



# **iJRASET**

International Journal For Research in  
Applied Science and Engineering Technology



---

# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

---

**Volume:** 2017    **Issue:** conference    **Month of publication:** December 2017

**DOI:**

**[www.ijraset.com](http://www.ijraset.com)**

**Call:** ☎ 08813907089

**E-mail ID:** [ijraset@gmail.com](mailto:ijraset@gmail.com)

# Complementary Domination in Intuitionistic Fuzzy Graphs

Johnstephen J<sup>1</sup>, Vinothkumar N<sup>2</sup>

<sup>1</sup>Hindustan Institute of Technology – Coimbatore- Tamilnadu - India

<sup>2</sup>Bannari Amman Institute of Technology – Sathyamangalam – Erode- Tamilnadu - India

**Abstract:** Let  $G$  be an intuitionistic fuzzy graph. Let  $u$  and  $v$  be two vertices of  $G$ . Let  $D$  be a minimal dominating set of  $G(V, \sigma, \mu)$ . If  $V-D$  contains a dominating set  $D'$ , then  $D'$  is called a complementary dominating set of  $G$  with respect to  $D$ .

The complementary intuitionistic fuzzy domination number  $\gamma_{cf}(G)$  of a fuzzy graph  $G$  is the minimum cardinality of a complementary dominating set of  $G$ . The aim of this study is to define the complementary domination set and complementary domination number  $\gamma(G)$ . Further discusses the properties and bounds of the complementary domination number of the intuitionistic fuzzy graphs.

**Key words:** Intuitionistic Fuzzy Graph, Intuitionistic Fuzzy Domination, Intuitionistic Complementary Fuzzy Domination and Intuitionistic Complementary Fuzzy Domination number

## I. INTRODUCTION

The first definition of fuzzy graphs was proposed by Kafmann, from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another elaborated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by A. Somasundaram, S. Somasundaram and A. Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs. C. Natarajan and S.K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. Somasundaram, A & Somasundaram, S introduced the concepts of domination and total domination in fuzzy graphs and determined the domination number for several classes of fuzzy graphs and obtained bounds for the same. Somasundaram, A studied several operations on fuzzy graphs such as union, join, composition, Cartesian product and obtained their domination parameters. Nair, PS & Cheng, SC discussed the concepts of clique and fuzzy cliques in fuzzy graphs. Various properties of fuzzy cliques and a characterization of fuzzy cliques were also presented. Monderson, JN & Yao, YY examined the properties of various types of fuzzy cycles, fuzzy trees, fuzzy bridges, and fuzzy cut nodes in fuzzy graphs. Nagoor Gani, A & Basheer Ahmed, M examined the properties of various types of degree, order and size of fuzzy graphs and compared the relationship between degree, order and size of fuzzy graphs. In this paper we develop the complementary efficient domination in fuzzy graphs and investigate the property of domination parameter in fuzzy graph.

## II. PRELIMINARIES

An intuitionistic fuzzy graph (IFG) is of the form  $G=(V,E)$ , where  $V=\{v_1, v_2, \dots, v_n\}$  such that  $\mu_1: V \rightarrow [0,1]$ ,  $\gamma_1: V \rightarrow [0,1]$  denote the degree of membership and nonmembership of the element  $v_i \in V$  respectively and  $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$  for every  $v_i \in V, (i=1,2,\dots,n)$ .  $E \subseteq V \times V$  where

$\mu_2: V \times V \rightarrow [0,1]$ , and  $\gamma_2: V \times V \rightarrow [0,1]$  are such that

$\mu_2(v_i, v_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$ ,  $\gamma_2(v_i, v_j) \leq \gamma_1(v_i) \vee \gamma_1(v_j)$  and  $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$

An arc  $(v_i, v_j)$  of an IFG  $G$  is called an *strong arc* if  $\mu_2(v_i, v_j) = \mu_1(v_i) \wedge \mu_1(v_j)$ ,  $\gamma_2(v_i, v_j) = \gamma_1(v_i) \wedge \gamma_1(v_j)$ .

Let  $G = (V,E)$  be an IFG. Then the *cardinality of  $G$*  is defined to be

$$|G| = \left| \sum_{v_i \in V} \frac{(1 + \mu_1(v_i) - \gamma_1(v_i))}{2} \right| + \left| \sum_{v_i \in V} \frac{(1 + \mu_2(v_i, v_j) - \gamma_1(v_i, v_j))}{2} \right|$$

Let  $G = (V, E)$  be an IFG. The *vertex cardinality* of  $G$  is defined to be  $|G| = \left| \sum_{v_i \in V} \frac{(1 + \mu_1(v_i) - \gamma_1(v_i))}{2} \right|$  for all  $v_i \in V, (i = 1, 2, \dots, n)$ . Let  $G = (V, E)$  be an IFG.

An *edge cardinality* of  $G$  is defined to be  $|G| = \left| \sum_{v_i \in V} \frac{(1 + \mu_2(v_i, v_j) - \gamma_1(v_i, v_j))}{2} \right|$  for all  $(v_i, v_j) \in V \times V$ , Let  $G = (V, E)$  be

an IFG. A set  $D \subseteq V$  is said to be a dominating set of  $G$  if every  $v \in V - D$  there exist  $u \in D$  such that  $u$  dominates  $v$ .

An intuitionistic fuzzy dominating set  $D$  of an IFG,  $G$  is called minimal dominating set of  $G$  if every node  $u \in D$ ,  $D - \{u\}$  is not a dominating set in  $G$ . An intuitionistic fuzzy domination number  $\gamma_{if}(G)$  of an IFG,  $G$  is the minimum vertex cardinality over all minimal dominating sets in  $G$ .

A set  $S \subseteq V$  in an IFG,  $G$  is said to be an independent if there is no strong between the vertices  $v \in S$ . An independent set  $S$  of IFG,  $G$  is said to be maximal independent set if every node  $v \in V - S$  then the set  $S \cup \{v\}$  is not an independent set in  $G$ . The minimum cardinality among all the maximal independent sets in an IFG,  $G$  is called the intuitionistic fuzzy independent number.

### III. COMPLEMENTRY DOMINATION

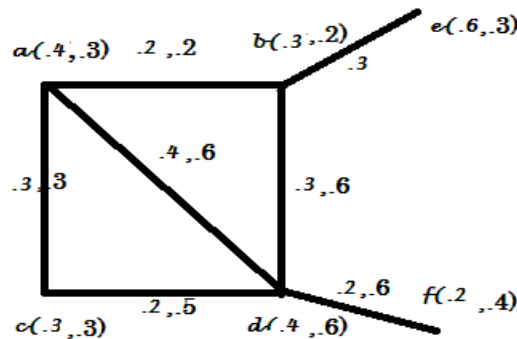
#### A. Definition 2.1

Let  $D$  be a minimal dominating set of an IFG,  $G(V, E)$ . If  $V - D$  contains a dominating set  $D'$ , then  $D'$  is called a complementary dominating set of  $G$  with respect to  $D$ .

#### B. Definition 2.2

The complementary intuitionistic fuzzy domination number  $\gamma_{cif}(G)$  of a fuzzy graph  $G$  is the minimum cardinality of a complementary dominating set of  $G$ .

#### 1) Example



Minimum complementary dominating set  $D' = \{a, e, f\}$  &  $\gamma_{cif}(G) = 2.8$

#### C. Proposition 2.1. For any fuzzy graph $G$ ,

$$\gamma_f(G) < \gamma_{cif}(G)$$

1) *Proof:* Clearly every complementary dominating set is a dominating set of  $G$ . Therefore  $\gamma_{if}(G) < \gamma_{cif}(G)$ .

D. Proposition 2.2. For any intuitionistic fuzzy graph of order  $p$ ,  $\gamma_{cif}(G) \leq p - \Delta_N(G) \leq p - \Delta_E(G)$ .

1) *Proof:* Let  $u$  be a vertex of degree  $\Delta_N(G)$  in a fuzzy graph  $G$ . Then clearly  $V-N(u)$  is a complementary dominating set of  $G$ . Therefore  $\gamma_{cif}(G) \leq |V - N(u)| = p - \Delta_N(G)$ . Since  $\Delta_N(G) > \Delta_E(G)$ , therefore  $\gamma_{cif}(G) \leq p - \Delta_N(G) \leq p - \Delta_E(G)$ .

E. *Proposition 2.3.* For any intuitionistic fuzzy graph  $G$ ,  $\gamma_{cif}(G) \leq \beta_{if}(G)$ .

1) *Proof :* Let  $D$  be the minimum dominating set of a fuzzy graph  $G$  and  $I$  be a maximum independent set in  $V-D$ . Every vertex in  $V - (D \cup I)$  must be adjacent to exactly one vertex in  $I$ . If every vertex in  $D$  is adjacent to at least one vertex in  $I$ , then  $I$  is the complementary dominating set of  $G$ .

Otherwise  $D' \subset D$  be such that no vertex in  $D'$  is adjacent to an edge in  $I$ . then every vertex in  $D'$  is adjacent to at least one vertex in  $V - (D \cup I)$  so far each vertex in  $D'$  pick in  $V - (D \cup I)$  a vertex adjacent to it and let  $R$  be the set of all vertex. Then  $|R|_{if} \leq |D'|_{if}$  and  $\gamma_{cif}(G) \leq |I \cup R| \leq |I| + |D'| \leq \beta_{if}(G)$ .

F. *Theorem 2.1.* Let  $G$  be an intuitionistic fuzzy graph, then  $\gamma_f(G) \leq \gamma_{cif}(G)$ .

1) *Proof:*  $D'$  is a minimum intuitionistic fuzzy dominating set with respect to  $D$ .  $D'$  is also a dominating set but not minimum dominating set, therefore  $|D|_{if} \leq |D'|_{if} \Rightarrow \gamma_{if}(G) \leq \gamma_{cif}(G)$ .

G. *Theorem 2.2.* For any fuzzy graph  $G$ , then  $\gamma_{cif}(G) \leq \frac{O(G)}{2}$ .

1) *Proof:* Let  $D'$  be the minimum complementary dominating set with respect to  $D$ .  $D' \subseteq V - D$

$$\Rightarrow |D'|_{if} \leq |V|_{if} - |D|_{if}$$

$$\Rightarrow \gamma_{cif}(G) \leq O(G) - \gamma_{if}(G). \text{ [Since } \gamma_{if}(G) \leq \frac{O(G)}{2} \text{]}$$

$$\Rightarrow \gamma_{cif}(G) \leq O(G) - \frac{O(G)}{2}$$

$$\Rightarrow \gamma_{cif}(G) \leq \frac{O(G)}{2}.$$

Hence proved.

2) *Remark:*  $\gamma_{if}(G) + \gamma_{cif}(G) \leq O(G)$

H. *Theorem 2.3.* Let  $G$  be a fuzzy graph.  $D$  is the minimum dominating set of  $G$ . If  $\langle V-D \rangle$  does not contain any strong arc, then

$$\gamma_{if}(G) + \gamma_{cif}(G) = O(G).$$

1) *Proof:* Let  $G$  be an intuitionistic fuzzy graph.  $D$  is the minimum dominating set of  $G$ . Then  $|D|_f = \gamma_f(G)$ . If  $\langle V - D \rangle$  does not contain any strong arc, therefore every vertex in  $\langle V - D \rangle$  is dominate itself.

$$\Rightarrow |D|_{if} = |V - D'|_{if}$$

$$\Rightarrow \gamma_{cif}(G) = |V|_{if} - |D|_{if}$$

$$\Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) = O(G).$$

Hence proved.

I. *Theorem 2.4.* Let  $G$  be a strong intuitionistic fuzzy graph. Let  $D$ , be the minimum complementary dominating set with respect to  $D$ , then

$$\Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) < O(G).$$

1) *Proof:* Let  $G$  be a strong fuzzy graph. Let  $D$ , be the minimum complementary dominating set with respect to  $D$ , therefore  $D' \subseteq V - D$ . Assume  $G$  is strong fuzzy graph then  $\langle V - D \rangle$  is also a strong fuzzy graph. Therefore a vertex in  $\langle V - D \rangle$  has the strong arc between some other vertices. So we have  $D' \subset V-D$ .

$$\begin{aligned} \Rightarrow |D'|_{if} &< |V - D|_{if} \\ \Rightarrow \gamma_{cif}(G) &< |V|_{if} - |D|_{if} \\ \Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) &< O(G). \end{aligned}$$

Hence proved.

J. **Theorem 2.5.** Let G be a complete intuitionistic fuzzy graph D' be the inverse fuzzy dominating set with respect to D of G.

Then

$$\gamma_{cif}(G) + \gamma_{if}(G) = \sigma_1 + \sigma_2.$$

Here  $\sigma_1, \sigma_2$  be the least two membership values of the vertices in G.

I) **Proof:** Let  $\sigma_1(G) = u, \sigma_2(G) = v$  (say), be the least two membership values of vertices in G. Let G be the complete intuitionistic fuzzy graph. D' is the minimum complementary dominating set with respect to D. Therefore  $|D'|_{if} = \gamma_{if}(G)$ .

$$\Rightarrow \gamma_{if}(G) = \sigma_1 \text{ [ since G is complete]}$$

G is a complete fuzzy graph. Therefore  $\langle V-D \rangle$  is also a complete graph.  $D' \in V - D$

$$\Rightarrow |D'|_{if} = \gamma_{cif}(G) = \sigma_2$$

$$\Rightarrow \gamma_{cif}(G) + \gamma_{if}(G) = \sigma_1 + \sigma_2.$$

K. **Theorem 2.6.** Let G be an intuitionistic fuzzy graph, G does not contain any strong arc, then  $\gamma_{cif}(G) = 0$ .

I) **Proof:** Let G be an intuitionistic fuzzy graph, G does not contain any strong arc. Let D' be the minimum complementary dominating set with respect to D in G.

$$\Rightarrow |D'|_{if} = |V|_{if} = O(G)$$

There is no strong arc in G. Therefore every vertex dominates itself.  $D' \in V - D$

$$\Rightarrow |D'|_{if} = |V - D|_{if}$$

$$\Rightarrow \gamma_{cif}(G) \leq |V|_{if} - |D|_{if}$$

$$\Rightarrow \gamma_{cif}(G) = 0. \text{ Hence proved.}$$

L. **Theorem 2.7** Let  $G_1$  and  $G_2$  are an intuitionistic fuzzy graph and  $D_1'$  and  $D_2'$  be a complementary dominating set with respect to  $D_1$  and  $D_2$  respectively. Then  $G_1 \times G_2$  is a fuzzy graph and we have  $D_1 \times (V_2 - D_2)$  or  $D_2 \times (V_1 - D_1)$  is a complementary dominating set of  $G_1 \times G_2$ .

I) **Proof:** Let  $D_1^{-1}$  and  $D_2^{-1}$  be a complementary intuitionistic fuzzy dominating set with respect to  $D_1$  and  $D_2$  of  $G_1$  and  $G_2$  respectively. We know that  $\gamma_f(G_1 \times G_2) = \min \{ |D_1 \times V_2|_f, |V_1 \times D_2|_f \}$ . Therefore we note that the complementary dominating set  $D_1 \times D_2' \subseteq V_1 \times V_2 - (D_1 \times V_2)$  or  $D_1 \times D_2' \subseteq V_1 \times V_2 - (D_2 \times V_1)$ .  $D_1, D_2$  are minimum dominating set of  $G_1$  and  $G_2$  respectively.

The sub graph  $\langle V_1 \times V_2 - (D_1 \times D_2) \rangle$  of a fuzzy graph  $G_1 \times G_2$ .  $D_1 \times V_2$  and  $D_2 \times V_1$  are minimum dominating set of  $G_1 \times G_2$ . Therefore  $D_1 \times V_2$  dominates  $D_2 \times V_1$  and vice versa.

The sub graph  $\langle V_1 \times V_2 - (D_1 \times D_2) \rangle$  in  $G_1 \times G_2$ .  $D_2 \times V_1$  is a dominating set of  $G_1 \times G_2$ . Therefore sub graph  $\langle (V_1 - D_1) \times V_2 \rangle$  in  $G_1 \times G_2$ .  $D_2$  dominates  $V_2$ . Therefore  $D_2 \times (V_1 - D_1)$  is a dominating set of  $\langle (V_1 - D_1) \times V_2 \rangle$ . Therefore  $D_2 \times (V_1 - D_1)$  is a complementary fuzzy dominating set of  $G_1 \times G_2$ . Similarly  $D_1 \times (V_2 - D_2)$  is a complementary fuzzy dominating set of  $G_1 \times G_2$ .

#### IV. CONCLUSION

In this paper, deals complementary domination and complementary domination number for intuitionistic fuzzy graphs. Some interesting properties of these new concepts are proved. Further, discuss the bounds in operations on fuzzy graphs.

#### LITERATURE CITED

- [1] Mordeson, JN & Nair, PS 2000, 'Fuzzy Graphs and Fuzzy Hyper graphs', Physica-Verlag.
- [2] Natarajan, C & Ayyaswamy, SK 2010, 'On strong(weak) domination in fuzzy graphs', International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering, Vol.4, no.7, pp.1035-1037.
- [3] Somasundaram, A & Somasundaram, S 1998, 'Domination in Fuzzy Graphs', Pattern Recognition Lett., vol.19, pp. 787-791.
- [4] Somasundaram, A 2005, 'Domination in Fuzzy Graphs-II', Journal of Fuzzy Mathematics, vol.13, no.2, pp. 281-288.

- [5] Somasundaram, A 2005, 'Domination in products of fuzzy graphs', International Journal of Uncertainty, Fuzziness and Knowledge Based Systems, vol.13, no.2, pp.195-204.
- [6] Zimmerman, HJ 1996, Fuzzy Set Theory and its Applications, Allied Publishers.
- [7] Vinothkumar, N & Geetharamani, G 2011, 'Strong (weak) domination in fuzzy graphs', 'CIIT International Journal of Fuzzy Systems', vol.3, no.3, pp.110-114.





10.22214/IJRASET



45.98



IMPACT FACTOR:  
7.129



IMPACT FACTOR:  
7.429



# INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Call : 08813907089  (24\*7 Support on Whatsapp)