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Border and Exterior of Soft Semi #gα Closed Sets in Soft Bi-Topological Spaces

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Abstract*: In this paper, we introduce a new class of closed sets via, soft semi #gα-closed sets in bi-topological spaces. And also we study the concepts of border and exterior of soft semi #gα-closed sets in bi-topological spaces which are denoted by*$(1,2)^*$ *soft semi #gα-bd (F,A) and* $(1,2)^*$ *soft semi #gα-ext (F,A), where (F,A)is any soft set of (X,E) and also investigate their basic properties.*

Keywords: $(1,2)^*$***soft semi #gα--closed set,*** $(1,2)^*$***soft semi #gα--open set,*** $(1,2)^*$***soft semi #gα—interior,*** $(1,2)^*$***soft semi #gα—closure,*** $(1,2)^*$***soft semi #gα—border and*** $(1,2)^*$***soft semi #gα—exterior.***

I. INTRODUCTION

In 1963, the concepts of bi-topological spaces was originally initiated by J.C. Kelly[3]. The theory of generalized closed sets in topological spaces which was found by Levine[8] in 1970. The concepts of generalized and semi generalized closed sets was introduced and studied by Lellis[7] in classical topology. He defined a bi-topological space (X,τ_1,τ_2) to be a set X with two topologies τ_1 *and* τ_2 on X and initiated the systematic study of bi-topological spaces. The soft theory is rapidly processing in different field of mathematics. It was first proposed by Russian researcher Molodtsov[9] in 1999.Muhammad Shabir and Manazza Naz [10] introduced soft topological spaces in 2011. It was defined over an initial universe with a fixed set of parameters. N. Cagman and S. Karatas[2] introduced topology on a set called “soft topology” and initiated the theory of soft topological spaces in 2013.In this paper we defined and examined the basic properties of $(1,2)^*$ soft semi #gα-border and $(1,2)^*$ soft semi #gα-exterior in soft bi-topological spaces and study their properties.

II. PRELIMINARIES

In this section we have presented some of the basic definitions and results of soft set, soft topological space, bi-topological space to use in the sequel. Throughout this paper, X is an initial universe, E is the set of parameters, P(X) is the power set of X, and A⊆ X.

A. Definition 2.1.

Let $\tilde{\tau}$ be the collection of soft sets over X, then $\tilde{\tau}$ is called a soft topology on X if$\tilde{\tau}$ satisfies the following axioms:
$\emptyset$, $\tilde{X}$ belongs to $\tilde{\tau}$.
The union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
The triplet (X,$\tilde{\tau}$,E) is called a soft topological space over X. For simplicity, we can take the soft topological space (X,$\tilde{\tau}$,E) as X throughout the work.

B. Definition 2.2.

A set X together with two different topologies is called bi-topological space. It is denoted by (X,τ_1,τ_2).

C. Definition 2.3.

A soft set (F,A) of a soft topological space (X,$\tilde{\tau}$,E) is called

1) soft $\alpha -$ closed [4] if $\tilde{s}cl(\tilde{s}\,int\,(\tilde{s}cl(F,A)))$ $\tilde{\subseteq}$ (F,A). The complement of soft α-closed set is called soft α-open.
2) soft semi – closed [2] if $\tilde{s}int(\tilde{s}cl(F,A))$ $\tilde{\subseteq}$ (F,A). The complement of soft semi – closed set is called soft semi-open.
3) soft g-closed [5] if $\tilde{s}cl($F,A$)$ $\tilde{\subseteq}$ (U,E), whenever (F,A) $\tilde{\subseteq}$ (U,E) and (U,E) is soft open in (X , $\tilde{\tau}$, E). The complement of soft g-closed set is called soft g-open.

4) soft $g^{\#}$ α- closed [6] if $\tilde{s}\,\alpha cl$(F,A) ⊆̃ (U,E), whenever (F,A) ⊆̃ (U,E) and (U,E) is soft g-open in (X , $\tilde{\tau}$, E). The complement of soft $g^{\#}$ α -closed set is called soft $g^{\#}$ α -open.soft #gα- closed [8] if $\tilde{s}\,\alpha cl$(F,A) ⊆̃ (U,E), whenever (F,A) ⊆̃ (U,E) and (U,E) is soft $g^{\#}$α- open in (X , $\tilde{\tau}$, E). The complement of soft #gα -closed set is called soft #gα -open.
5) soft semi #gα-closed [9] if $\tilde{s}\,scl$(F,A) ⊆̃ (U,E), whenever (F,A) ⊆̃ (U,E) and (U,E) is soft #gα- open in (X , $\tilde{\tau}$, E). The complement of soft semi #gα -closed set is called soft semi #gα –open
6) The union of all soft semi #gα open sets [10] each contained in a set (F,A) of (X , $\tilde{\tau}$, E) is called soft semi #gα interior of (F,A) which is denoted by $\tilde{s}$ semi #gα-int(F,A)
7) The intersection of all soft semi #gα- closed sets [10], each containing a set (F,A) of (X , $\tilde{\tau}$, E) is called soft semi #gα-closure of (F,A), which is denoted by $\tilde{s}$ semi #gα-closure of (F,A).

D. Definition 2.4.

Let X be a non-empty soft set on the universe X ,$\tilde{\tau}_1$,$\tilde{\tau}_2$ are different soft topologies on $\tilde{X}$. Then ($\tilde{X}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$)is called a soft bi-topological space.

E. Definition 2.5.

Let F_A ∈S(U). Power soft set of F_A is defined by ,$\tilde{P}$ (F_A) = {F_{Ai} ⊆̃ F_A: i ∈ I}
And its cardinality is defined by$|\tilde{P}\ (F_A)|$ = 2 $\sum_{x\in E}|f_A(x)|$ where $|f_A(X)|$ is cardinality of $f_A(X)$.

F. Example 2.6.

Let U = {u_1,u_2,u_3} , E = {x_1,x_2} and F_E = X = {(x_1 ,{u_1,u_2,u_3}), (x_2 ,{u_1,u_2,u_3})}. And let ($\tilde{X}$, $\tilde{\tau}_1$, $\tilde{\tau}_2$) be a soft bi-topological space , where $\tilde{\tau}_1$ = {∅, $_2$, $_3$, $_5$, X } , $\tilde{}_2$ = { , $_2$, $_8$, $_{14}$, X } , then $\widetilde{}_{1,2}$ soft open sets are { , $_2$ $_3$, $_5$, $_8$, $_{14}$, $_{17}$, $_{32}$, X }and $\widetilde{}_{1,2}$ soft closed sets are { , $_4$ $_6$, $_7$, $_{12}$, $_{31}$, $_{44}$, $_{46}$,X }.
Then ,

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$_3$= {($_1$,{ $_2$})}
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$_{12}$ = {($_2$,{ $_2$, $_3$})}
$_{13}$= {($_1$,{ $_3$, $_1$})}
$_{14}$= {($_1$,{ $_1$} ,($_2$,{ $_1$})}
$_{15}$= {($_1$,{ $_1$} ,($_2$,{ $_2$})}
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$_{18}$ = {($_1$,{ $_2$} ,($_2$,{ $_2$})}
$_{19}$= {($_1$,{ $_2$} ,($_2$,{ $_1$, $_2$})}
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${}_{52}= \{({}_1 ,\{ {}_1, {}_2, {}_3\},({}_2,\{ {}_2, {}_3\})\}$

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${}_{56}= \{({}_1 ,\{ {}_1, {}_2\},({}_2,\{ {}_1, {}_2,u_3\})\}$

${}_{57}= \{({}_1 ,\{ {}_3\},({}_2,\{ {}_1, {}_2, {}_3\})\}$

${}_{58}= \{({}_1 ,\{ {}_2, {}_3\},({}_2,\{ {}_1, {}_2, {}_3\})\}$

${}_{59}= \{({}_1 , {}_1, {}_3\},({}_2,\{ {}_1, {}_2, {}_3\})\}$

${}_{60}= \{({}_1 \{ {}_1, {}_3\},({}_2,\{ {}_1, {}_3\})\}$

${}_{61}=\{({}_1 \{ {}_1, {}_2\},({}_2,\{ {}_1, {}_3\})\}$

${}_{62}= \{({}_1 \{ {}_2, {}_3\},({}_2,\{ {}_1, {}_3\})\}$

${}_{63}= \{ ({}_2,\{ {}_1, {}_2, {}_3\})\}$

$_{64}$= {($_1$,{ $_1$, $_2$, $_3$}, ($_2$,{ $_1$, $_2$, $_3$})} = X. Are all soft subsets of . So |$\tilde{}$ ( ) | = 2^6 = 64

G. Definition 2.7.

A soft set (F,A) of a soft bi-topological space ($\tilde{}$, $\tilde{}_1$, $\tilde{}_2$,E) is called $(1,2)^*$soft semi #gα-closed if $\tilde{}$scl(F,A)⊆̃ (U,E), whenever (F,A) ⊆̃ (U,E) and (U,E) is $(1,2)^*$ soft #gα-open in ($\tilde{}$, $\tilde{}_1$, $\tilde{}_2$,E). The complement of $(1,2)^*$ soft semi #gα-closed set is called $(1,2)^*$ soft semi #gα-open.

H. Theorem 2.8.

If (F,A) and $\tilde{}_{1,2}$(G,B) are soft subset of (X,E), then

1) (F,A) is $(1,2)^*$ soft semi #gα-open iff $(1,2)^*$ soft semi #gα-int(F,A) ≅ (F,A).
2) $(1,2)^*$ soft semi #gα-int (F,A) is $(1,2)^*$ soft semi #gα − open
3) (F,A) is $(1,2)^*$ soft semi #gα-closed iff$(1,2)^*$ soft semi #gα-cl(F,A) ≅ (F,A)
4) $(1,2)^*$soft semi #gα-cl (F,A) is $(1,2)^*$ soft semi #gα-closed.
5) $(1,2)^*$ soft semi #gα-cl ((X,E)\(F,A)) ≅ (X,E)\ $(1,2)^*$ soft semi #gα-int(F,A).
6) $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)) ≅ (X,E)\ $(1,2)^*$ soft semi #gα-cl(F,A).
7) If (F,A) is $(1,2)^*$ soft semi #gα-open in ($\tilde{}$, $\tilde{}_1$, $\tilde{}_2$,E) and $\tilde{}_{1,2}$(G,B) is $(1,2)^*$ soft semi #gα-open in ($\tilde{}$, $\tilde{}_1$, $\tilde{}_2$,E), then (F,A)∩̃ $\tilde{}_{1,2}$(G,B) is $(1,2)^*$ soft semi #gα-open in ($\tilde{}$, $\tilde{}_1$, $\tilde{}_2$,E)
8) A point x ∈̃ $(1,2)^*$ soft semi #gα-cl (F,A) iff every $(1,2)^*$ soft semi #gα-open set in (X,E) containing x intersects (F,A).
9) Arbitary intersection of $(1,2)^*$ soft semi #gα-closed sets in ($\tilde{}$, $\tilde{}_1$, $\tilde{}_2$,E) is also $(1,2)^*$ soft semi #gα-closed in ($\tilde{}$, $\tilde{}_1$, $\tilde{}_2$,E).

Proof We know that ,$(1,2)^*$ soft union of all $(1,2)^*$ soft open sets contained in (F,A) is called $\tilde{}$ int(F,A). So $(1,2)^*$ soft semi #gα-open sets also is in (F,A).

Since $(1,2)^*$ soft semi #gα-int (F,A) are$(1,2)^*$ soft open sets. Therefore $(1,2)^*$ soft semi #gα-int(F,A) is $(1,2)^*$ soft semi #gα-open sets.

Let the $(1,2)^*$ soft intersection of all $(1,2)^*$ soft closed sets containing (F,A) is called $\tilde{}$ cl(F,A). So $(1,2)^*$ soft semi #gα-closed sets is in (F,A). (iv) is similar to (iii).

$(1,2)^*$ soft complement of (X,E) and (F,A) is equal to $(1,2)^*$ soft complement of (X,E) and$(1,2)^*$ soft semi #gα-int (F,A).

Similarly (vi) can be proved. (vii) and (viii) are follow from the definition of $(1,2)^*$ soft interior. (ix) obivious from the definition of $(1,2)^*$ soft closure.

I. Definition 2.9.

For any soft subset (F,A) of $\tilde{}_{1,2}$(X,E),

The soft border of (F,A) is defined by soft bd (F,A) ≅ (F,A)\ $(1,2)^*$ soft int(F,A).

The soft exterior of (F,A) is defined by soft ext (F,A) ≅ $(1,2)^*$ soft int((X,E) \ (F,A)).

III.SOFT SEMI #G-ALPHA BORDER AND EXTERIOR OF A SET IN BI-TOPOLOGICAL SPACES

In this section, we introduce and study the concepts of border and exterior of soft semi #gα-closed sets in soft bi-topological spaces.

A. Definition 3.1.

For any soft subset (F,A) of $\tilde{}_{1,2}$(X,E) , $(1,2)^*$ soft semi #gα-border of (F,A) is defined by

$(1,2)^*$soft semi #gα-bd (F,A) ≅ (F,A)\ $(1,2)^*$ soft semi #gα- int(F,A).

B. Theorem 3.2.

In a soft bi-topological space (X, $\tilde{}_{1,2}$,E), for any soft subset (F,A) of (X,E), the following statements hold.

$(1,2)^*$soft semi #gα-bd (□)≅ $(1,2)^*$ soft semi #gα-bd (X,E) ≅□.

$(1,2)^*$soft semi #gα-bd (F,A) ⊆̃ (F,A).

(F,A)≅ $(1,2)^*$ soft semi #gα- int (F,A) ∪̃ $(1,2)^*$ soft semi #gα-bd (F,A).

$(1,2)^*$soft semi #gα-int (F,A)∩̃ $(1,2)^*$ soft semi #gα-bd (F,A) ≅□.

soft semi #gα-int (F,A) ≅ (F,A) \$(1,2)^*$ soft semi #gα-bd (F,A).

Proof Let us take $(1,2)^*$soft semi #gα-bd (□) is in $(1,2)^*$ soft semi #gα-bd (X,E) and is an empty set.

$(1,2)^*$soft semi #gα-bd (F,A) it should be any subset of (F,A).
Since soft union of $(1,2)^*$ soft semi #gα-bd (F,A) and $(1,2)^*$ soft semi #gα-int(F,A) are for any subset of (F,A).
The soft intersection of $(1,2)^*$soft semi #gα-bd (F,A) and $(1,2)^*$ soft semi #gα-int (F,A) are empty set.
The soft complement of $(1,2)^*$soft semi #gα-bd (F,A) is $(1,2)^*$ soft semi #gα-int (F,A).

C. Theorem 3.3.

$(1,2)^*$ soft semi #gα-int($(1,2)^*$ soft semi #gα-bd (F,A)) $\cong$□.(F,A) is $(1,2)^*$ soft semi #gα-open if and only if $(1,2)^*$ soft semi #gα-bd (F,A) $\cong$□.
soft semi #gα-bd ($(1,2)^*$ soft semi #gα-int(F,A))$\cong$□.
$(1,2)^*$ soft semi #gα-bd ($(1,2)^*$ soft semi #gα-bd(F,A)) $\cong$ $(1,2)^*$ soft semi #gα-bd (F,A).
soft semi #gα-bd (F,A) $\cong$ (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-cl((X,E) \ (F,A)).
Proof:Let x $\tilde{\in}$ $(1,2)^*$soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd (F,A)). Then x $\tilde{\in}$ $(1,2)^*$ soft semi #gα-bd(F,A), since $(1,2)^*$ soft semi #gα-bd (F,A) $\tilde{\subseteq}$ (F,A), x $\tilde{\in}$ $(1,2)^*$ soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd (F,A)) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-int (F,A). Therefore x $\tilde{\in}$ $(1,2)^*$ soft semi #gα-int (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-bd(F,A) which is contradiction to the above theorem (iv). Thus (i) is proved.
(F,A) is $(1,2)^*$ soft semi #gα-open iff $(1,2)^*$ soft semi #gα-int(F,A) $\cong$ (F,A) [Theorem 2.8(i)]. But $(1,2)^*$ soft semi #gα-bd (F,A) $\cong$ (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) implies $(1,2)^*$ soft semi #gα-bd (F,A)$\cong$□. This proves (ii) and (iii).
And when (F,A) $\cong$ $(1,2)^*$ soft semi #gα-bd (F,A) Definition 3.1 becomes $(1,2)^*$ soft semi #gα-bd ($(1,2)^*$ soft semi #gα-bd (F,A)) $\cong$ $(1,2)^*$ soft semi #gα-bd (F,A) \ $(1,2)^*$ soft semi #gα-int ($(1,2)^*$ soft semi #gα-bd(F,A)). Using (iii), we will get (iv).
(v) $(1,2)^*$ soft semi #gα-bd (F,A) $\cong$ (F,A) \ $(1,2)^*$ soft semi #gα-int (F,A) $\cong$ (F,A) $\tilde{\cap}$ ($\sim_{1,2}$(X,E) \ $(1,2)^*$ soft semi #gα-int (F,A)) $\cong$ (F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-cl((X,E) \ (F,A)) [Theorem 2.8 (v)]. Hence (v) is also proved.

D. Definition.3.4.

For any soft subset (F,A) of $\sim_{1,2}$(X,E) , its $(1,2)^*$ soft semi #gα-exterior is defined by, $(1,2)^*$ soft semi #gα-ext (F,A) $\cong$ $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)).

E. Theorem 3.5.

For any $\cong$ $(1,2)^*$soft subets (F,A) and $\sim_{1,2}$(G,B) of $\sim_{1,2}$(X,E), in soft bi-topological space (X, $\sim_{1,2}$,E), the following statements hold.
$(1,2)^*$soft semi #gα-ext (□) $\cong$ $(1,2)^*$ soft semi #gα-ext (X,E) $\cong$ $\sim$.
If (F,A) $\tilde{\subseteq}$ $\sim_{1,2}$(G,B), then $(1,2)^*$ soft semi #gα-ext (G,B) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-ext (F,A).
$(1,2)^*$soft semi #gα-ext (F,A) is $(1,2)^*$ soft semi #gα-open.
(F,A) is $(1,2)^*$ soft semi #gα-closed if and only if $(1,2)^*$ soft semi #gα-ext (F,A) $\cong$ $\sim_{1,2}$(X,E) \ (F,A).
$(1,2)^*$ soft semi #gα-ext (F,A) $\cong$(X,E)\ $(1,2)^*$ soft semi #gα-cl(F,A).
Proof:Let us take (i) $(1,2)^*$soft semi #gα-ext (□) is in $(1,2)^*$ soft semi #gα-ext (X,E) and is an empty set.
And (ii) if any soft subset of (F,A) is contained in $\sim_{1,2}$(G,B) then, $(1,2)^*$ soft semi #gα-ext (F,A) is always contained in $(1,2)^*$ soft semi #gα-ext (G,B).
Since $(1,2)^*$soft semi #gα-int (F,A) is $(1,2)^*$ soft semi #gα-open, proof of (iii) is follow from the definition 3.4.
is$(1,2)^*$ soft semi #gα-cl (F,A) is $(1,2)^*$ soft semi #gα-closed.
Since $(1,2)^*$ soft semi #gα-int((X,E) \ (F,A))$\cong$ (X,E) \ $(1,2)^*$ soft semi #gα-cl(F,A), (v) follows from definition 3.4.

F. Theorem 3.6.

$(1,2)^*$ soft semi #gα-ext ($(1,2)^*$soft semi #gα-ext (F,A)) $\cong$ $(1,2)^*$ soft semi #gα-int($(1,2)^*$soft semi #gα-cl(F,A)).
(F,A) is $(1,2)^*$ soft semi #gα-regular, then $(1,2)^*$ soft semi #gα-ext($(1,2)^*$soft semi #gα-ext (F,A))$\cong$ (F,A).
$(1,2)^*$soft semi #gα-ext (F,A) $\cong$ $(1,2)^*$soft semi #gα-ext ((X,E) \($(1,2)^*$soft semi #gα-ext (F,A)).
$(1,2)^*$soft semi #gα-int (F,A) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-ext ($(1,2)^*$ soft semi #gα-ext (F,A)).
Proof:Since $(1,2)^*$ soft semi #gα- int (X,E)\(F,A)) $\cong$ (X,E)\ $(1,2)^*$ soft semi #gα-ext(F,A), (i) follows from definition 3.4.Similarly (ii) can be proved. If (F,A) is $(1,2)^*$ soft semi #gα-regular, from the above theorem (iv), we have $(1,2)^*$ soft semi #gα-ext (F,A) $\cong$(X,E)\(F,A) which is also $(1,2)^*$ soft semi #gα-regular.

Thus $(1,2)^*$soft semi #gα-ext(soft semi #gα-ext (F,A)) $\cong$(F,A), (ii) is proved. (iii) $(1,2)^*$ soft semi #gα-ext((X,E) \ $(1,2)^*$ soft semi #gα-ext (F,A)) $\cong$ $(1,2)^*$ soft semi #gα-ext ((X,E)\ $(1,2)^*$ soft semi #gα-int ((X,E)\(F,A))) $\cong$ $(1,2)^*$ soft semi #gα-int ((X,E)\((X,E)\ $(1,2)^*$ soft semi #gα-int((X,E)\(F,A)))) $\cong$ $(1,2)^*$ soft semi #gα-int(($1,2)^*$ soft semi #gα-int(X,E)\(F,A)$\cong$ (($1,2)^*$ soft semi #gα-int (X,E)\(F,A)) $\cong$ $(1,2)^*$ soft semi #gα-ext(F,A). Hence (iii) is proved.
Since (F,A) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-cl(F,A), using (i) (iv) can be proved.

G. Theorem 3.7.

(X,E) $\cong$ $(1,2)^*$ soft semi #gα-int(F,A) $\tilde{\cup}$ $(1,2)^*$ soft semi #gα-ext (F,A) $\tilde{\cup}$ $(1,2)^*$ soft semi #gα-fr (B,E).
$(1,2)^*$ soft semi #gα-ext ((F,A) $\tilde{\cup}$ $\tilde{\ }_{1,2}$(G,B)) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-ext(F,A) $\tilde{\cap}$ $(1,2)^*$ soft semi #gα-ext (B,E).
$(1,2)^*$ soft semi #gα-ext ((F,A) $\tilde{\cap}$ $\tilde{\ }_{1,2}$(G,B) $\tilde{\subseteq}$ $(1,2)^*$ soft semi #gα-ext(F,A) $\tilde{\cup}$ $(1,2)^*$ soft semi #gα-ext (B,E).
Proof If we know that complement of $(1,2)^*$soft semi #gα-int (X,E) is in (F,A) and union of $(1,2)^*$ soft semi #gα-int(F,A) is $\tilde{\ }_{1,2}$(X,E).
And then any subset of (F,A) of $\tilde{\ }_{1,2}$(,) , its exterior is complement of $\tilde{\ }_{1,2}$(X,E) , its exterior is complement of $\tilde{\ }_{1,2}$(X,E) and (F,A). So union of all$(1,2)^*$ soft semi #gα-interior , exterior and frontier is in $\tilde{\ }_{1,2}$(X,E). Hrence (i) is proved.
Proof of (ii) union of all exterior of (F,A) and $\tilde{\ }_{1,2}$(G,B) is contained in intersection of $(1,2)^*$ soft semi #gα-ext(F,A) and $(1,2)^*$ soft semi #gα-ext (B,E). Hence (ii) is proved.
And next , proof of (iii) intersection of all exterior of (F,A) and $\tilde{\ }_{1,2}$(G,B) is contained in union of all $(1,2)^*$ soft semi #gα-ext(F,A) and $(1,2)^*$ soft semi #gα-ext (B,E). Hence (ii) is proved.
Example 3.8.
Let U = $\{\ _1,\ _2,\ _3\}$, E = $\{\ _1,\ _2\}$ and = X = $\{(\ _1,\{\ _1,\ _2,\ _3\}), (\ _2,\{\ _1,\ _2,\ _3\})\}$. And let (X , $\tilde{\ }_1$, $\tilde{\ }_2$) be a soft bi-topological space , where $\tilde{\ }_1$ = $\{\ ,\ _2,\ _3,\ _5,$ X $\}$, $\tilde{\ }_2$ = $\{\ ,\ _2,\ _8,\ _{14},$ X $\}$, then $\tilde{\ }_{1,2}$ soft open sets are $\{\ ,\ _2\ _3,\ _5,\ _8,\ _{14},\ _{17},\ _{32}$, X$\}$ and $\tilde{\ }_{1,2}$ soft closed sets are $\{\ ,\ _4\ _6,\ _7,\ _{12},\ _{31},\ _{44},\ _{46}$, X $\}$. Consider a $(1,2)^*$soft semi #gα-closed set $\tilde{\ }$ = $\{(\ _1,\{\ _1,\ _2,\ _3\}), (\ _2,\{\ _1\})\}$
$(1,2)^*$soft semi #gα-border :
$\{\ \{(\ _1,\{\ _1,\ _2,\ _3\}), (\ _2,\{\ _1\})\}\ \}$
$(1,2)^*$soft semi #gα-exterior :
$\{\ \{(\ _1,\{\ _1,\ _2,\ _3\}), (\ _2,\{\ _1\})\}$

IV. CONCLUSIONS

In this paper ,Border and Exterior of soft semi gα-closed sets in soft bi-topological spaces were introduced and studied with already existing sets in soft bi-topological spaces. The scope for further research can be focused on the applications of soft bi-topological spaces.

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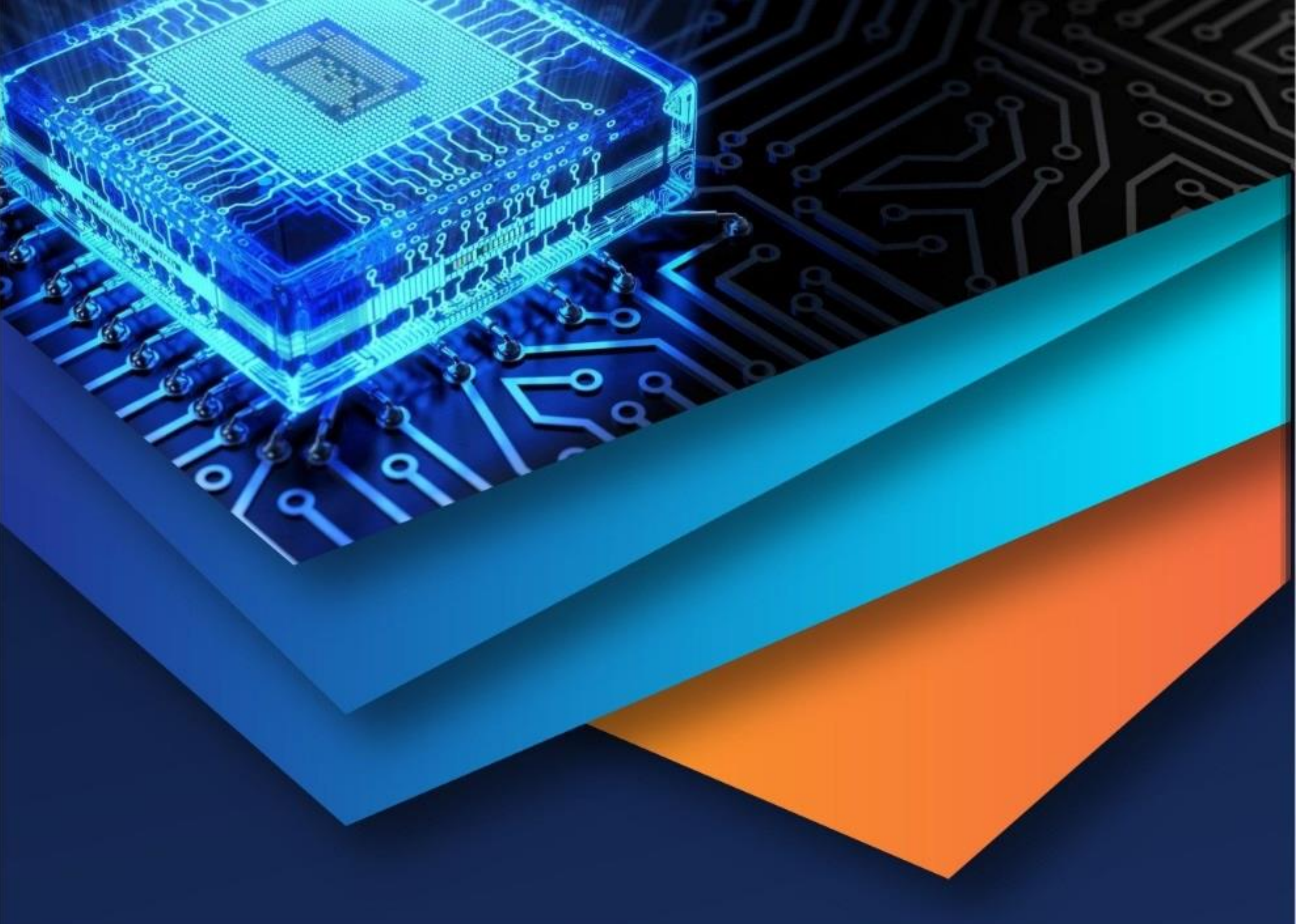

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