Intuitionistic Fuzzy Hx Right and Left Ideal of a Hx Ring

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Abstract : In this paper, we define the notion of intuitionistic fuzzy HX right and eft ideal of a HX ring and some of their related properties are investigated. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an HX ring and discuss some of its properties. We introduce the concept of an image, pre-image of an intuitionistic fuzzy HX ring and discuss in detail a series of homomorphic and anti homomorphic properties of an intuitionistic fuzzy HX ring.
Keywords: intuitionistic fuzzy set, fuzzy HX ring, intuitionistic fuzzy HX right and left ideal, homomorphism and anti homomorphism of an intuitionistic fuzzy HX right and left ideal, image and pre-image of an intuitionistic fuzzy set.

I. INTRODUCTION

In 1965, Zadeh [14] introduced the concept of fuzzy subset μ of a set X as a function from X into the closed unit interval [0, 1] and studied their properties. Fuzzy set theory is a useful tool to describe situations in which the data or imprecise or vague and it is applied to logic, set theory, group theory, ring theory, real analysis, measure theory etc. In 1967, Rosenfeld [13] defined the idea of fuzzy subgroup and gave some of its properties. Li Hong Xing [5] introduced the concept of HX group. In 1982 Wang-jin Liu[7] introduced the concept of fuzzy ring and fuzzy ideal. With the successful upgrade of algebraic structure of group many researchers considered the algebraic structure of some other algebraic systems in which ring was considered as first. In 1988, Professor Li Hong Xing [6] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [2,3] gave the structures of HX ring on a class of ring. R.Muthuraj et.al[12]., introduced the concept of fuzzy HX ring. In this paper we define a new algebraic structure of an intuitionistic fuzzy HX right and left ideal of a HX ring and investigate some related properties. We define the necessity and possibility operators of an intuitionistic fuzzy subset of an intuitionistic fuzzy HX ring and discuss some of its properties. Also we introduce the image and pre-image of an intuitionistic fuzzy set in an intuitionistic fuzzy HX right and left ideal of a HX ring and discuss some of its properties.

II. PRELIMINARIES

In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, R = (R ,+, ·) is a Ring, e is the additive identity element of R and xy, we mean x.y

A. Definition

Let R be a ring. In $2^R - \{\phi\}$, a non-empty set $\vartheta \subset 2^R - \{\phi\}$ with two binary operation ' + ' and '.' is said to be a HX ring on R if ϑ is a ring with respect to the algebraic operation defined by

1) $A + B = \{a + b \,/\, a \in A \text{ and } b \in B\}$, which its null element is denoted by Q , and the negative element of A is denoted by – A.

2) $AB = \{ab \,/\, a \in A \text{ and } b \in B\}$,

3) $A (B + C) = AB + AC$ and $(B + C) A = BA + CA$.

B. Definition

Let R be a ring. Let μ be a fuzzy ring defined on R. Let $\vartheta \subset 2^R - \{\phi\}$ be a HX ring. A fuzzy subset λ^μ of ϑ is called a fuzzy HX ring on ϑ or a fuzzy ring induced by μ if the following conditions are satisfied. For all $A,B \in \vartheta$,

1) $\lambda^\mu (A - B) \geq \min \{ \lambda^\mu (A), \lambda^\mu (B) \}$,

2) $\lambda^\mu (AB) \geq \min \{ \lambda^\mu (A), \lambda^\mu (B) \}$

where $\lambda^\mu (A) = \max\{ \mu(x) \,/\, \text{ for all } x \in A \subseteq R \}$.

III. PROPERTIES OF AN INTUITIONISTIC FUZZY HX RIGHT AND LEFT IDEAL

A. Definition

Let R be a ring. Let H = { ⟨x, μ(x), η(x)⟩ / x∈R } be an intuitionistic fuzzy set defined on a ring R, where $\mu : R \to [0,1]$, $\eta : R \to [0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An intuitionistic fuzzy subset $\lambda^H = \{ \langle A, \lambda^\mu(A), \lambda^\eta(A)\rangle / A \in \Re$ and $0 \le \lambda^\mu(A) + \lambda^\eta(A) \le 1\}$ of $\Re$ is called an intuitionistic fuzzy HX right ideal on $\Re$ or an intuitionistic fuzzy right ideal induced by H if the following conditions are satisfied. For all A,B ∈$\Re$,

$\lambda^\mu(A - B) \ge \min\{ \lambda^\mu(A), \lambda^\mu(B)\}$,

$\lambda^\mu(AB) \ge \lambda^\mu(A)$

$\lambda^\eta(A - B) \le \max\{ \lambda^\eta(A), \lambda^\eta(B)\}$

$\lambda^\eta(AB) \le \lambda^\eta(A)$.

where $\lambda^\mu(A) = \max\{ \mu(x) /$ for all $x \in A \subseteq R\}$ and $\lambda^\eta(A) = \min\{\eta(x) /$ for all $x \in A \subseteq R\}$.

B. Definition

Let R be a ring. Let H = { ⟨x, μ(x), η(x)⟩ / x∈R } be an intuitionistic fuzzy set defined on a ring R, where $\mu : R \to [0,1]$, $\eta : R \to [0,1]$ such that $0 \le \mu(x) + \eta(x) \le 1$. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. An intuitionistic fuzzy subset $\lambda^H = \{ \langle A, \lambda^\mu(A), \lambda^\eta(A)\rangle / A \in \Re$ and $0 \le \lambda^\mu(A) + \lambda^\eta(A) \le 1\}$ of $\Re$ is called an intuitionistic fuzzy HX left ideal on $\Re$ or an intuitionistic fuzzy left ideal induced by H if the following conditions are satisfied. For all A,B ∈$\Re$,

1) $\lambda^\mu(A - B) \ge \min\{ \lambda^\mu(A), \lambda^\mu(B)\}$,
2) $\lambda^\mu(AB) \ge \lambda^\mu(B)$
3) $\lambda^\eta(A - B) \le \max\{ \lambda^\eta(A), \lambda^\eta(B)\}$
4) $\lambda^\eta(AB) \le \lambda^\eta(B)$.

where $\lambda^\mu(A) = \max\{ \mu(x) /$ for all $x \in A \subseteq R\}$ and $\lambda^\eta(A) = \min\{ \eta(x) /$ for all $x \in A \subseteq R\}$

C. Theorem

If H is an intuitionistic fuzzy right ideal of a ring R then the intuitionistic fuzzy subset λ^H is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

1) *Proof:* Let H be an intuitionistic fuzzy right ideal of R.

$$\begin{aligned}\min\{\lambda^\mu(A), \lambda^\mu(B)\} &= \min\{\max\{\mu(x)/\text{for all } x \in A \subseteq R\}, \max\{\mu(y)/\text{for all } y \in B \subseteq R\}\} \\ &= \min\{ \mu(x_0), \mu(y_0)\} \\ &\le \mu(x_0 - y_0) \\ &\le \max\{ \mu(x-y) /\text{for all } x-y \in A-B \subseteq R\} \\ &\le \lambda^\mu(A - B)\end{aligned}$$

$$\lambda^\mu(A-B) \ge \min\{\lambda^\mu(A), \lambda^\mu(B)\}.$$

$$\begin{aligned}\lambda^\mu(AB) &= \max\{ \mu(xy) / \text{for all } x \in A \subseteq R \text{ and } y \in B \subseteq R\} \\ &= \mu(x_0 y_0), \text{ for } x_0 \in A \text{ and } y_0 \in B. \\ &\ge \mu(x_0) \\ &= \max\{ \mu(x) / \text{for all } x \in A \subseteq R\} \\ &\ge \lambda^\mu(A).\end{aligned}$$

$$\lambda^\mu(AB) \ge \lambda^\mu(A).$$

$$\begin{aligned}\max\{\lambda^\eta(A), \lambda^\eta(B)\} &= \max\{\min\{ \eta(x)/\text{for all } x \in A \subseteq R\}, \min\{\eta(y)/\text{for all } y \in B \subseteq R\}\} \\ &= \max\{ \eta(x_0), \eta(y_0)\} \\ &\ge \eta(x_0 - y_0) \\ &\ge \min\{ \eta(x-y) /\text{for all } x-y \in A-B \subseteq R\} \\ &\ge \lambda^\eta(A - B)\end{aligned}$$

$$\lambda^\eta(A-B) \le \max\{\lambda^\eta(A), \lambda^\eta(B)\}.$$

$$\lambda^\eta(AB) = \min\{ \eta(xy) / \text{for all } x \in A \subseteq R \text{ and } y \in B \subseteq R\}$$

$= \eta(x_0 y_0)$, for $x_0 \in A$ and $y_0 \in B$.

$\leq \eta(x_0)$

$= \min\{\eta(x) \,/\, \text{for all } x \in A \subseteq R\}$

$\leq \lambda^{\eta}(A)$.

$\lambda^{\eta}(AB) \leq \lambda^{\eta}(A)$.

Hence, λ^H is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

D. Theorem

If H is an intuitionistic fuzzy left ideal of a ring R then the intuitionistic fuzzy subset λ^H is an intuitionistic fuzzy HX left ideal of a HX ring $\Re$.

1) Proof: It is clear.

E. Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ^G and λ^H be any two intuitionistic fuzzy HX right ideals of a HX ring $\Re$ then their intersection, $\gamma^G \cap \lambda^H$ is also an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

1) Proof: Let $G = \{\langle x, \alpha(x), \beta(x)\rangle / x \in R\}$ and $H = \{\langle x, \mu(x), \eta(x)\rangle / x \in R\}$ be any two intuitionistic fuzzy sets defined on a ring R.

Then, $\gamma^G = \{\langle A, \gamma^{\alpha}(A), \gamma^{\beta}(A)\rangle / A \in \Re\}$ and $\lambda^H = \{\langle A, \lambda^{\mu}(A), \lambda^{\eta}(A)\rangle / A \in \Re\}$ be any two intuitionistic fuzzy HX right ideals of a HX ring $\Re$.

$\gamma^G \cap \lambda^H = \{\langle A, (\gamma^{\alpha} \cap \lambda^{\mu})(A), (\gamma^{\beta} \cup \lambda^{\eta})(A)\rangle / A \in \Re\}$

Let $A, B \in \Re$

a) $(\gamma^{\alpha} \cap \lambda^{\mu})(A-B) = \min\{\gamma^{\alpha}(A-B), \lambda^{\mu}(A-B)\}$

$\geq \min\{\min\{\gamma^{\alpha}(A), \gamma^{\alpha}(B)\}, \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}\}$

$= \min\{\min\{\gamma^{\alpha}(A), \lambda^{\mu}(A)\}, \min\{\gamma^{\alpha}(B), \lambda^{\mu}(B)\}\}$

$= \min\{(\gamma^{\alpha} \cap \lambda^{\mu})(A), (\gamma^{\alpha} \cap \lambda^{\mu})(B)\}$

$(\gamma^{\alpha} \cap \lambda^{\mu})(A-B) \geq \min\{(\gamma^{\alpha} \cap \lambda^{\mu})(A), (\gamma^{\alpha} \cap \lambda^{\mu})(B)\}$.

b) $(\gamma^{\alpha} \cap \lambda^{\mu})(AB) = \min\{\gamma^{\alpha}(AB), \lambda^{\mu}(AB)\}$

$\geq \min\{\gamma^{\alpha}(A), \lambda^{\mu}(A)\}$

$= (\gamma^{\alpha} \cap \lambda^{\mu})(A)$.

$(\gamma^{\alpha} \cap \lambda^{\mu})(AB) \geq (\gamma^{\alpha} \cap \lambda^{\mu})(A)$.

c) $(\gamma^{\beta} \cup \lambda^{\eta})(A-B) = \max\{\gamma^{\beta}(A-B), \lambda^{\eta}(A-B)\}$

$\leq \max\{\max\{\gamma^{\beta}(A), \gamma^{\beta}(B)\}, \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}\}$

$= \max\{\max\{\gamma^{\beta}(A), \lambda^{\eta}(A)\}, \max\{\gamma^{\beta}(B), \lambda^{\eta}(B)\}\}$

$= \max\{(\gamma^{\beta} \cup \lambda^{\eta})(A), (\gamma^{\beta} \cup \lambda^{\eta})(B)\}$

$(\gamma^{\beta} \cup \lambda^{\mu})(A-B) \leq \max\{(\gamma^{\beta} \cup \gamma^{\eta})(A), (\gamma^{\beta} \cup \lambda^{\eta})(B)\}$.

d) $(\gamma^{\beta} \cup \lambda^{\eta})(AB) = \max\{\gamma^{\beta}(AB), \lambda^{\eta}(AB)\}$

$\leq \max\{\gamma^{\beta}(A), \lambda^{\eta}(A)\}$

$= (\gamma^{\beta} \cup \lambda^{\eta})(A)$.

$(\gamma^{\beta} \cup \lambda^{\eta})(AB) \leq (\gamma^{\beta} \cup \lambda^{\eta})(A)$.

Hence, $\gamma^G \cap \lambda^H$ is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

F. Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ^G and λ^H be any two intuitionistic fuzzy HX left ideals of a HX ring $\Re$ then their intersection, $\gamma^G \cap \lambda^H$ is also an intuitionistic fuzzy HX left ideal of a HX ring $\Re$.

1) Proof: Let $G = \{\langle x, \alpha(x), \beta(x)\rangle / x \in R\}$ and $H = \{\langle x, \mu(x), \eta(x)\rangle / x \in R\}$ be any two intuitionistic fuzzy sets defined on a ring R.

Then, $\gamma^G = \{\langle A, \gamma^{\alpha}(A), \gamma^{\beta}(A)\rangle / A \in \Re\}$ and $\lambda^H = \{\langle A, \lambda^{\mu}(A), \lambda^{\eta}(A)\rangle / A \in \Re\}$ be any two intuitionistic fuzzy HX left ideals of a HX ring $\Re$.

$\gamma^G \cap \lambda^H = \{\langle A, (\gamma^{\alpha} \cap \lambda^{\mu})(A), (\gamma^{\beta} \cup \lambda^{\eta})(A)\rangle / A \in \Re\}$

Let $A, B \in \Re$

a) $(\gamma^{\alpha} \cap \lambda^{\mu})(A–B) = \min\{\gamma^{\alpha}(A–B), \lambda^{\mu}(A–B)\}$

$\geq \min\{\min\{\gamma^{\alpha}(A), \gamma^{\alpha}(B)\}, \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}\}$

$= \min\{\min\{\gamma^{\alpha}(A), \lambda^{\mu}(A)\}, \min\{\gamma^{\alpha}(B), \lambda^{\mu}(B)\}\}$

$= \min\{(\gamma^{\alpha} \cap \lambda^{\mu})(A), (\gamma^{\alpha} \cap \lambda^{\mu})(B)\}$

$(\gamma^{\alpha} \cap \lambda^{\mu})(A–B) \geq \min\{(\gamma^{\alpha} \cap \lambda^{\mu})(A), (\gamma^{\alpha} \cap \lambda^{\mu})(B)\}$.

b) $(\gamma^{\alpha} \cap \lambda^{\mu})(AB) = \min\{\gamma^{\alpha}(AB), \lambda^{\mu}(AB)\}$

$\geq \min\{\gamma^{\alpha}(B), \lambda^{\mu}(B)\}$

$= (\gamma^{\alpha} \cap \lambda^{\mu})(B)$.

$(\gamma^{\alpha} \cap \lambda^{\mu})(AB) \geq (\gamma^{\alpha} \cap \lambda^{\mu})(B)$.

c) $(\gamma^{\beta} \cup \lambda^{\eta})(A–B) = \max\{\gamma^{\beta}(A–B), \lambda^{\eta}(A–B)\}$

$\leq \max\{\max\{\gamma^{\beta}(A), \gamma^{\beta}(B)\}, \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}\}$

$= \max\{\max\{\gamma^{\beta}(A), \lambda^{\eta}(A)\}, \max\{\gamma^{\beta}(B), \lambda^{\eta}(B)\}\}$

$= \max\{(\gamma^{\beta} \cup \lambda^{\eta})(A), (\gamma^{\beta} \cup \lambda^{\eta})(B)\}$

$(\gamma^{\beta} \cup \lambda^{\mu})(A–B) \leq \max\{(\gamma^{\beta} \cup \gamma^{\eta})(A), (\gamma^{\beta} \cup \lambda^{\eta})(B)\}$.

d) $(\gamma^{\beta} \cup \lambda^{\eta})(AB) = \max\{\gamma^{\beta}(AB), \lambda^{\eta}(AB)\}$

$\leq \max\{\gamma^{\beta}(B), \lambda^{\eta}(B)\}$

$= (\gamma^{\beta} \cup \lambda^{\eta})(B)$.

$(\gamma^{\beta} \cup \lambda^{\eta})(AB) \leq (\gamma^{\beta} \cup \lambda^{\eta})(B)$.

Hence, $\gamma^{G} \cap \lambda^{H}$ is an intuitionistic fuzzy HX left ideal of a HX ring $\Re$.

G. Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ^{G} be an intuitionistic fuzzy HX ring and λ^{H} be an intuitionistic fuzzy HX right (left) ideals of a HX ring $\Re$ then their intersection, $\gamma^{G} \cap \lambda^{H}$ is also an intuitionistic fuzzy HX right (left) ideal of a HX ring $\Re$.

1) Proof: It is clear.

H. Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ^{G} be an intuitionistic fuzzy HX ring and λ^{H} be an intuitionistic fuzzy HX ideal of a HX ring $\Re$ then their intersection, $\gamma^{G} \cap \lambda^{H}$ is also an intuitionistic fuzzy HX ideal of a HX ring $\Re$.

I. Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ^{G} and λ^{H} be any two intuitionistic fuzzy HX right ideals of a HX ring $\Re$ then their union, $\gamma^{G} \cup \lambda^{H}$ is also an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

1) Proof: Let $G = \{\langle x, \alpha(x), \beta(x)\rangle / x \in R\}$ and $H = \{\langle x, \mu(x), \eta(x)\rangle / x \in R\}$ be any two intuitionistic fuzzy sets defined on a ring R.

Then, $\gamma^{G} = \{\langle A, \gamma^{\alpha}(A), \gamma^{\beta}(A)\rangle / A \in \Re\}$ and $\lambda^{H} = \{\langle A, \lambda^{\mu}(A), \lambda^{\eta}(A)\rangle / A \in \Re\}$ be any two intuitionistic fuzzy HX right ideals of a HX ring $\Re$.Then,

$\gamma^{G} \cup \lambda^{H} = \{\langle A, (\gamma^{\alpha} \cup \lambda^{\mu})(A), (\gamma^{\beta} \cap \lambda^{\eta})(A)\rangle / A \in \Re\}$

Let $A, B \in \Re$

a) $(\gamma^{\alpha} \cup \lambda^{\mu})(A–B) = \max\{\gamma^{\alpha}(A–B), \lambda^{\mu}(A–B)\}$

$\geq \max\{\min\{\gamma^{\alpha}(A), \gamma^{\alpha}(B)\}, \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}\}$

$= \min\{\max\{\gamma^{\alpha}(A), \lambda^{\mu}(A)\}, \max\{\gamma^{\alpha}(B), \lambda^{\mu}(B)\}\}$

$= \min\{(\gamma^{\alpha} \cup \lambda^{\mu})(A), (\gamma^{\alpha} \cup \lambda^{\mu})(B)\}$

$(\gamma^{\alpha} \cup \lambda^{\mu})(A–B) \geq \min\{(\gamma^{\alpha} \cup \lambda^{\mu})(A), (\gamma^{\alpha} \cup \lambda^{\mu})(B)\}$.

b) $(\gamma^{\alpha} \cup \gamma^{\mu})(AB) = \max\{\gamma^{\alpha}(AB), \lambda^{\mu}(AB)\}$

$\geq \max\{\gamma^{\alpha}(A), \lambda^{\mu}(A)\}$

$= (\gamma^{\alpha} \cup \lambda^{\mu})(A)$.

$(\gamma^{\alpha} \cup \lambda^{\mu})(AB) \geq (\gamma^{\alpha} \cup \lambda^{\mu})(A)$.

c) $(\gamma^{\beta}\cap \lambda^{\eta})$ (A–B) = min { γ^{β}(A–B) , λ^{η}(A–B)}

≤ min { max{γ^{β}(A), γ^{β}(B)}, max{ λ^{η}(A), λ^{η} (B)}}

= max {min {γ^{β}(A), λ^{η}(A)}, min{γ^{β}(B), λ^{η} (B)}}

= max { $(\gamma^{\beta}\cap \gamma^{\eta})$(A) , $(\gamma^{\beta}\cup \lambda^{\eta})$(B)}

$(\gamma^{\beta}\cap \lambda^{\mu})$ (A–B) ≤ max { $(\gamma^{\beta}\cap \gamma^{\eta})$(A) , $(\gamma^{\beta}\cap \lambda^{\eta})$(B)}.

d) $(\gamma^{\beta}\cap \lambda^{\eta})$ (AB) = min { γ^{β}(AB) , λ^{η}(AB)}

≤ min{ γ^{β}(A), λ^{η}(A)}

= $(\gamma^{\beta}\cap \lambda^{\eta})$ (A) .

$(\gamma^{\beta}\cap \lambda^{\eta})$(AB) ≤ $(\gamma^{\beta}\cap \lambda^{\eta})$ (A).

Hence, $\gamma^{G}\cup \lambda^{H}$ is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

J. Theorem

Let G and H be any two intuitionistic fuzzy sets on R. Let γ^{G} and λ^{H} be any two intuitionistic fuzzy HX left ideals of a HX ring $\Re$ then their union, $\gamma^{G}\cup \lambda^{H}$ is also an intuitionistic fuzzy HX left ideal of a HX ring $\Re$.

1) *Proof:* It is clear.

K Theorem

Let G and H be any two intuitionistic fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings. If γ^{G} and λ^{H} are any two intuitionistic fuzzy HX right ideals of $\Re_1$ and $\Re_2$ respectively then, $\gamma^{G}\times \lambda^{H}$ is also an intuitionistic fuzzy HX right ideal of a HX ring $\Re_1\times \Re_2$.

1) *Proof:* Let G = { ⟨x, α(x), β(x)⟩ / x∈ R_1 } and H = { ⟨x, μ(x), η(x)⟩ / x∈ R_2 } be any two intuitionistic fuzzy sets defined on a ring R_1 and R_2 respectively.

Then, γ^{G} = { ⟨A, γ^{α}(A), γ^{β}(A)⟩ / A∈$\Re_1$ } and λ^{H} = { ⟨A, λ^{μ}(A), λ^{η}(A)⟩ / A∈$\Re_2$ } be any two intuitionistic fuzzy HX right ideal of a HX ring $\Re_1$ and $\Re_2$ respectively. Then,

$(\gamma^{G}\times \lambda^{H})$ = {⟨(A,B), $(\gamma^{\alpha}\cap \lambda^{\mu})$(A, B), $(\gamma^{\beta}\cup \lambda^{\eta})$(A, B)⟩ /(A, B) ∈ $\Re_1 \times \Re_2$},

where $(\gamma^{\alpha}\cap \lambda^{\mu})$(A, B) = min {$\gamma^{\alpha}$(A), λ^{μ}(B)}, for all (A, B) ∈ $\Re_1 \times \Re_2$,

$(\gamma^{\beta}\cup \lambda^{\eta})$(A, B) = max {$\gamma^{\beta}$(A), λ^{η}(B)}, for all (A, B) ∈ $\Re_1 \times \Re_2$.

Let A, B ∈ $\Re_1\times \Re_2$, where A = (C,D), B = (E,F) ,

a) $(\gamma^{\alpha}\cap \lambda^{\mu})$ (A – B)= min { γ^{α}(C – E) , λ^{μ}(D – F)}

≥ min { min {γ^{α}(C), γ^{α}(E)}, min{ λ^{μ}(D), λ^{μ} (F)}}

= min {min {γ^{α}(C), λ^{μ}(D)}, min{γ^{α}(D), λ^{μ} (E)}}

= min { $(\gamma^{\alpha}\cap \lambda^{\mu})$(C, D) , $(\gamma^{\alpha}\cap \lambda^{\mu})$(E , F)}

$(\gamma^{\alpha}\cap \lambda^{\mu})$ (A – B)≥ min { $(\gamma^{\alpha}\cap \lambda^{\mu})$(A) , $(\gamma^{\alpha}\cap\lambda\lambda^{\mu})$(B)}.

b) $(\gamma^{\alpha}\cap \lambda^{\mu})$ (AB) = $(\gamma^{\alpha}\cap \lambda^{\mu})$ ((C , D) · (E, F))

= $(\gamma^{\alpha}\cap \lambda^{\mu})$ (C E, DF)

= min { γ^{α}(CE) , λ^{μ}(DF)}

≥ min { γ^{α}(C), λ^{μ}(D)}

= $(\gamma^{\alpha}\cap \lambda^{\mu})$(C, D)

$(\gamma^{\alpha}\cap \lambda^{\mu})$ (AB) ≥ $(\gamma^{\alpha}\cap \lambda^{\mu})$(A).

c) $(\gamma^{\beta}\cup \lambda^{\eta})$ (A – B)= max { γ^{β}(C – E) , λ^{η}(D – F)}

≤ max{ max {γ^{β}(C), γ^{β}(E)},max{ λ^{η}(D), λ^{η} (F)}}

= max {max {γ^{β}(C), λ^{η}(D)},max{γ^{β}(E), λ^{η} (F)}}

= max { $(\gamma^{\beta}\cup \lambda^{\eta})$(C, D) , $(\gamma^{\beta}\cup \lambda^{\eta})$(E, F)}

$(\gamma^{\beta}\cup \lambda^{\mu})$ (A – B)≤ max { $(\gamma^{\beta}\cup \gamma^{\eta})$(A) , $(\gamma^{\beta}\cup \lambda^{\eta})$(B)}.

d) $(\gamma^{\beta}\cup \lambda^{\eta})$ (AB) = $(\gamma^{\beta}\cup \lambda^{\eta})$ ((C , D) · (E, F))

= $(\gamma^{\beta}\cup \lambda^{\eta})$ (C E , DF)

= max { γ^{β}(CE) , λ^{η}(DF)}

≤ max{γ^{β}(C), λ^{η}(D)}

$= \quad \max\{\gamma^{\beta}(C), \lambda^{\eta}(D)\}$

$= \quad (\gamma^{\beta} \cup \lambda^{\eta})(C, D)$

$(\gamma^{\beta} \cup \lambda^{\mu})(AB) \quad \leq \quad (\gamma^{\beta} \cup \gamma^{\eta})(C, D).$

Hence, $\gamma^G \times \lambda^H$ is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

K. Theorem

Let G and H be any two intuitionistic fuzzy sets of R_1 and R_2 respectively. Let $\Re_1 \subset 2^{R_1} - \{\phi\}$ and $\Re_2 \subset 2^{R_2} - \{\phi\}$ be any two HX rings. If γ^G and λ^H are any two intuitionistic fuzzy HX left ideals of $\Re_1$ and $\Re_2$ respectively then, $\gamma^G \times \lambda^H$ is also an intuitionistic fuzzy HX left ideal of a HX ring $\Re_1 \times \Re_2$.

1) Proof: It is clear.

L. Definition

Let $\lambda^H = \{\langle A, \lambda^{\mu}(A), \lambda^{\gamma}(A) \rangle / \text{ for all } A \in \vartheta\}$ be an intuitionistic fuzzy subset of a HX ring ϑ . We define the following "necessity" and "possibility" operations :

$\Box \lambda^H = \{ \langle A, \lambda^{\mu}(A), 1 - \lambda^{\mu}(A) \rangle / A \in \vartheta\}.$

$\Diamond \lambda^H = \{ \langle A, 1 - \lambda^{\gamma}(A), \lambda^{\gamma}(A) \rangle / A \in \vartheta\}$

M. Theorem

Let H be an intuitionistic fuzzy set on R. Let λ^H be an intuitionistic fuzzy HX right ideal of a HX ring $\Re$ then $\Box\lambda^H$ is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

1) Proof: Let λ^H be an intuitionistic fuzzy HX right ideal of a HX ring $\Re$. Then,

a) $\lambda^{\mu}(A - B) \quad \geq \quad \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

b) $\lambda^{\mu}(AB) \quad \geq \quad \lambda^{\mu}(A$

c) $\lambda^{\eta}(A - B) \quad \leq \quad \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}$

d) $\lambda^{\eta}(AB) \quad \leq \quad \lambda^{\eta}(A).$

Now, $\lambda^{\mu}(A - B) \quad \geq \quad \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

$1 - \lambda^{\mu}(A - B) \quad \leq \quad 1 - \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$

$\leq \quad \max\{1 - \lambda^{\mu}(A), 1 - \lambda^{\mu}(B)\}.$

That is, $1 - \lambda^{\mu}(A - B) \quad \leq \quad \max\{1 - \lambda^{\mu}(A), 1 - \lambda^{\mu}(B)\}.$

We have, $\lambda^{\mu}(AB) \quad \geq \quad \lambda^{\mu}(A)$

$1 - \lambda^{\mu}(AB) \quad \leq \quad 1 - \lambda^{\mu}(A).$

Hence, $\Box\lambda^H$ is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

N. Theorem

Let H be an intuitionistic fuzzy set on R. Let λ^H be an intuitionistic fuzzy HX left ideal of a HX ring $\Re$ then $\Box\lambda^H$ is an intuitionistic fuzzy HX left ideal of a HX ring $\Re$

O. Theorem

Let H be an intuitionistic fuzzy set on R. Let λ^H be an intuitionistic fuzzy HX right ideal of a HX ring $\Re$ then $\Diamond\lambda^H$ is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

1) Proof: Let λ^H be an intuitionistic fuzzy HX right ideal of a HX ring $\Re$. Then,

a) $\lambda^{\mu}(A - B) \quad \geq \quad \min\{\lambda^{\mu}(A), \lambda^{\mu}(B)\}$,

b) $\lambda^{\mu}(AB) \quad \geq \quad \lambda^{\mu}(A)$

c) $\lambda^{\eta}(A - B) \quad \leq \quad \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}$

d) $\lambda^{\eta}(AB) \quad \leq \quad \lambda^{\eta}(A).$

Now, $\lambda^{\eta}(A - B) \quad \leq \quad \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}$

$1 - \lambda^{\eta}(A - B) \quad \geq \quad 1 - \max\{\lambda^{\eta}(A), \lambda^{\eta}(B)\}$

$\geq \quad \min\{1 - \lambda^{\eta}(A), 1 - \lambda^{\eta}(B)\}.$

That is, $1 - \lambda^{\eta}(A - B) \quad \geq \quad \min\{1 - \lambda^{\eta}(A), 1 - \lambda^{\eta}(B)\}.$

We have, $\lambda^{\eta}(AB) \quad \leq \quad \lambda^{\eta}(A)$

$1 - \lambda^{\eta}(AB) \quad \geq \quad 1 - \lambda^{\eta}(A).$

Hence, $\Diamond\lambda^H$ is an intuitionistic fuzzy HX right ideal of a HX ring $\Re$.

P. Theorem

H be an intuitionistic fuzzy set on R. Let λ^H be an intuitionistic fuzzy HX left ideal of a HX ring $\Re$ then $\Diamond\lambda^H$ is an intuitionistic fuzzy HX left ideal of a HX ring $\Re$.

1) Proof: It is clear.

III. HOMOMORPHISM AND ANTI HOMOMORPHISM OF AN INTUITIONISTIC FUZZY HX RIGHT AND LEFT IDEAL OF A HX RING

In this section, we introduce the concept of an image, pre-image of an intuitionistic fuzzy subset of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre-images of an intuitionistic fuzzy HX right and left ideal of a HX ring $\Re$.

A. Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ^H be an intuitionistic fuzzy HX right ideal of $\Re_1$ then $f(\lambda^H)$ is an intuitionistic fuzzy HX right ideal of $\Re_2$, if λ^H has a supremum property and λ^H is f-invariant.

1) Proof: Let $H = \{ \langle x, \mu(x), \eta(x)\rangle / x\in R_1 \}$ be an intuitionistic fuzzy sets defined on a ring R_1.

Then, $\lambda^H = \{ \langle X, \lambda^\mu(X), \lambda^\eta(X)\rangle / X\in\Re_1 \}$ be an intuitionistic fuzzy HX right ideal of a HX ring $\Re_1$. Then, $f(\lambda^H) = \{ \langle f(X), f(\lambda^\mu)(f(X)), f(\lambda^\eta)(f(X))\rangle / X\in\Re_1 \}$.

There exist $X,Y\in \Re_1$ such that $f(X) , f(Y) \in \Re_2$,

$$\begin{aligned}
(f(\lambda^\mu))(f(X)-f(Y)) &= (f(\lambda^\mu))(f(X-Y)) ,\\
&= \lambda^\mu(X-Y)\\
&\geq \min\{\lambda^\mu(X) , \lambda^\mu(Y)\}\\
&= \min\{(f(\lambda^\mu))(f(X)) , (f(\lambda^\mu))(f(Y))\}\\
(f(\lambda^\mu))(f(X)-f(Y)) &\geq \min\{(f(\lambda^\mu))(f(X)), (f(\lambda^\mu))(f(Y))\}
\end{aligned}$$

$$\begin{aligned}
(f(\lambda^\mu))(f(X)f(Y)) &= (f(\lambda^\mu))(f(XY))\\
&= \lambda^\mu(XY)\\
&\geq \lambda^\mu(X)\\
&= (f(\lambda^\mu))(f(X))\\
(f(\lambda^\mu))(f(X)f(Y)) &\geq f(\lambda^\mu))(f(X)).\\
(f(\lambda^\eta))(f(X)-f(Y)) &= (f(\lambda^\eta))(f(X-Y)) ,\\
&= \lambda^\eta(X-Y)\\
&\leq \max\{\lambda^\eta(X) , \lambda^\eta(Y)\}\\
&= \max\{(f(\lambda^\eta))(f(X)) , (f(\lambda^\eta))(f(Y))\}\\
(f(\lambda^\eta))(f(X)-f(Y)) &\leq \max\{(f(\lambda^\eta))(f(X)), (f(\lambda^\eta))(f(Y))\}
\end{aligned}$$

$$\begin{aligned}
(f(\lambda^\eta))(f(X)f(Y)) &= (f(\lambda^\eta))(f(XY))\\
&= \lambda^\eta(XY)\\
&\leq \lambda^\eta(X)\\
&= (f(\lambda^\eta))(f(X))\\
(f(\lambda^\eta))(f(X)f(Y)) &\leq f(\lambda^\eta))(f(X)).
\end{aligned}$$

Hence, $f(\lambda^H)$ is an intuitionistic fuzzy HX right ideal of $\Re_2$.

B. Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ^H be an intuitionistic fuzzy HX left ideal of $\Re_1$ then $f(\lambda^H)$ is an intuitionistic fuzzy HX left ideal of $\Re_2$, if λ^H has a supremum property and λ^H is f-invariant.

1) Proof: It is clear.

C. Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ^G be an intuitionistic fuzzy HX right ideal of $\Re_2$, then $f^{-1}(\gamma^G)$ is an intuitionistic fuzzy HX right ideal of $\Re_1$.

1) *Proof:* Let $G = \{ \langle y, \alpha(y), \beta(y)\rangle / y \in R_2 \}$ be an intuitionistic fuzzy sets defined on a ring R_2.

Then, $\gamma^G = \{ \langle Y, \gamma^\alpha(Y), \gamma^\beta(Y)\rangle / Y \in \Re_2 \}$ be an intuitionistic fuzzy HX right ideal of a HX ring $\Re_2$. Then, $f^{-1}(\gamma^G) = \{ \langle X, f^{-1}(\gamma^\alpha)(X), f^{-1}(\gamma^\beta)(X)\rangle / X \in \Re_1 \}$.

For any $X,Y \in \Re_1$, $f(X), f(Y) \in \Re_2$,

$$(f^{-1}(\gamma^\alpha))(X-Y) = \gamma^\alpha(f(X-Y))$$
$$= \gamma^\alpha(f(X) - f(Y))$$
$$\geq \min\{\gamma^\alpha(f(X)), \gamma^\alpha(f(Y))\}$$
$$= \min\{(f^{-1}(\gamma^\alpha))(X), (f^{-1}(\gamma^\alpha))(Y)\}$$
$$(f^{-1}(\gamma^\alpha))(X-Y) \geq \min\{(f^{-1}(\gamma^\alpha))(X), (f^{-1}(\gamma^\alpha))(Y)\}.$$

$$(f^{-1}(\gamma^\alpha))(XY) = \gamma^\alpha(f(XY))$$
$$= \gamma^\alpha(f(X)\,f(Y))$$
$$\geq \gamma^\alpha(f(X))$$
$$= (f^{-1}(\gamma^\alpha))(X)$$
$$(f^{-1}(\gamma^\alpha))(XY) \geq (f^{-1}(\gamma^\alpha))(X).$$

$$(f^{-1}(\gamma^\beta))(X-Y) = \gamma^\beta(f(X-Y))$$
$$= \gamma^\beta(f(X) - f(Y))$$
$$\leq \max\{\gamma^\beta(f(X)), \gamma^\beta(f(Y))\}$$
$$= \max\{(f^{-1}(\gamma^\beta))(X), (f^{-1}(\gamma^\beta))(Y)\}$$
$$(f^{-1}(\gamma^\beta))(X-Y) \leq \max\{(f^{-1}(\gamma^\beta))(X), (f^{-1}(\gamma^\beta))(Y)\}.$$

$$(f^{-1}(\gamma^\beta))(XY) = \gamma^\beta(f(XY))$$
$$= \gamma^\beta(f(X)\,f(Y))$$
$$\leq \gamma^\beta(f(X))$$
$$= (f^{-1}(\gamma^\beta))(X)$$
$$(f^{-1}(\gamma^\beta))(XY) \leq (f^{-1}(\gamma^\beta))(X).$$

Hence, $f^{-1}(\gamma^G)$ is an intuitionistic fuzzy HX right ideal of $\Re_1$.

D. Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \Re_1 \rightarrow \Re_2$ be a homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ^G be an intuitionistic fuzzy HX left ideal of $\Re_2$, then $f^{-1}(\gamma^G)$ is an intuitionistic fuzzy HX left ideal of $\Re_1$.

1) *Proof:* It is clear.

E. Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \rightarrow \Re_2$ be an anti homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ^H be an intuitionistic fuzzy HX right ideal of $\Re_1$ then $f(\lambda^H)$ is an intuitionistic fuzzy HX left ideal of $\Re_2$, if λ^H has a supremum property and λ^H is f-invariant.

1) *Proof:* Let $H = \{ \langle x, \mu(x), \eta(x)\rangle / x \in R \}$ be an intuitionistic fuzzy sets defined on a ring R_1. Then, $\lambda^H = \{ \langle X, \lambda^\mu(X), \lambda^\eta(X)\rangle / X \in \Re \}$ be an intuitionistic fuzzy HX right ideal of a HX ring $\Re_1$. Then, $f(\lambda^H) = \{ \langle f(X), f(\lambda^\mu)(f(X)), f(\lambda^\eta)(f(X))\rangle / X \in \Re \}$.

There exist $X,Y \in \Re_1$ such that $f(X), f(Y) \in \Re_2$, *a)* $(f(\lambda^{\mu}))(f(X)-f(Y)) = (f(\lambda^{\mu}))(f(X-Y))$,

$= \lambda^{\mu}(X-Y)$

$= \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$

$= \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$

$(f(\lambda^{\mu}))(f(X)-f(Y)) \geq \min\{(f(\lambda^{\mu}))(f(X)), (f(\lambda^{\mu}))(f(Y))\}$

b) $(f(\lambda^{\mu}))(f(X)f(Y)) = (f(\lambda^{\mu}))(f(YX))$

$= \lambda^{\mu}(YX)$

$\geq \lambda^{\mu}(Y)$

$= (f(\lambda^{\mu}))(f(Y))$

$(f(\lambda^{\mu}))(f(X)f(Y)) \geq (f(\lambda^{\mu}))(f(Y))$.

c) $(f(\lambda^{\eta}))(f(X)-f(Y)) = (f(\lambda^{\eta}))(f(X-Y))$,

$= \lambda^{\eta}(X-Y)$

$\leq \max\{\lambda^{\eta}(X), \lambda^{\eta}(Y)\}$

$= \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$

$(f(\lambda^{\eta}))(f(X)-f(Y)) \leq \max\{(f(\lambda^{\eta}))(f(X)), (f(\lambda^{\eta}))(f(Y))\}$

d) $(f(\lambda^{\eta}))(f(X)f(Y)) = (f(\lambda^{\eta}))(f(YX))$

$= \lambda^{\eta}(YX)$

$\leq \lambda^{\eta}(Y)$

$= (f(\lambda^{\eta}))(f(Y))$

$(f(\lambda^{\eta}))(f(X)f(Y)) \leq (f(\lambda^{\eta}))(f(Y)$.

Hence, $f(\lambda^H)$ is an intuitionistic fuzzy HX left ideal of $\Re_2$.

F. Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f: \Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let H be an intuitionistic fuzzy subset of R_1. Let λ^H be an intuitionistic fuzzy HX left ideal of $\Re_1$ then $f(\lambda^H)$ is an intuitionistic fuzzy HX right ideal of $\Re_2$, if λ^H has a supremum property and λ^H is f-invariant.

1) Proof: It is clear.

G. Theorem

Let $\Re_1$ and $\Re_2$ be any two HX rings on R_1 and R_2 respectively. Let $f: \Re_1 \to \Re_2$ be an anti homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ^G be an intuitionistic fuzzy HX right ideal of $\Re_2$, then $f^{-1}(\gamma^G)$ is an intuitionistic fuzzy HX left ideal of $\Re_1$.

1) Proof: Let $G = \{\langle y, \alpha(y), \beta(y)\rangle / y \in R_2\}$ be an intuitionistic fuzzy sets defined on a ring R_2.

Then, $\gamma^G = \{\langle Y, \gamma^{\alpha}(Y), \gamma^{\beta}(Y)\rangle / Y \in \Re_2\}$ be an intuitionistic fuzzy HX right ideal of a HX ring $\Re_2$. Then, $f^{-1}(\gamma^G) = \{\langle X, f^{-1}(\gamma^{\alpha})(X), f^{-1}(\gamma^{\beta})(X)\rangle / X \in \Re_1\}$.

For any $X, Y \in \Re_1$, $f(X), f(Y) \in \Re_2$,

$(f^{-1}(\gamma^{\alpha}))(X-Y) = \gamma^{\alpha}(f(X-Y))$

$= \gamma^{\alpha}(f(X)-f(Y))$

$\geq \min\{\gamma^{\alpha}(f(X)), \gamma^{\alpha}(f(Y))\}$

$= \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$

$(f^{-1}(\gamma^{\alpha}))(X-Y) \geq \min\{(f^{-1}(\gamma^{\alpha}))(X), (f^{-1}(\gamma^{\alpha}))(Y)\}$.

$(f^{-1}(\gamma^{\alpha}))(XY) = \gamma^{\alpha}(f(XY))$

$= \gamma^{\alpha}(f(Y)f(X))$

$\geq \gamma^{\alpha}(f(Y))$

$= (f^{-1}(\gamma^{\alpha}))(Y)$

$(f^{-1}(\gamma^{\alpha}))(XY) \geq (f^{-1}(\gamma^{\alpha}))(Y).$

$$
\begin{aligned}
(f^{-1}(\gamma^{\beta}))(X-Y) &= \gamma^{\beta}(f(X-Y)) \\
&= \gamma^{\beta}(f(X)-f(Y)) \\
&\leq \max\{\gamma^{\beta}(f(X)), \gamma^{\beta}(f(Y))\} \\
&= \max\{(f^{-1}(\gamma^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}
\end{aligned}
$$

$(f^{-1}(\gamma^{\beta}))(X-Y) \leq \max\{(f^{-1}(\gamma^{\beta}))(X), (f^{-1}(\gamma^{\beta}))(Y)\}.$

$$
\begin{aligned}
(f^{-1}(\gamma^{\beta}))(XY) &= \gamma^{\beta}(f(XY)) \\
&= \gamma^{\beta}(f(Y)\,f(X)) \\
&\leq \gamma^{\beta}(f(Y)) \\
&= (f^{-1}(\gamma^{\beta}))(Y)
\end{aligned}
$$

$(f^{-1}(\gamma^{\beta}))(XY) \leq (f^{-1}(\gamma^{\beta}))(Y).$

Hence, $f^{-1}(\gamma^{G})$ is an intuitionistic fuzzy HX left ideal of $\mathfrak{R}_1$.

H. Theorem

Let $\mathfrak{R}_1$ and $\mathfrak{R}_2$ be any two HX rings on R_1 and R_2 respectively. Let $f : \mathfrak{R}_1 \rightarrow \mathfrak{R}_2$ be a homomorphism on HX rings. Let G be an intuitionistic fuzzy subset of R_2. Let γ^{G} be an intuitionistic fuzzy HX left ideal of $\mathfrak{R}_2$, then $f^{-1}(\gamma^{G})$ is an intuitionistic fuzzy HX right ideal of $\mathfrak{R}_1$.

1) *Proof:* It is clear.

IV. ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to all our friends for their help to make this paper a successful one.

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