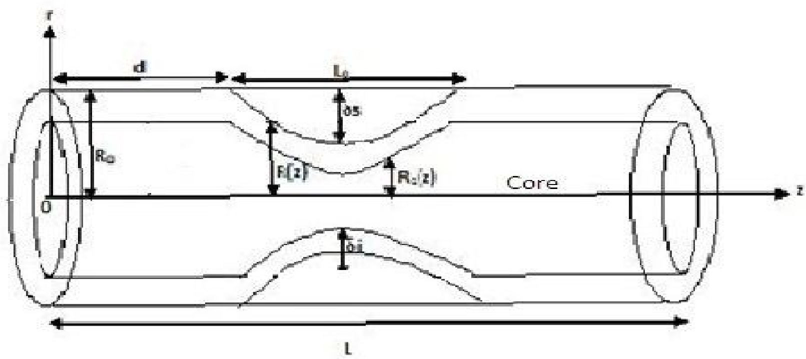


Fig. 1. Flow Geometry of stenosed artery.

The consistency function $\bar{\mu}(\bar{r})$ may be written as

$$\bar{\mu}(\bar{r}) = \bar{\mu}_c \text{ for } 0 \le \bar{r} \le \bar{R}_1(\bar{z}) \quad (1)$$

$$= \bar{\mu}_p \text{ for } \bar{R}_1(\bar{z}) \le \bar{r} \le \bar{R}(\bar{z}) \quad (2)$$

Where $\bar{\mu}_c$ and $\bar{\mu}_p$ are the viscosities of the central core fluid and the plasma respectively and $\bar{R}_1(\bar{z}) \le \bar{r} \le \bar{R}(\bar{z})$ are the radii of the central core region and the artery in the stenotic region.

We assumed that blood is incompressible, suspension of erythrocytes in plasma and has uniform dense throughout but the viscosity μ(r) varies in the radial direction. According to Einstein's formula for the variable viscosity of blood taken to be

$$\mu(\mathrm{r}) = \mu_0\left[1 + \alpha \mathrm{h}(\mathrm{r})\right] \quad (3)$$

where μ_0 is the coefficient of viscosity of plasma, α is a constant and h(r) stands for the hematocrit. The analysis will be carried out by using the flowing empirical forula for hematocrit.

$$h(r) = H\left(1 - \left(\frac{r}{R_0}\right)^m\right) \quad (4)$$

in which R_0 represents the radius of a normal arterial segment, H is the maximum hematocrit at the center of the artery and (m≥2) a parameter that determines the exact shape of the velocity profile for blood. The shape of the hematocrit profile given by equation (4) is valid only for very dilute suspensions of erythrocytes, which are considered to be of spherical shape.

The non dimensional variables are

©IJRASET (UGC Approved Journal): All Rights are Reserved

$$r=\frac{\overline{r}}{R_0}, z=\frac{\overline{z}}{z_0}, R=\frac{\overline{R}}{R_0}, R_1=\frac{\overline{R_1}}{R_0}, p=\frac{\overline{p}}{\rho_p U_0^2}, u_c=\frac{\overline{\mu_c}}{U_0}, u_p=\frac{\overline{\mu_p}}{U_0}, v_c=\frac{\overline{v_c z_0}}{U_0\delta_0}, v_p=\frac{\overline{v_p z_0}}{U_0\delta_0}, \delta_s=\frac{\overline{\delta_s}}{R_0}$$

where $\overline{u}$ and $\overline{v}$ are velocity components in the axial $\overline{z}$ and radial $\overline{r}$ directions, $\overline{p}$ the pressure, $\overline{\rho}$ is the density, $\overline{R_0}$ is the radius of the normal artery, $\overline{z_c}$ the one-fourth length of the stenosis $\overline{L_0 U_0}$ the average region and $\overline{\delta_s}$ is the maximum height of the stenosis (Fig. 1). The quantities in the peripheral layer and in the central core are denoted by subscripts p and c, respectively '-over a letter denotes the corresponding dimensional quantity. As per discussion made by Young(1968), the appropriate equations describing the flow in the case of a mild stnosis $(\delta_s / R << 1)$, subject to the additional conditions (a) $\mathrm{Re}_p(\delta_s / L_0) << 1$ (b) $2R_0 / L_0 \sim o(1)$,

Central core region $0 \le r \le R_1(z)$,

The non-dimentional governing equations of motion in the core region is,

$$0=-\frac{\partial p}{\partial z}+\frac{\beta}{\mathrm{Re}_p\,\mu'(r)}\left[\frac{\partial^2 u_c}{\partial r^2}+\frac{1}{r}\frac{\partial u_c}{\partial r}\right]+\sigma_c B_0^2 u_c \tag{5}$$

$$0=-\frac{\partial p}{\partial r} \tag{6}$$

Peripheral region $R_1(z) \le r \le R(z)$,

The non-dimentional governing equations of motion in the peripheral region is,

$$0=-\frac{\partial p}{\partial z}+\frac{\beta}{\mathrm{Re}_p\,\mu'(r)}\left[\frac{\partial^2 u_p}{\partial r^2}+\frac{1}{r}\frac{\partial u_p}{\partial r}\right]+\sigma_p B_0^2 u_p+\frac{\mu(r)}{k}u_p \tag{7}$$

$$0=-\frac{\partial p}{\partial r} \tag{8}$$

The non-dimentional boundary conditions are

$$\left.\begin{array}{l} u_p = u_s \text{ at } r = R(z) \\ u_p = u_c \text{ at } r = R_1(z) \\ \tau_p = \tau_c \text{ at } r = R_1(z) \\ \dfrac{\partial u_c}{\partial r} = 0 \text{ at } r = 0 \\ \dfrac{\partial u_p}{\partial r} = 0 \text{ at } r = 0 \end{array}\right\} \tag{9}$$

where $\left(u_s=\frac{\overline{u_s}}{U_0}\right)$ is the non-dimensional slip velocity (axial) and τ is the shear stress. The geometry of the stenosis (non-dimensional form) is given by,

$$\frac{R(z)}{R_0}=\begin{cases} 1-\frac{\delta}{R_0}[1+\cos\frac{2\pi}{l_0}(z-d-\frac{l_0}{2})];\ when\ d \le z \le d+l_0 \\ 1 \qquad ;\ \text{otherwise} \end{cases} \tag{10}$$

©IJRASET (UGC Approved Journal): All Rights are Reserved

where Where R(z) is the radius of the artery in the stenotic region, R_0 is the radius of the normal artery, l_0 the length of stenosis, d length of non-stenosis and δ the maximum height of stenosis. Where $R_1(z)$ is the radius of the central core region of constricted artery.

III. METHOD OF SOLUTION

With the use equations (3) and (4), the governing equation (5) reduces to ,

$$\frac{\partial p}{\partial z}=\frac{z_0\mu_c(r)}{U_0R_0^2\rho_p}\left[\frac{\partial^2 u_c}{\partial r^2}+\frac{1}{r}\frac{\partial u_c}{\partial r}-\sigma_c B_0^2 u_c\right]$$

$$\frac{\partial p}{\partial z}=\frac{z_0\mu_c(r)}{U_0R_0^2\rho_p}\left[\frac{1}{r}\frac{\partial}{\partial r}r(a_1-a_2r^m)\frac{\partial u_c}{\partial r}-M_c^2u_c\right]$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}r(a_1-a_2r^m)\frac{\partial u_c}{\partial r}\right]-M_c^2u_c=\frac{R_0^2U_0\rho_p}{\mu_0z_0}\frac{\partial p}{\partial z} \tag{11}$$

With the use equations (3) and (4), the governing equation (7) reduces to,

$$\frac{\partial p}{\partial z}=\frac{z_0\mu_p(r)}{U_0R_0^2\rho_p}\left[\frac{\partial^2 u_p}{\partial r^2}+\frac{1}{r}\frac{\partial u_p}{\partial r}-\sigma_p B_0^2 u_p-\frac{\mu(r)}{k'}u_p\right]$$

$$\frac{\partial p}{\partial z}=\frac{z_0\mu_p(r)}{U_0R_0^2\rho_p}\left[\frac{1}{r}\frac{\partial}{\partial r}r(a_1-a_2r^m)\frac{\partial u_p}{\partial r}-M_p^2u_p-\frac{1}{k}(a_1-a_2r^m)u_p\right]$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}r(a_1-a_2r^m)\frac{\partial u_p}{\partial r}-M_p^2u_p-\frac{1}{k}(a_1-a_2r^m)u_p\right]=\frac{R_0^2U_0\rho_p}{\mu_0z_0}\frac{\partial p}{\partial z} \tag{12}$$

where $a_1=1+a_2$, $a_2=\alpha H$.

where $\mu'=\frac{\overline{\mu_p}}{\overline{\mu_c}},\beta=\frac{\overline{z_0}}{\overline{R_0}},\mathrm{Re}=\frac{\overline{U_0R_0\rho_p}}{\overline{\mu_p}},M_c^2=\frac{\sigma_c(B_0R_0)}{\mu_0},M_p^2=\frac{\sigma_p(B_0R_0)}{\mu_0}$ and $k=\frac{k'}{R_0^2}$ The Equations (11) and (12) can be solved subjected to the boundary conditions (9) using Frobenius method, Then only admissible series solution of the Equations (11) and (12) will exists and can put in the form

$$u_c=C_1\sum_{n=0}^{\infty}A_nr^n+\frac{R_0^2U_0\rho_p}{4a_1\mu_0z_0}\frac{dp}{dz}\sum_{n=0}^{\infty}B_nr^{n+2} \tag{13}$$

and

$$u_p=C_2\sum_{n=0}^{\infty}A_nr^n+\frac{R_0^2U_0\rho_p}{4a_1\mu_0z_0}\frac{dp}{dz}\sum_{n=0}^{\infty}B_nr^{n+2} \tag{14}$$

where C_1, C_2 , A_n and B_n are arbitrary constants.

To find the arbitrary constants C_1 and C_2 , we use the slip boundary condition (9) and obtained as

©IJRASET (UGC Approved Journal): All Rights are Reserved

$$C_1 = \frac{u_p - \frac{R_0^2 U_0 \rho_p}{4 a_1 \mu_0 z_0} \frac{dp}{dz} \sum_{n=0}^{\infty} B_n \left(\frac{R_1}{R_0}\right)^{n+2}}{\sum_{n=0}^{\infty} A_n \left(\frac{R_1}{R_0}\right)^n} \quad (15)$$

and

$$C_2 = \frac{u_s - \frac{R_0^2 U_0 \rho_p}{4 a_1 \mu_0 z_0} \frac{dp}{dz} \sum_{n=0}^{\infty} B_n \left(\frac{R}{R_0}\right)^{n+2}}{\sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^n} \quad (16)$$

Substituting the value of u_c and u_p from equations (13) and (14) in to equations (10) and (11) we get,

$$C_1\left[\sum_{n=0}^{\infty} n(n-1)(a_1 - a_2 r^m) A_n r^{n-2} + \sum_{n=0}^{\infty} n(a_1 - (m-1)a_2 r^m) A_n r^{n-2} - M_c^2 \sum_{n=0}^{\infty} A_n r^n\right] +$$
$$\frac{u_p - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz}\left[\sum_{n=0}^{\infty}(n+1)(n+2)(a_1 - a_2 r^m) B_n r^n + \sum_{n=0}^{\infty}(n+2)(a_1 - (m-1)a_2 r^m) B_n r^n - M_c^2 \sum_{n=0}^{\infty} B_n r^n\right] = \frac{u_p - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz} \quad (17)$$

and

$$C_2\left[\sum_{n=0}^{\infty} n(n-1)(a_1 - a_2 r^m) A_n r^{n-2} + \sum_{n=0}^{\infty} n(a_1 - (m-1)a_2 r^m) A_n r^{n-2} - \left(M_p^2 + \frac{a_1}{k}\right)\sum_{n=0}^{\infty} A_n r^n + \sum_{n=0}^{\infty}\frac{a_2}{k} A_n r^{n+m}\right] +$$
$$\frac{u_s - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz}\left[\sum_{n=0}^{\infty}(n+1)(n+2)(a_1 - a_2 r^m) B_n r^n + \sum_{n=0}^{\infty}(n+2)(a_1 - (m-1)a_2 r^m) B_n r^n - \left(M_p^2 + \frac{a_1}{k}\right)\sum_{n=0}^{\infty} B_n r^n + \sum_{n=0}^{\infty}\frac{a_2}{k} B_n r^{n+m}\right] \quad (18)$$
$$= \frac{u_s - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz}$$

Equating the coefficient of $C_1, C_2, \frac{u_p - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz}$ and $\frac{u_s - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz}$ in equation (15) and (16) we have

$$\left[\sum_{n=0}^{\infty} n(n-1)(a_1 - a_2 r^m) A_n r^{n-2} + \sum_{n=0}^{\infty} n(a_1 - (m-1)a_2 r^m) A_n r^{n-2} - M_c^2 \sum_{n=0}^{\infty} A_n r^n\right] = 0$$

and

$$\left[\sum_{n=0}^{\infty} n(n-1)(a_1 - a_2 r^m) A_n r^{n-2} + \sum_{n=0}^{\infty} n(a_1 - (m-1)a_2 r^m) A_n r^{n-2} - \left(M_p^2 + \frac{a_1}{k}\right)\sum_{n=0}^{\infty} A_n r^n + \sum_{n=0}^{\infty}\frac{a_2}{k} A_n r^{n+m}\right] = 0 \quad (19)$$

$$\frac{u_p - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz}\left[\sum_{n=0}^{\infty}(n+1)(n+2)(a_1 - a_2 r^m) B_n r^n + \sum_{n=0}^{\infty}(n+2)(a_1 - (m-1)a_2 r^m) B_n r^n - M_c^2 \sum_{n=0}^{\infty} B_n r^n\right] = 1 \quad (20)$$

$$\frac{u_s - R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0}\frac{dp}{dz}\left[\sum_{n=0}^{\infty}(n+1)(n+2)(a_1 - a_2 r^m) B_n r^n + \sum_{n=0}^{\infty}(n+2)(a_1 - (m-1)a_2 r^m) B_n r^n - \left(M_p^2 + \frac{a_1}{k}\right)\sum_{n=0}^{\infty} A_n r^n + \sum_{n=0}^{\infty}\frac{a_2}{k} A_n r^{n+m}\right] = 1 \quad (21)$$

Hence the constants A_n and B_n are obtained by equating the coefficients of r^{n+1} and r^{n-1} from both side of equations (17), (18) and (19) respectively, we get

©IJRASET (UGC Approved Journal): All Rights are Reserved

$$A_{n+1}=\frac{a_2(n+1)^2A_{n+1-m}-ma_2A_{n+1-m}+\left(M_c^2+M_p^2+\frac{a_1}{k}\right)A_{n-1}-\frac{a_2}{k}A_{n-1-m}}{a_1(1+n)^2} \tag{22}$$

$$B_{n+1}=\frac{a_2(3+n)^2-a_2mB_{n+1-m}+\left(M_c^2+M_p^2+\frac{a_1}{k}\right)B_{n-1}-\frac{a_2}{k}B_{n-1-m}}{a_1(3+n)^2} \tag{23}$$

with $A_0=B_0=1$ (24)

Substituting the expressions of C_1 and C_2 in the equations (13) and (14), we have the velocity profile for the arterial segment in the radial direction is

$$u_c=u_s+\frac{R_0^2U_0\rho_p}{4a_1\mu_0z_0}q(z)\left(\frac{\left[\sum_{n=0}^{\infty}B_n\left(\frac{R}{R_0}\right)^{n+2}\sum_{n=0}^{\infty}A_nr^n-\sum_{n=0}^{\infty}B_nr^{n+2}\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n\right]}{\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n}-\frac{\left[\sum_{n=0}^{\infty}B_n\left(\frac{R_1}{R_0}\right)^{n+2}\sum_{n=0}^{\infty}A_nr^n-\sum_{n=0}^{\infty}B_nr^{n+2}\sum_{n=0}^{\infty}A_n\left(\frac{R_1}{R_0}\right)^n\right]}{\sum_{n=0}^{\infty}A_n\left(\frac{R_1}{R_0}\right)^n}\right) \tag{25}$$

and

$$u_p=u_s+\frac{R_0^2U_0\rho_p}{4a_1\mu_0z_0}q(z)\frac{\left[\sum_{n=0}^{\infty}B_n\left(\frac{R}{R_0}\right)^{n+2}\sum_{n=0}^{\infty}A_nr^n-\sum_{n=0}^{\infty}B_nr^{n+2}\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n\right]}{\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n} \tag{26}$$

where $q(z)=-\frac{dp}{dz}$. The total velocity profile is $U=u_c+u_p$

$$U=2u_s+\frac{R_0^2U_0\rho_p}{4a_1\mu_0z_0}q(z)\left(2\frac{\left[\sum_{n=0}^{\infty}B_n\left(\frac{R}{R_0}\right)^{n+2}\sum_{n=0}^{\infty}A_nr^n-\sum_{n=0}^{\infty}B_nr^{n+2}\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n\right]}{\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n}-\frac{\left[\sum_{n=0}^{\infty}B_n\left(\frac{R_1}{R_0}\right)^{n+2}\sum_{n=0}^{\infty}A_nr^n-\sum_{n=0}^{\infty}B_nr^{n+2}\sum_{n=0}^{\infty}A_n\left(\frac{R_1}{R_0}\right)^n\right]}{\sum_{n=0}^{\infty}A_n\left(\frac{R_1}{R_0}\right)^n}\right) \tag{27}$$

The flow rate through the core region in the stenosed artery Q_c is defined as

$$Q_c=2\int_0^{R_1(z)}ru_cdr \tag{28}$$

which on using Equation (25), gives

$$Q_c=2\left(\frac{R_1}{R_0}\right)u_s+\frac{R_0^2U_0\rho_p}{2a_1\mu_0z_0}q(z)\left(\frac{\left[\sum_{n=0}^{\infty}B_n\left(\frac{R}{R_0}\right)^{n+2}\sum_{n=0}^{\infty}A_n\frac{\left(\frac{R_1}{R_0}\right)^{n+2}}{n+2}-\sum_{n=0}^{\infty}B_n\frac{\left(\frac{R_1}{R_0}\right)^{n+4}}{n+4}\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n\right]}{\sum_{n=0}^{\infty}A_n\left(\frac{R}{R_0}\right)^n}-\frac{\left[\sum_{n=0}^{\infty}B_n\left(\frac{R_1}{R_0}\right)^{n+2}\sum_{n=0}^{\infty}A_n\frac{\left(\frac{R_1}{R_0}\right)^{n+2}}{n+2}-\sum_{n=0}^{\infty}B_n\frac{\left(\frac{R_1}{R_0}\right)^{n+4}}{n+4}\sum_{n=0}^{\infty}A_n\left(\frac{R_1}{R_0}\right)^n\right]}{\sum_{n=0}^{\infty}A_n\left(\frac{R_1}{R_0}\right)^n}\right) \tag{29}$$

©IJRASET (UGC Approved Journal): All Rights are Reserved

Similarly, the flow rate through peripheral layer in the stenosed artery Q_p can be written as

$$Q_p = 2\int_{R_1(z)}^{R(z)} r u_c \, dr$$

$$Q_p = 2\left(\frac{R}{R_0}-\frac{R_1}{R_0}\right)u_s + \frac{R_0^2 U_0 \rho_p}{2a_1\mu_0 z_0} q(z) \frac{\left[\sum_{n=0}^{\infty} B_n \left(\frac{R}{R_0}\right)^{n+2} \sum_{n=0}^{\infty} A_n \frac{\left(\frac{R}{R_0}-\frac{R_1}{R_0}\right)^{n+2}}{n+2} - \sum_{n=0}^{\infty} B_n \frac{\left(\frac{R}{R_0}-\frac{R_1}{R_0}\right)^{n+4}}{n+4} \sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}\right]}{\sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}} \tag{30}$$

The total flow rate of the stenosed artery is $Q = Q_c + Q_p$

$$Q = 2\left(\frac{R_1}{R_0}\right)u_s + \frac{R_0^2 U_0 \rho_p}{2a_1\mu_0 z_0} q(z) \left(\frac{\left[\sum_{n=0}^{\infty} B_n \left(\frac{R}{R_0}\right)^{n+2} \sum_{n=0}^{\infty} A_n \frac{\left(\frac{R_1}{R_0}\right)^{n+2}}{n+2} - \sum_{n=0}^{\infty} B_n \frac{\left(\frac{R_1}{R_0}\right)^{n+4}}{n+4} \sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}\right]}{\sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}} - \frac{\left[\sum_{n=0}^{\infty} B_n \left(\frac{R_1}{R_0}\right)^{n+2} \sum_{n=0}^{\infty} A_n \frac{\left(\frac{R_1}{R_0}\right)^{n+2}}{n+2} - \sum_{n=0}^{\infty} B_n \frac{\left(\frac{R_1}{R_0}\right)^{n+4}}{n+4} \sum_{n=0}^{\infty} A_n \left(\frac{R_1}{R_0}\right)^{n}\right]}{\sum_{n=0}^{\infty} A_n \left(\frac{R_1}{R_0}\right)^{n}} \right) +$$

$$2\left(\frac{R}{R_0}-\frac{R_1}{R_0}\right)u_s + \frac{R_0^2 U_0 \rho_p}{2a_1\mu_0 z_0} q(z) \frac{\left[\sum_{n=0}^{\infty} B_n \left(\frac{R}{R_0}\right)^{n+2} \sum_{n=0}^{\infty} A_n \frac{\left(\frac{R}{R_0}-\frac{R_1}{R_0}\right)^{n+2}}{n+2} - \sum_{n=0}^{\infty} B_n \frac{\left(\frac{R}{R_0}-\frac{R_1}{R_0}\right)^{n+4}}{n+4} \sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}\right]}{\sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}} \tag{31}$$

The wall shear stress on the endothelial surface is given by

$$\tau = \left[-\mu(r)\frac{dU}{dr}\right]_{r=R(z)}$$

$$\tau = \frac{R_0^2 U_0 \rho_p}{4a_1\mu_0 z_0} q(z)\, 2 \left(\frac{\left[\sum_{n=0}^{\infty} B_n \left(\frac{R}{R_0}\right)^{n+2} \sum_{n=0}^{\infty} A_n n \left(\frac{R}{R_0}\right)^{n-1} - \sum_{n=0}^{\infty} B_n (n+2) \left(\frac{R}{R_0}\right)^{n+1} \sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}\right]}{\sum_{n=0}^{\infty} A_n \left(\frac{R}{R_0}\right)^{n}} - \frac{\left[\sum_{n=0}^{\infty} B_n \left(\frac{R_1}{R_0}\right)^{n+2} \sum_{n=0}^{\infty} A_n n \left(\frac{R}{R_0}\right)^{n-1} - \sum_{n=0}^{\infty} B_n (n+2) \left(\frac{R}{R_0}\right)^{n+1} \sum_{n=0}^{\infty} A_n \left(\frac{R_1}{R_0}\right)^{n}\right]}{\sum_{n=0}^{\infty} A_n \left(\frac{R_1}{R_0}\right)^{n}} \right) \tag{32}$$

©IJRASET (UGC Approved Journal): All Rights are Reserved

IV. RESULTS AND DISCUSSION

The analytical expression for velocity component, Volumetric flow rate and wall shear stress are obtained. The effect of magnetic field , Hartmann number and maximum Hematocrit at the center of the arterial segment on velocity, flow rate and pressure gradiant are computed graphically. In this section we are discuss the effect of various parameter on the flow characteristics graphically with the use of following numerical data which is applicable to blood.

d=0.25, H=0.2, m=2, M=2.5, α=2.5, l=0.5.

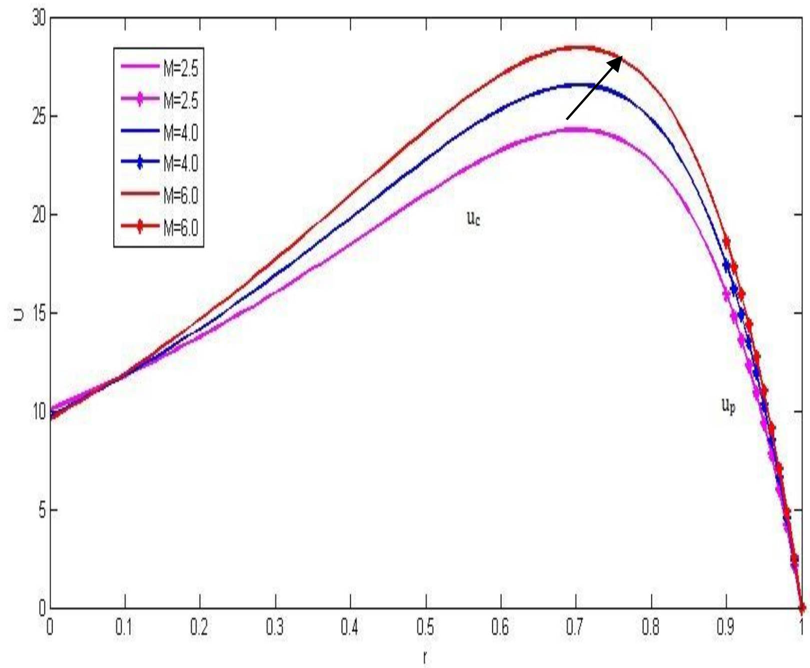


Fig. 2 :Velocity distribution length of the stenosis at z=2.5 with r for different values Hartmann number M, when hemotocirt H=0.2, frequency parameter α=2.5.

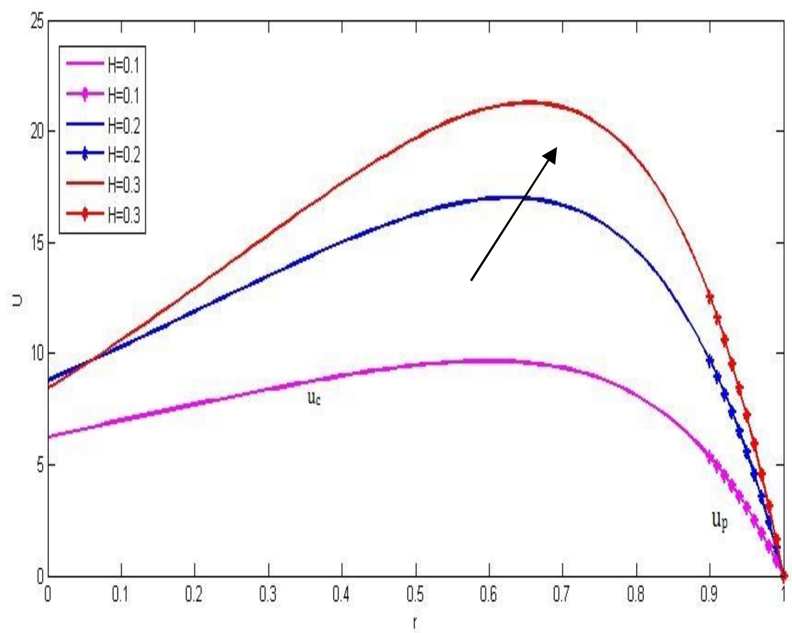


Fig. 3 : Velocity distribution length of the stenosis at z=2.5 with r for different values hematocrit H, when Hartmann number M=2.5, frequency parameter α=2.5

©IJRASET (UGC Approved Journal): All Rights are Reserved

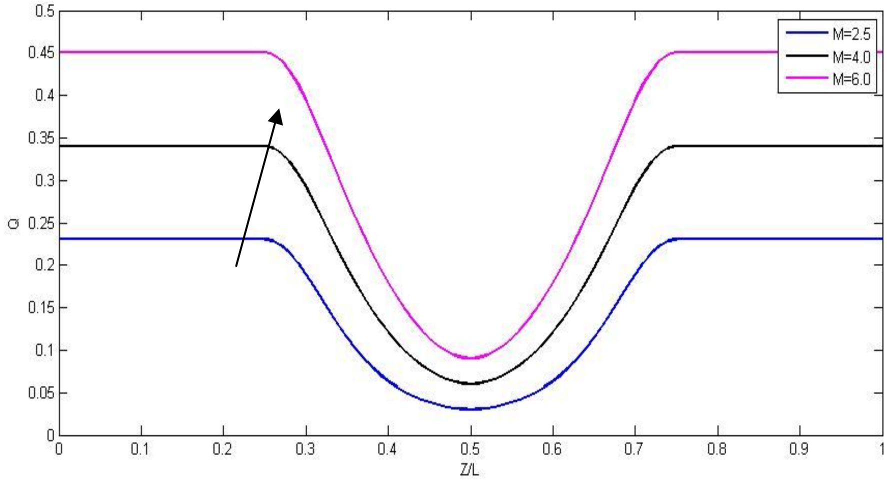


Fig. 4: Flow rate Q with varying length of the stenosis Z/L for different Hartmann number M, Hematocrit H=0.2, frequency parameter α=2.5.

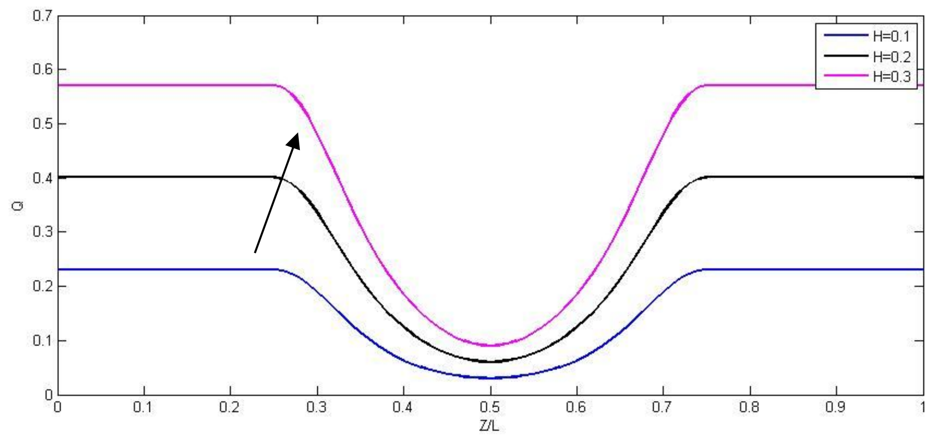
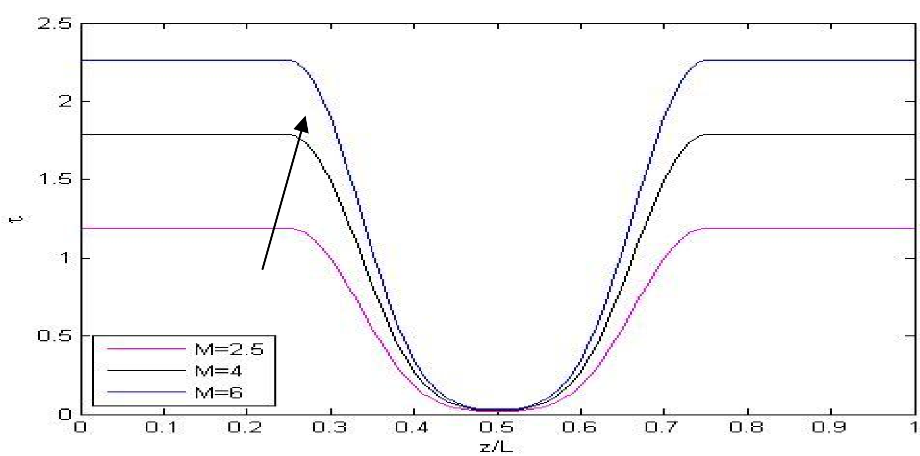


Fig. 5: Flow rate Q with varying length of the stenosis Z/L for different Hematocrit H, Hartmann number M=2.5, frequency parameter α=2.5.



6: Wall shear stress **τ** with varying length of the stenosis Z/L for different Hartmann number M, Hematocrit H=0.2, frequency parameter α=2.5.

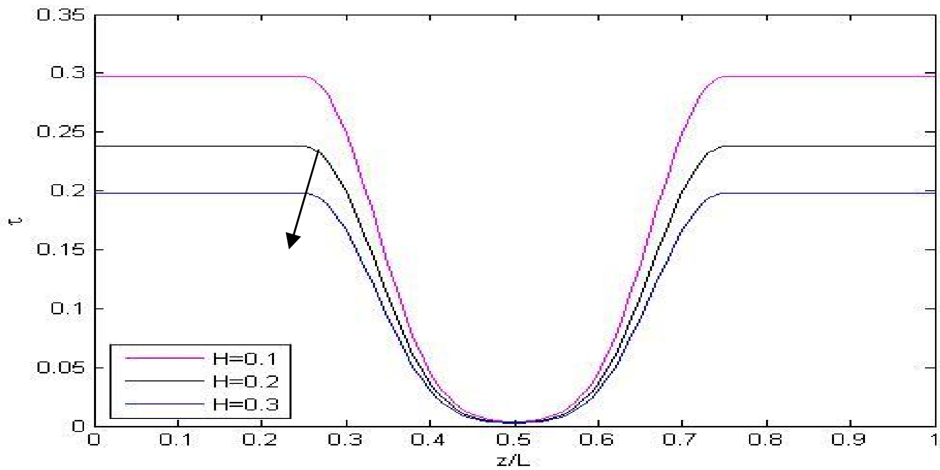


Fig. 7: Wall shear stress **τ** with varying length of the stenosis Z/L for different Hematocrit H, Hartmann number M=2.5, frequency parameter α=2.5.

Figure 2 & 3 shows that the axial velocity profile gradually increases with increase of Hartmann number M and Hematocrit H. This fact lies within the hematocrit as the blood viscosity is high in the stenosed artery due to the aggregation of blood cells rather than low viscosity in the plasma near the arterial wall. Figure 4 & 5 shows that the volume flow rate increases with the increase of Hartmann number and hematocrit H. Figure 6 and 7 gives the distribution of the wall shear stress for different values of the hematocrit H and Hartmann number M. We observe from figure 6 shows that wall shear stress increases as the increase of Hartmann number M. Figure 6 shows that wall shear stress decreases as the increase of Hematocrit H. One can note from these figures that pressure gradiant is low at the throat of the secondary stenosis as well as at downstream of the artery.

V. CONCLUSION

A two-layered model of blood flow through a stenosed artery with variable viscosity peripheral layer thickness and variable slip velocity at the wall has been considered. The model consists of a cored surrounded by a peripheral plasma layer. Both the fluids (core and peripheral layer) are assumed to be non-Newtonian having different viscosities. The analytic expressions for peripheral layer thickness and core region with variable viscosities of the slip velosity, flow rate and wall shear stress have been obtained (27, 31 and 32). A theoretical study of blood flow through a stenosed artery in the presence magnetic field has been carried out. In this study the variable viscosity of blood depending on hematocrit. The problem is solved analytically by using the Frobenius method. Finally we can conclude that further potential improvement of the model are anticipated. Since the hematocrit positively affects blood pressure, further study should examine the other factors such as diet, tobacco, smoking, overweight etc. from a cardiovascular point of view. Moreover on the basis of the present results, it can be concluded that the flow of blood and pressure can be controlled by the application of an external magnetic field. All the flow characteristics are found to be affected by the influence of applied magnetic field with profile velocity stenosis.

REFERENCES

[1] V.P. Srivastava, R. Srivastava, Particulate suspension blood flow through a narrow catheterized artery, Comput. Math. Appl. 58 (2009) 227–238
[2] V.P. Srivastava, R. Rastogi, Blood flow through a stenosed catheterized artery: effects of hematocrit and stenosis shape, Comput. Math. Appl. 59 (2010) 1377–1385
[3] V.P. Srivastava, R. Rastogi, R. Vishnoi, A two-layered suspension blood flow through an overlapping stenosis, Comput. Math. Appl. 60 (2010) 432–441
[4] Subramaniya Hariharan, On the normal stress effects of incompressible non-Newtonian fluids, J. Franklin Inst. 312 (1981) 109–118
[5] F.J. Testa, G. Rosen, Theory for incompressible fluid turbulence, J. Franklin Inst. 297 (1974) 127–133
[6] J.S. Lee, Y.C. Fung, Flow in locally constricted tubes at low Reynolds number, J. Appl. Mech.—Trans. ASME 37 (1970) 9–16.
[7] G. Bugliarello, J. Sevilla, Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes, Biorheology 7 (1970) 85–107
[8] P. Chaturani,R. Ponalagusamy, A two-layered model for blood flow through stenosed arteries, in:Proceedings of the 11th National Conference on Fluid Mechanics and Fluid Power, B.H.E.L(R&D),Hyderabad,India,1982,16–22
[9] V.P. Srivastava, M. Saxena, Two-layered model of Casson fluid flow through stenotic blood vessels: applications to cardiovascular system, J. Biomech. 27 (1994) 921–928
[10] J.B. Shukla, R.S. Parihar, S.P. Gupta, Biorhelogical aspects of blood flow through artery with mild stenosis: effects of peripheral layer, Biorheology 17 (1980) 403–410.
[11] J.B. Shukla, R.S. Parihar, S.P. Gupta, Effects of peripheral layer viscosity on blood flow through the arterywith mild stenosis, Bull. Math. Biol. 42 (1980) 797–805

©IJRASET (UGC Approved Journal): All Rights are Reserved

[12] Ponalagusamy, R. Tamil Selvi A study on two-layered model (Casson–Newtonian) for blood flow through an arterial stenosis: Axially variable slip velocity at the wall.Journal of the Franklin Institute 348 (2011) 2308–2321

[13] D. Philip, Peeyush Chandra, Flow of Eringen fluid (simple micro fluid) through an artery with mild stenosis, Int. J. Eng. Sci. 34 (1996) 87–99

[14] R. Ponalagusamy, Blood flow Through an Artery with Mild stenosis: A Two-layered Model, Different Shapes of Stenoses and Slip Velocity at the wall, Journal Applied sciences 7(7). 1071-1077, (2007). ISSN : 1812-5654.

[15] G.C. Shit, M.Roy and A. Sinha, Mathematical modeling of blood flow through a tapered overlapping stenosed artery with variable viscosity, Applied Bionics and Biomechanics 11 (2014) 185-195. DOI : 10.3233/ABB-140102.