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COBB-Douglas, Constant Elasticity of Substitution (CES) and Transcendental Logarithmic Production Functions in Non-linear Type of Special Functions

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Abstract: In the literature, many Statisticians and Econometricians have described the various nonlinear statistical models in the context of production function analysis. It is no exaggeration to say that in econometric analysis, the Cobb-Douglas, Constant Elasticity of substitution (CES) and transcendental logarithmic (known as Translog) production functions are the most frequently used nonlinear type of special functions. They have been construed as the simple types of meaningful functions in both theoretical and empirical studies of production functions and have received privileged attention in the analysis of economic growth.

I. INTRODUCTION

The production analysis deals with the structural relationships between inputs and outputs of Firms. A production function is a technique which indicates the technology involved in the process of production. Mathematically, a production function can be represented by $Y = f(X_1, X_2, \ldots, X_n)$, where Y is output and X's are inputs such as labour, capital, land, raw material, energy etc. In other words the production function is a technological relationship between output and inputs by disaggregation. If output is a flow variable, all inputs must be expressed in flow terms. It is not as easy and trivial as it sounds. Generally, a production function depicts a relationship between a flow of output over a defined time interval and a flow of inputs over the same or previous time. Nonlinear production function models have been used in a wide variety of contexts and the problem of statistical inference in these models based on measurements arises in almost every corner of econometric research. The Cobb-Douglas production function model lasted 60 years before generalizations began to appear. A growing dissatisfaction with the descriptive accuracy of the function and with its econometric implications has spawned a multitude of alternative functional forms during the past four decades. First came the CES functional form, followed by several Variable Elasticity of Substitution (VES) forms and more flexible forms such as Translog and Frontier functional forms. The Cobb-Douglas form has been generalized in other ways, as have the CES, VES and Translog generalizations themselves.

II. CONCEPTS IN THE PRODUCTION FUNCTION ANALYSIS

The various concepts frequently used by Econometricians in the context of study of production function analysis are :

A. Isoquants

In the case of a firm producing a single output from two inptus, the production function may be defined as

$$y = f(x_1, x_2) \qquad \ldots (2.1)$$

Where y is the maximum possible level of output Y;

$x_1$and x_2 are the levels of the two inputs X_1 and X_2 respectively;

and f(.,.) is a function that is generally assumed to be continuously differentiable, so that the partial derivatives are continuous

Consider, $f(x_1,x_2) = y^0 = \text{constant} \qquad \ldots (2.2)$

Totally differentiating (2.2) gives

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$$\frac{\partial f}{\partial x_1}\,dx_1 + \frac{\partial f}{\partial x_2}\,dx_2 = 0 \qquad \dots (2.3)$$

$$\Rightarrow \qquad \frac{dx_2}{dx_1} = -\left[\frac{\partial f/\partial x_1}{\partial f/\partial x_2}\right] = -\left(\frac{f_1}{f_2}\right) \qquad \dots (2.4)$$

Where $f_1 = \frac{\partial f}{\partial x_1}$ and $f_2 = \frac{\partial f}{\partial x_2}$ are called marginal products of inputs X_1 and X_2 respectively.

$\frac{\partial x_2}{\partial x_1}$ represents the rate at which the entrepreneur can substitute input X_2 for X_1 (or X_1 for X_2) in order to maintain specific output level y^0.

The ratio of the marginal products of two inputs X_1 and X_2 is known as the Marginal Rate of Technical Substitution (MRTS) between two inputs X_1 and X_2. i.e., MRTS = $\frac{f_1}{f_2}$.

B. *Homogeneous Production Function And Euler's Theorem*

When the inputs are changed by the some proportion k, the function Y changes by some power of 'k'. This property renders the function is homogeneous production function. The power of the multiplicant k in the transformed function defines the degree of homogeneity of the function. It should be noted that the sum of the powers of the variables (inputs) in each term on the R.H.S is constant in a homogeneous function. If the sum is zero, then the degree of homogeneity of the function is zero.When the degree of homogeneity is one, then the function is known as the "Linearly Homogeneous Function.". Suppose $Z = f(x_1, x_2)$ is assumed to be a linearly homogeneous production function , then the following three properties hold good for all the values of inputs x_1 and x_2

1) The given linearly homogeneous functions can be written in either of the following two forms :

$$Z = x_1\phi\left(\frac{x_2}{x_1}\right), \text{ or } Z = x_2\psi\left(\frac{x_1}{x_2}\right)$$

whereϕ and ψ are some functions of a single variable.

2) The partial derivatives, $\frac{\partial Z}{\partial x_1}$ and $\frac{\partial Z}{\partial x_2}$ are functions of the ratio of x_1 to x_2

3) Linearly homogeneous functions satisfy 'Euler's theorem'.

C. *Euler's Theorem*

The value of homogeneous function can always be written as a sum of terms, each of these being the product of the first partial derivative and the corresponding input variable.

If $Z = f(x_1,x_2)$ be a linearly homogeneous function, then it satisfies the Euler's theorem as follows.

$$x_1\frac{\partial Z}{\partial x_1} + y\,\frac{\partial Z}{\partial x_2} = Z$$

D. *Elasticity Of Substitution Between Factors Of Production (σ)*

Every firm is interested in knowing of the extent of substitutability between the factors of production in order to maximise profit or minimise cost looking to the prices which the factors command in the market. The measure of substitution between the factors is known as Elasticity of Technical Substitution. This is a pure number and measures the extent to which the substitution between the factors can take place. Since the substitution depends mainly on the slope or MRTS, Elasticity of Substitution is defined as the proportionate change in the ratio between the factors due to proportionate change in MRTS.

Consider the production function $y = f(x_1,x_2)$

The elasticity of substitution between the two factors is given by

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$$\sigma = \frac{d\ln\left(\frac{x_2}{x_1}\right)}{d\ln\left(\frac{Mp_{x_1}}{Mp_{x_2}}\right)} = \frac{d\ln\left(\frac{x_2}{x_1}\right)}{d\ln\left(MRTS_{x_1\,x_2}\right)} \qquad \ldots (3.1)$$

Here, the numerator involves the ratio of X_2 to X_1 while the denominator involves the ratio of the Marginal product of X_1 to that of X_2, ensuring that σ is nonnegative. It gives a measure of the curvature of the Isoquants.The magnitude of σ is an indication of the case with which product can be maintained by substituting one factor for another. Higher the value of σ, the higher the degree of substitutability between the factors.

The value of σ lies between 0 and ∞.

$\sigma = 0 \Rightarrow$ two factors (or inputs) are incapable of substitution;

$\sigma = \infty \Rightarrow$ two factors are perfect substitutes;

$\sigma \to 0 \Rightarrow$ the shape of Isoquant tend to be an angle of 90°;

$\sigma \to \propto \Rightarrow$ the shape of Isoquant tends to be flatter.

If $y = f(x_1, x_2)$ be the linear homogeneous function, then the elasticity of substitution between x_1 and x_2 is given by

$$\sigma = \left[\frac{\frac{\partial y}{\partial x_1}\frac{\partial y}{\partial x_2}}{y\,.\frac{\partial^2 y}{\partial x_1 \partial x_2}}\right] \qquad \ldots (3.2)$$

E. Returns To Scale

Returns to scale may be constant, increasing or decreasing. If we increase all factors (i.e., scale) in a given proportion and the output increases in the same proportion, returns to scale are said to be constant. Thus, if a doubling or trebling of all factors causes a doubling or trebling of outputs, returns to scale are constant. But if the increase in all factors leads to a more than proportionate increase in output, returns to scale are said to be increasing. Thus, if all factors are doubled and output increases by more than a double, then the returns to scale are increasing. On the other hand, if the increase in all factors leads to a less than proportionate increase in the output, returns to scale are decreasing.

The k^{th} degree homogeneous production function satisfies the condition

$f(mx_1, mx_2) = m^k f(x_1, x_2), \forall\, m > 0,$

$m = 1 \Rightarrow$ Constant returns to scale ;

$m>1 \Rightarrow$ Increasing returns to scale ;

$m<1 \Rightarrow$ Decreasing returns to scale.

Thus, homogeneous production function of degree one or linearly homogeneous production function exhibits constant returns to scale in the economy.

III. SOME NONLINEAR PRODUCTION FUNCTIONS

Some important nonlinear production function models which are frequently used in practice are :

A. Cobb–Douglas Nonlinear Production Function

One of the most widely used nonlinear production functions for empirical estimation is the Cobb – Douglas production function. Cobb and Douglas (1928) proposed the general FORM of production function as

$$Y = A\, X_1^{\alpha}\, X_2^{1-\alpha} \qquad \ldots (4.1)$$

Where Y is the value added byLabour (X_1) and fixed capital (X_2);

A ,α are the fixed positive parameters.

Another form of Cobb-Douglas production function is given by

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$$Y = A\ X_1^{\alpha}\ X_2^{\beta} \qquad \ldots (4.2)$$

Where Y is the output of the firm;
X_1 is labour input;
X_2 is capital input;
A is technological coefficient parameter;
αislabour input elasticity parameter;
and β is capital input elasticity parameter.

B. Constant Elasticity Of Substitution (Ces) Nonlinear Production Function

The CES production function was first developed by Arrow, Chenery, Minhas and Solow (1961) which includes the Cobb-Douglas production function as a special case. This function has constant elasticity of substitution between the factors of production. An essential form of CES production function is given by

$$y = A\left[\delta\ X_1^{-\rho} + (1-\delta)X_2^{-\rho}\right]^{-\frac{1}{\rho}} \qquad \ldots (4.3)$$

Where y = output of firm
X_1, X_2= Two factors of production (may be capital and labour inputs respectively)
A = Efficiency parameter or Scale parameter, A>0
δ = Distribution parameter, $0<\delta<1$
ρ = Substitution parameter, $\rho \geq -1$

It can be shown that this production function is linearly homogeneous production function. It shows constant returns to scale and justifies Euler's theorem. Though this function is linearly homogeneous, the value of elasticity of substitution (σ) is not unity. By introducing returns to scale parameter(ν), the general form of CES production function is given by

$$Y = A\left[\delta\ X_1^{-\rho} + (1-\delta)X_2^{-\rho}\right]^{-\frac{\nu}{\rho}},\quad \nu>0 \qquad \ldots (4.4)$$

C. Transcendental Logarithmic (Translog) Production Function

The translog production function was initiated by Christensen, Jorgenson and Lau (1973), which is quadratic in the logarithms of the variables. This function is quite flexible in approximating arbitrary production technologies interms of substitution possibilities. It provides a local approximation to any production frontier. For n inputs, the translog production functional form is given by

$$\ln y = a_0 + \sum_{i=1}^{n} \alpha_i \ln x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} \ln x_i \ln x_j \qquad \ldots (4.5)$$

Where x_i is the i[th] input and $\beta_{ij} = \beta_{ji}$.

This function is not invariant to a change of units. The translog production function is neither additive nor homogeneous and it can easily include multiple outputs and multiple inputs. The translog production function which contains an output (Y) and four inputs say Labour (X_1), Capital (X_2), Raw material (X_3) and Energy (X_4) is specified by $y = f(x_1, x_2, x_3, x_4)$. Assuming constant returns to scale with exogenous factor prices p_1, p_2, p_3 and p_4, and imposing symmetry on the second – order partial derivatives, gives the translog cost function as

$$\ln C = a_0 + \ln Y + \alpha_1 \ln p_1 + \alpha_2 \ln p_2 + \alpha_3 \ln p_3 + \alpha_4 \ln p_4$$

$$+ \frac{1}{2}\beta_{11}(\ln p_1)^2 + \beta_{12}(\ln p_1)(\ln p_2) + \beta_{13}(\ln p_1)(\ln p_3)$$

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$$+ \beta_{14} (\ln p_1)(\ln p_4) + \frac{1}{2}\beta_{22} (\ln p_2)^2 + \beta_{23} (\ln p_2)(\ln p_3)$$

$$+ \beta_{24} (\ln p_2)(\ln p_4) + \frac{1}{2}\beta_{33} (\ln p_3)^2 + \beta_{34} (\ln p_3)(\ln p_4)$$

$$+ \frac{1}{2}\beta_{44} (\ln p_4)^2 \qquad \dots (4.6)$$

The cost share equations are given by

$$\frac{\partial \ln C}{\partial \ln p_1} = S_1 = \alpha_1 + \beta_{11}\ln p_1 + \beta_{12}\ln p_2 + \beta_{13}\ln p_3 + \beta_{14}\ln p_4$$

$$\frac{\partial \ln C}{\partial \ln p_2} = S_2 = \alpha_2 + \beta_{12}\ln p_1 + \beta_{22}\ln p_2 + \beta_{23}\ln p_3 + \beta_{24}\ln p_4$$

$$\frac{\partial \ln C}{\partial \ln p_3} = S_3 = \alpha_3 + \beta_{13}\ln p_1 + \beta_{23}\ln p_2 + \beta_{33}\ln p_3 + \beta_{34}\ln p_4 \qquad \dots (4.7)$$

$$\frac{\partial \ln C}{\partial \ln p_4} = S_4 = \alpha_1 + \beta_{14}\ln p_1 + \beta_{24}\ln p_2 + \beta_{34}\ln p_3 + \beta_{44}\ln p_4$$

Since, the shares must sum to unity,

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1 \qquad \dots (4.8)$$

and the β's sum to zero in each column (and row).
i.e.,

$$\left.\begin{aligned} \beta_{11} + \beta_{12} + \beta_{13} + \beta_{14} = 0 \\ \beta_{12} + \beta_{22} + \beta_{23} + \beta_{24} = 0 \\ \beta_{13} + \beta_{23} + \beta_{33} + \beta_{34} = 0 \\ \beta_{14} + \beta_{24} + \beta_{34} + \beta_{44} = 0 \end{aligned}\right\} \qquad \dots (4.9)$$

By imposing the row-wise β constraints on the first three share equations gives the system.

$$\left.\begin{aligned} S_1 = \alpha_1 + \beta_{11} \ln\left(\frac{p_1}{p_4}\right) + \beta_{12} \ln\left(\frac{p_2}{p_4}\right) + \beta_{13} \ln\left(\frac{p_3}{p_4}\right) \\ S_2 = \alpha_2 + \beta_{12} \ln\left(\frac{p_1}{p_4}\right) + \beta_{22} \ln\left(\frac{p_2}{p_4}\right) + \beta_{23} \ln\left(\frac{p_3}{p_4}\right) \\ S_3 = \alpha_3 + \beta_{13} \ln\left(\frac{p_1}{p_4}\right) + \beta_{23} \ln\left(\frac{p_2}{p_4}\right) + \beta_{33} \ln\left(\frac{p_3}{p_4}\right) \end{aligned}\right\} \qquad \dots (4.10)$$

IV. CONCLUSIONS

In this paper, we proposes certain nonlinear regression models. A method of estimation has been proposed to a general nonlinear model with auto correlated disturbances. A model selection criterion has been derived for testing the no nested nonlinear hypotheses. A traditional simultaneous equations model for a Cobb-Douglas production function is specified and iterative indirect least squares and two stage least squares estimation methods have been developed in this paper.

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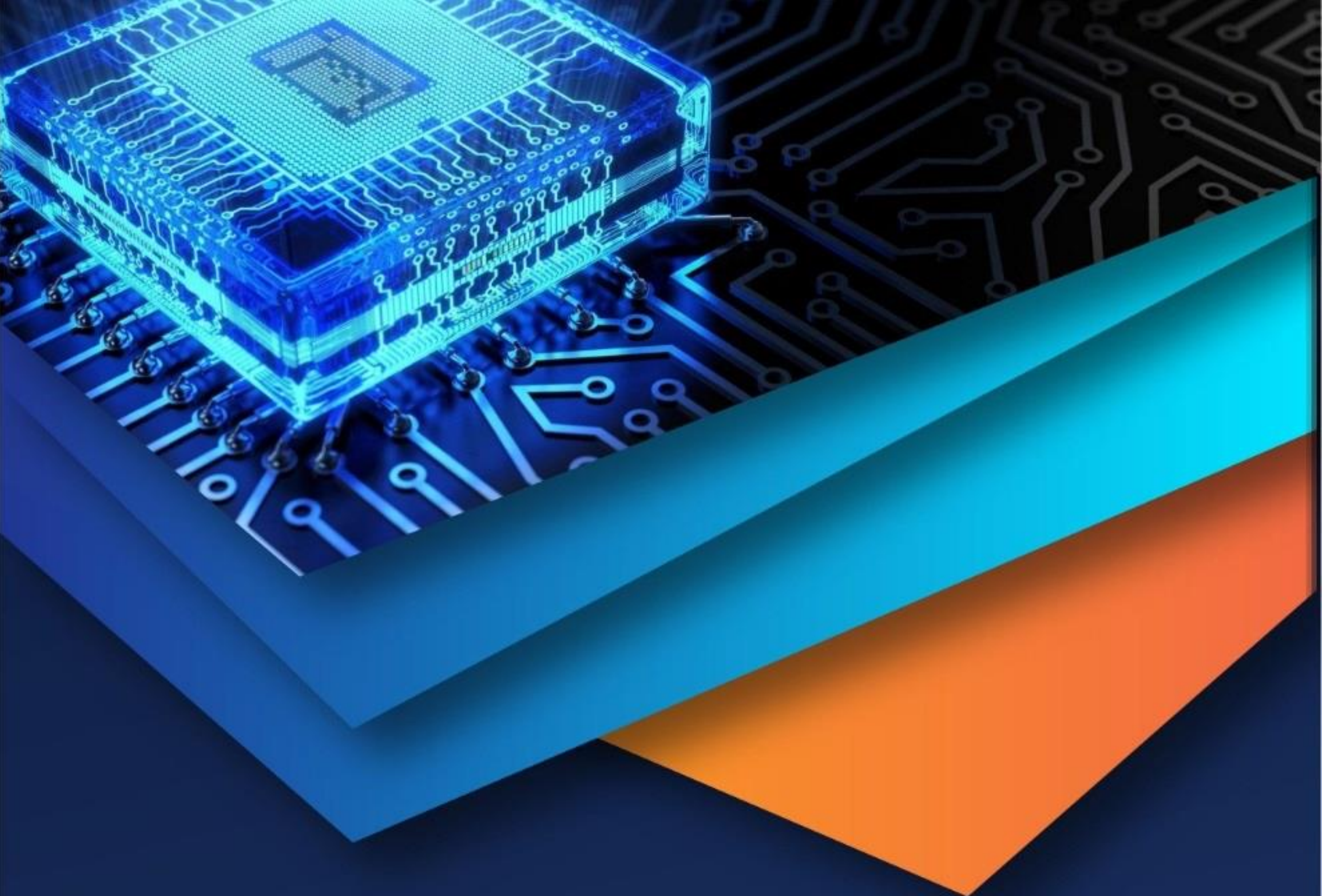

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