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# Two Wheeled Self Balancing Vehicle

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**Abstract:** *The main concept and the challenge of two wheeled self balancing vehicle is to bring out the stability of the vehicle. In this paper, the stability is analysed by comparing three types of controller in order to identify the desired stable output. A non-model controller of fuzzy mandian and modelled linear controller of PID and Linear quadratic regulator (LQR) is implemented. A common transfer function is derived from a state space equation of the system model. The same values are implemented in all the controllers to determine the better stability output.*

**Keywords:** *Stability, Fuzzy, PID, LQR, state space equation.*

## I. INTRODUCTION

Two wheel transporter moves from one place to another with simple operation. It is like a scooter but the wheels are parallel to each other. It was first invented by Dean kamen in 2007. It is widely used for its advantages over energy saving, simple structure, safety, consumer acceptance, parking impact and land use. It is a mode of transportation where it can be used by golfers, individual sightseeing, for a short distance travelling which reduces the human walk. The main key factor for this vehicle is stability. As the vehicle is running only in two wheels the stability of the wheel with the cart and the pendulum is must for the movement. For a slight movement of vehicle, it starts to fall down until we control the angle of the cart and the pendulum. It is similar to the operation of inverted Pendulum. In inverted pendulum, it is kept in opposite direction which is of pi angle. And that pendulum should be controlled from falling. The same concept is applied in the two wheeled inverted pendulum. In vehicle it consists of cart and the pendulum. Depending on the movement of the cart the pendulum should be balanced. If a person is travelling in a vehicle he should move front or back in order to balance the vehicle. When a person moves forward the vehicle moves forward, and to run the vehicle backwards the person should lean back words. To move the vehicle left or right the movable steering is connected to the cart. The angle between the pi angle and the deviation of the pendulum is known as pitch angle. And that pitch angle is sensed by the accelerometer and gyroscope. Depending on the capacity of the battery the vehicle distance is determined. These all functions are based on the controller.

In order to control the vehicle, we use fuzzy, PID and LQR controllers. Even though these controllers are changed period vice, the LQR controller shows the better stability than fuzzy and PID for this system. The system is a non-linear model, it is converted into a linear model using state space equation. By comparing the rise time and settling time of three controller's better stability is determined. The main concept is to reduce the complexity of the mathematical model and to bring out the good stability of the system. They replace the heavy vehicle used for personal short-range transportation. They are eco-friendly and reduces the gasoline consumption, reduces air pollution and consumption of fossil fuels. They are used in both indoor and outdoor environment. This system is carried out in MATLAB/SIMULINK.

## II. SYSTEM MODEL

The model has the cart and the pendulum

For simplification, let us assume it is of inverted pendulum attached to a cart.

$g$  = Gravity constant

$\Psi$  = Angle deviation

$L$  = Length of the pendulum

$m$  = Mass of the cart

$M$  = Mass of the pendulum

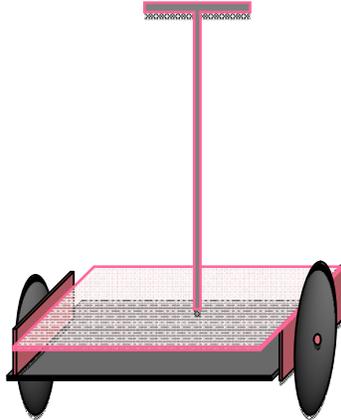
$x$  = Movement of direction

$f$  = Friction

$F_x$  = Force between cart and pendulum in x direction

$F_y$  = Force between cart and pendulum in y direction

A. Vehicle Model



Equation of the cart

$$F_{input} = m\ddot{x} + f\dot{x} + fx \quad \dots\dots\dots (1)$$

Equation of the pendulum

$$F_x = m\ddot{x} + ml\ddot{\theta}\cos\theta - Ml\dot{\theta}^2\sin\theta \dots\dots\dots (2)$$

when the force is perpendicular to the pendulum.

$$F_y\sin\theta + F_x\cos\theta - Mgsin\theta = Ml\ddot{\theta} + M\ddot{x}\cos\theta \quad \dots\dots\dots (3)$$

Torque acting on the centre of the pendulum is given as

$$-F_y l\sin\theta - F_x l\cos\theta = I\ddot{\theta} \quad \dots\dots\dots (4)$$

On combining cart and pendulum equation:

From equations (1) and (2)

$$F_{input} = (m+M)\ddot{x} + f\dot{x} + Ml\ddot{\theta}\cos\theta - Ml\dot{\theta}^2\sin\theta \quad \dots\dots\dots (5)$$

Combining (2) and (3)

$$(I + Ml^2)\ddot{\theta} + Mgl\sin\theta = -Ml\ddot{x}\cos\theta \quad \dots\dots\dots (6)$$

B. Linearizing the equations

To determine the transfer function from the equations (5) and (6) for the position x and angle deviation  $\Psi$

$$\cos\theta = \cos(\pi+\Psi) \approx -1 \quad \dots\dots\dots (7)$$

$$\sin\theta = \sin(\pi+\Psi) \approx -\Psi \quad \dots\dots\dots (8)$$

$$\theta^2 = \Psi^2 \approx 0 \quad \dots\dots\dots (9)$$

The linearized equation is from (5), (6), (7), (8), (9)

$$(I + Ml^2)\ddot{\Psi} - Mgl\Psi = Ml\ddot{x} \dots\dots\dots (10)$$

$$U = (m + M)\ddot{x} + f\dot{x} - Ml\ddot{\Psi} \dots\dots\dots (11)$$

Taking Laplace transform for (10) and (11), we get

$$(I + Ml^2)\Psi(s)s^2 - Mgl\Psi(s) = MlX(s)s^2 \quad \dots\dots\dots (12)$$

$$u(s) = (m + M) X(s)s^2 + f X(s)s - Ml\Psi(s)s^2 \quad \dots\dots\dots (13)$$

From (12)

$$X(s) = \left[ \frac{I+Ml}{Ml} - \frac{g}{s} \right] \Psi(s) \quad \dots\dots\dots (14)$$

Substitute (14) in (13)

$$u(s) = (m + M) \left[ \frac{I+Ml}{Ml} - \frac{g}{s} \right] \Psi(s)s^2 + \left[ \frac{I+Ml}{Ml} - \right] \Psi(s)s - Ml\Psi(s)s^2 \quad \dots\dots\dots (15)$$

On rearranging equation (15)

$$\Psi(s) = \frac{\frac{M}{q}s}{s + \frac{f(I+Ml)}{q} - \frac{(m+M)Mgl}{q} - \frac{fMgl}{q}} u(s)$$

where,

$$q = [(m + M) (I + Ml^2) - (Ml)^2]$$

Transfer function for cart position X (s) is

$$X(s) = \frac{(I+Ml)s - gMl}{s + f\left(\frac{I+Ml}{q}\right)s - \frac{(m+M)Mgl}{q} s - \frac{fmpgl}{q}} u(s)$$

In state space method,

$$\dot{x} = A x + B u$$

$$y = C x + D u$$

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \\ \dot{\psi} \\ \ddot{\psi} \end{pmatrix} = A \begin{pmatrix} x \\ \dot{x} \\ \psi \\ \dot{\psi} \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & A_{22} & A_{23} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & A_{42} & A_{43} & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ B_{12} \\ 0 \\ B_{14} \end{pmatrix}$$

Where,

$$A_{22} = \frac{-(I+Ml^2)f}{I(m+M)+mMl^2}$$

$$A_{23} = \frac{M^2gl^2}{I(m+M)+mMl^2}$$

$$A_{42} = \frac{-Mfl}{I(m+M)+mMl^2}$$

$$A_{43} = \frac{Mgl(m+m)}{I(m+M)+mMl^2}$$

$$B_{12} = \frac{I+Ml^2}{I(m+M)+mMl^2}$$

$$B_{14} = \frac{Ml}{I(m+M)+mMl^2}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The values for the parameters are given

Table 1

M(Kg)	M(Kg)	I(Kgm <sup>2</sup> )	L(m)	g(m/sec <sup>2</sup> )
1.5	0.2	0.055	0.05	9.8

The state space equation of the system is

$$\begin{pmatrix} \dot{x} \\ \dot{\dot{x}} \\ \dot{\psi} \\ \dot{\dot{\psi}} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0134 & 95.156 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1.425 & 180.79 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \psi \\ \dot{\psi} \end{pmatrix} + \begin{pmatrix} 0 \\ 0.9092 \\ 0 \\ 9.699 \end{pmatrix} u$$

$$Y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \\ \psi \\ \dot{\psi} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} u$$

The transfer function of the system is

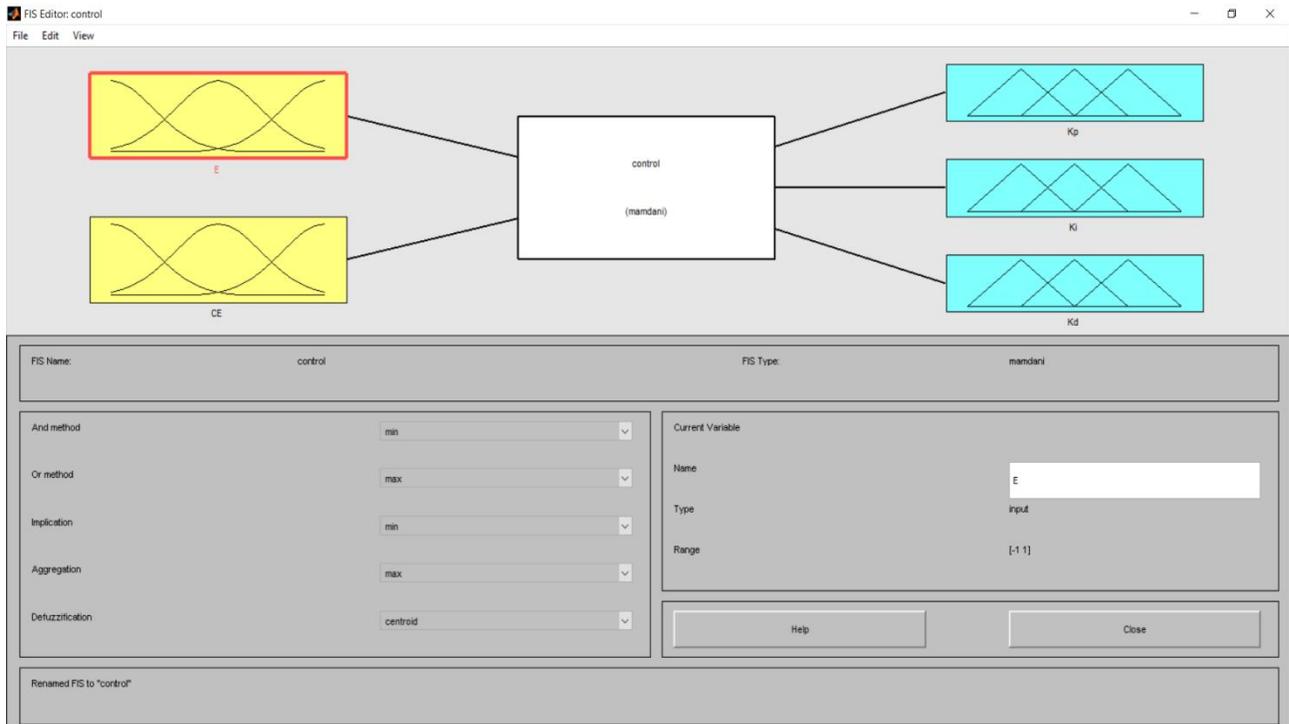
$$G(s) = \frac{102s+173.5}{s^3+2.64s^2-83.60s-307.2}$$

### III. DESIGN OFFUZZYCONTROLLER

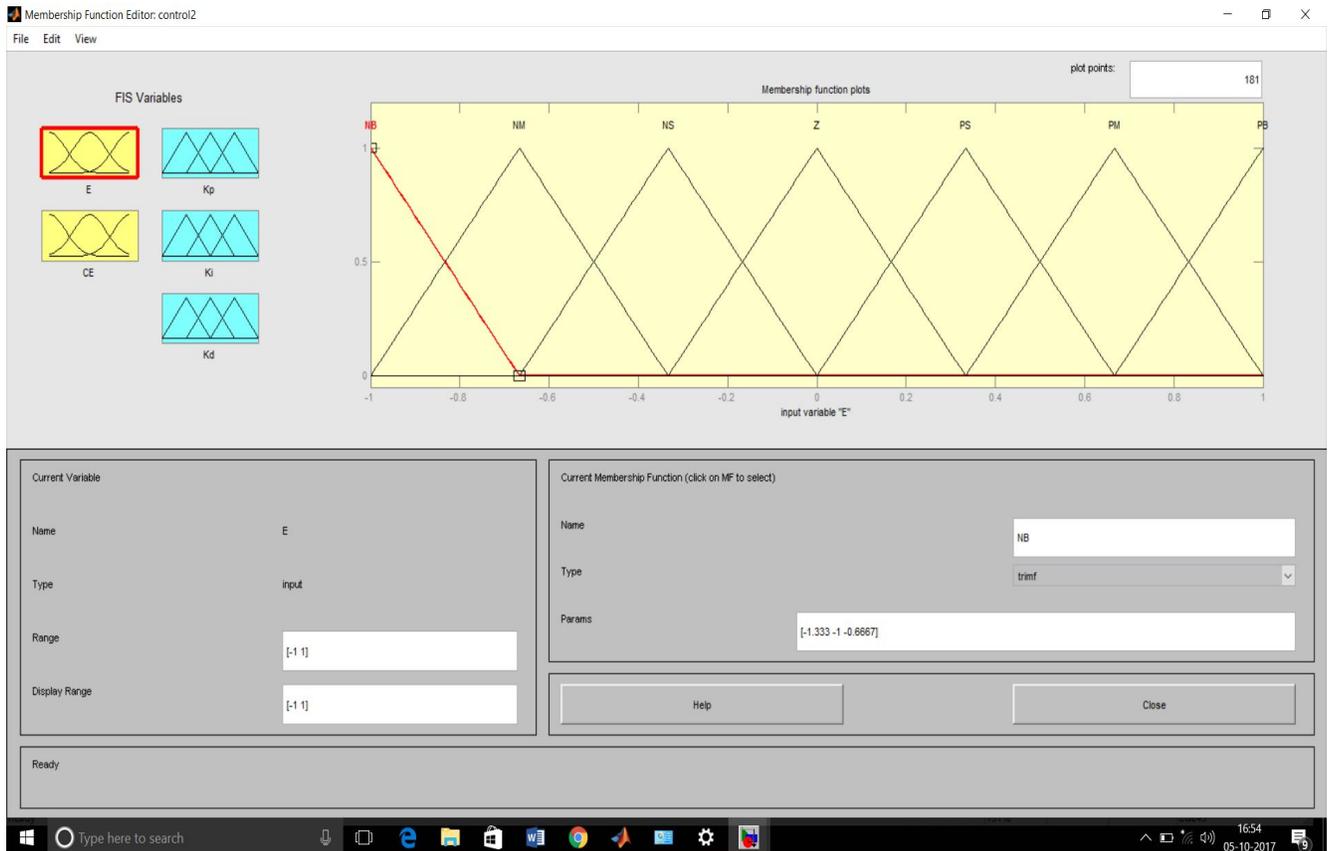
Fuzzy controller is of if, then rules. Values which are implemented are based on assumptions. There are two inputs for fuzzy as error and change in error. Those values are assigned with 7 inputs. The output obtained will be of three outputs, which denotes K<sub>p</sub>, K<sub>i</sub>, K<sub>d</sub> values.

The values assigned for the input and output are

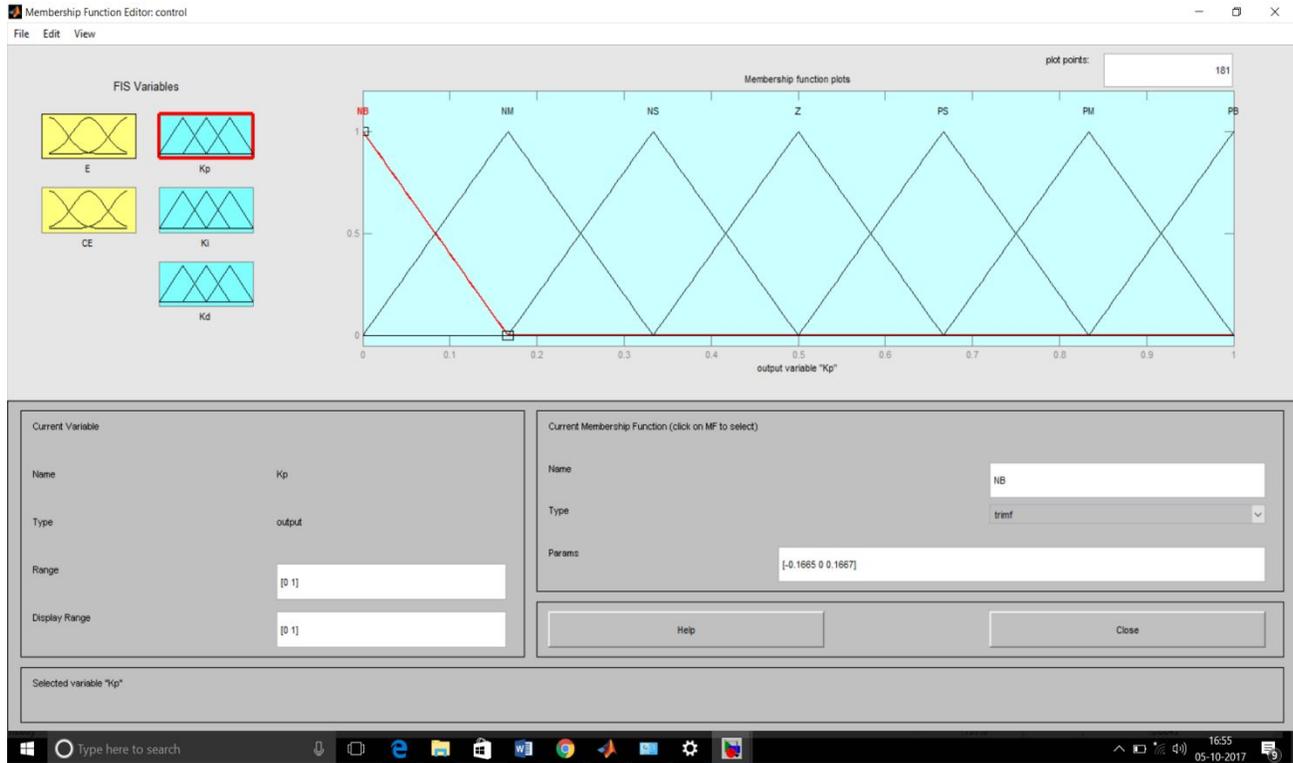
- NB - Negative Big
- NM - Negative Medium
- NS - Negative small
- ZO - Zero
- PS - Positive Small
- PM - Positive Medium
- PB - Positive Big



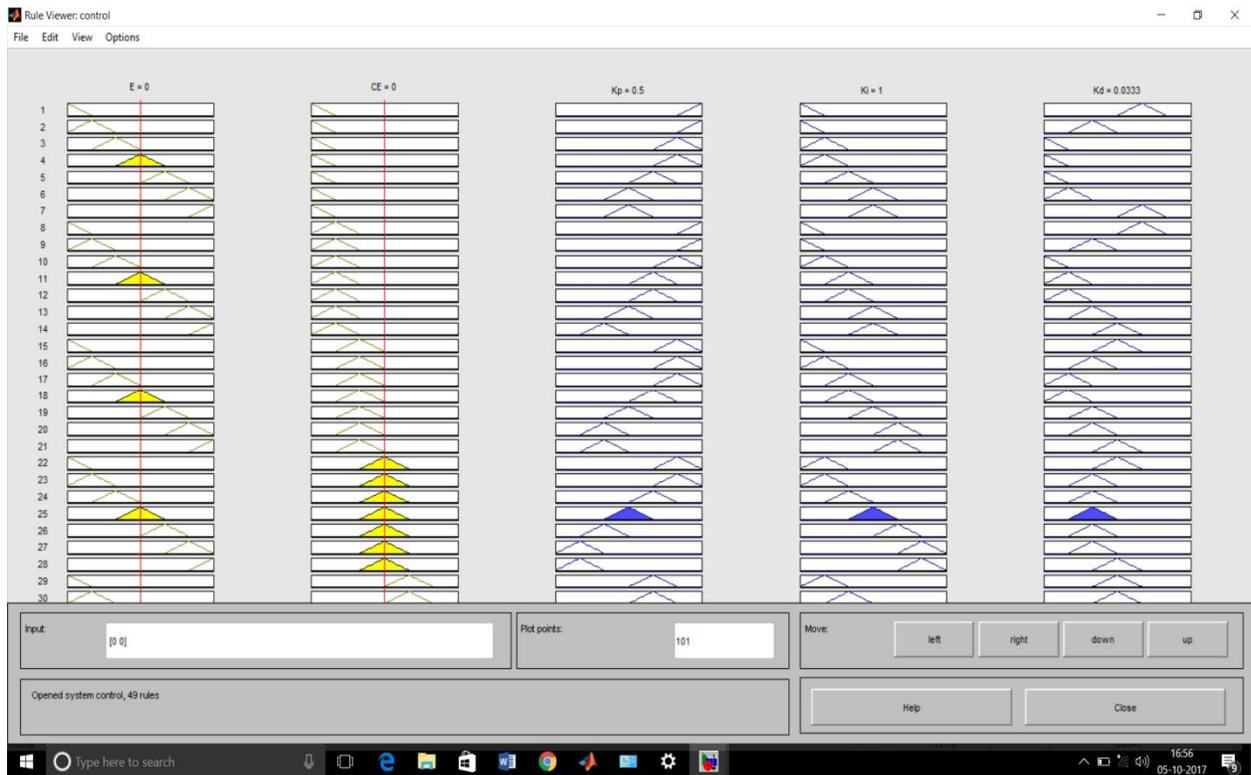
Fuzzy Control Structure



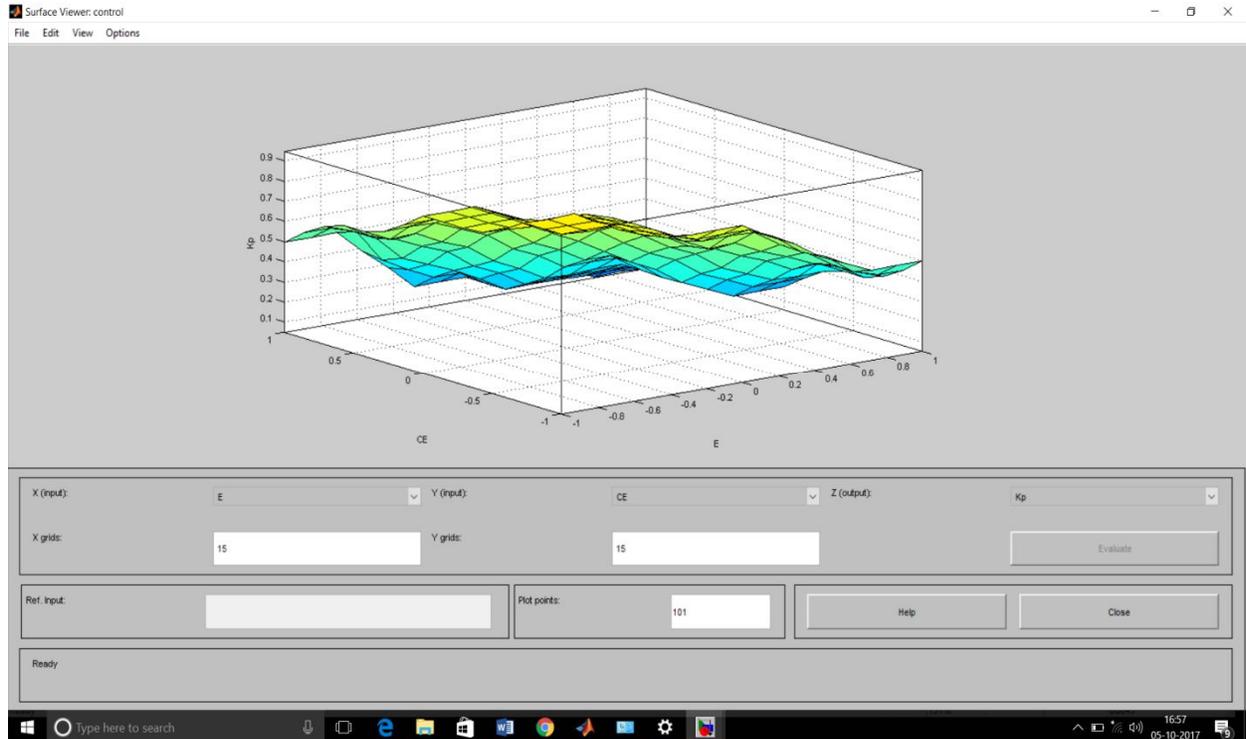
Fuzzy Input-Error And Change In Error



fuzzy output- $k_p$ ,  $k_i$ ,  $k_d$ .



Fuzzy Rules



Surface Output of the Fuzzy

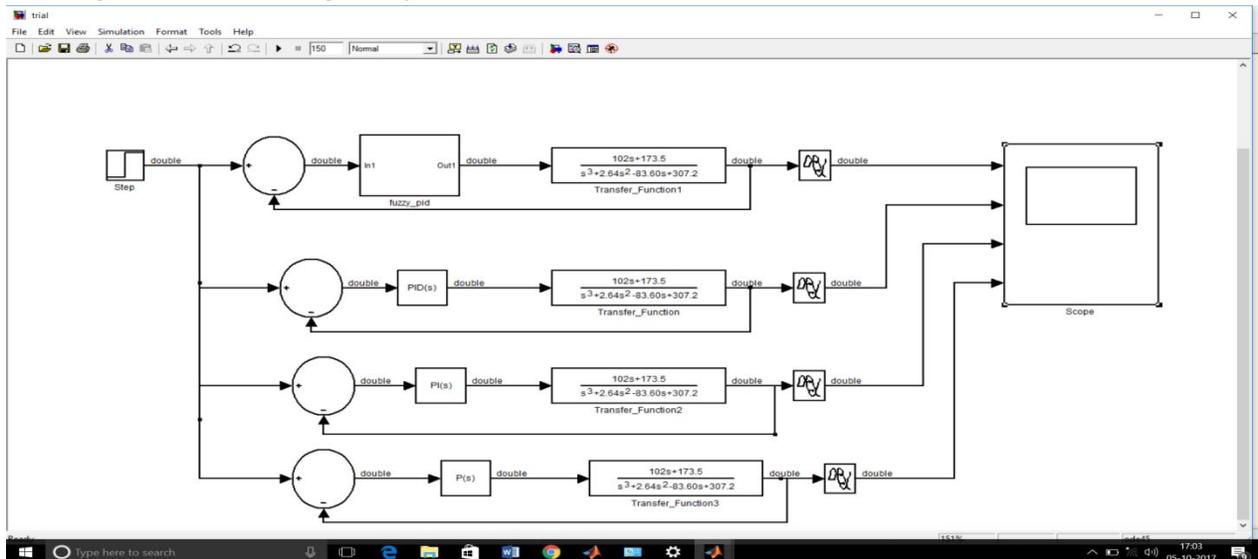
#### IV. DESIGN OF PID CONTROLLER

In PID controller the proportional, integral and derivative values are given, such that the cart should balance the pendulum. The control equation for PID controller is

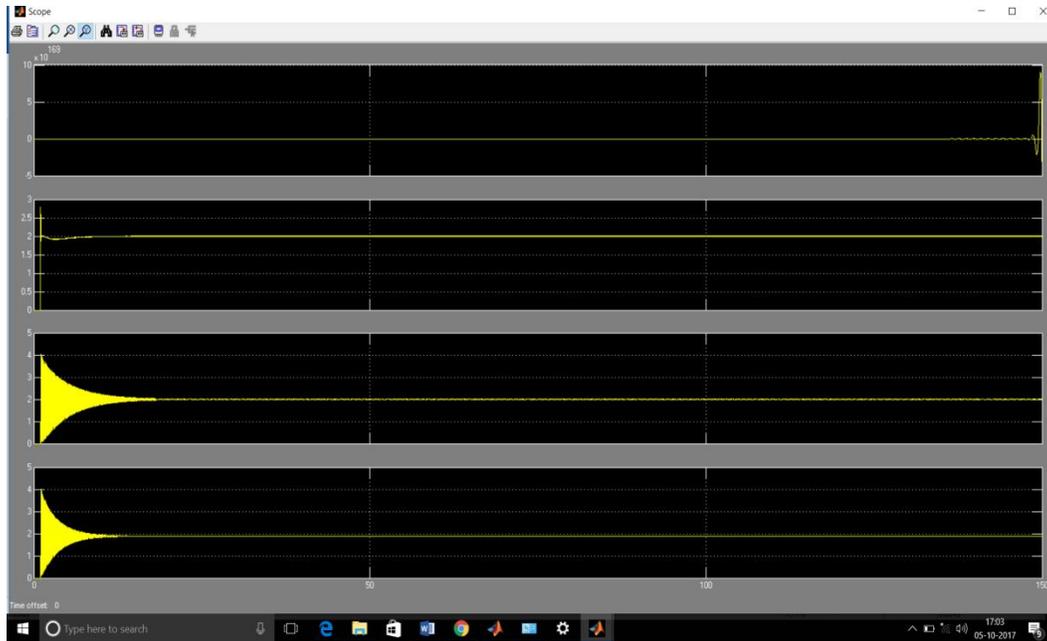
$$u(t) = K_p e(t) + \frac{1}{T_i} \int e(t) dt + T_d \frac{de(t)}{dt}$$

Here,

P changes the gain of the system. Increase in  $K_p$  value lowers the stability and have larger overshoot.  $K_i$  changes the response speed and adjust the time.  $K_d$  changes the response speed and the error of the system. Such that the value of the derivative should be greater than integrator. If  $K_d$  is too large the system becomes instable.



Simulation Ofpid, Pi, I And Fuzzy Controllers



Output of Fuzzy, Pid, Pi And P Controllers

### V. DESIGN OF LQR CONTROLLER

LQR is a linear quadratic regulator. Based on the linear state space mode.

$$\dot{x} = Ax + Bu$$

and the control law is given as

$$u = -Kx$$

K = state feedback gain

X = state vector

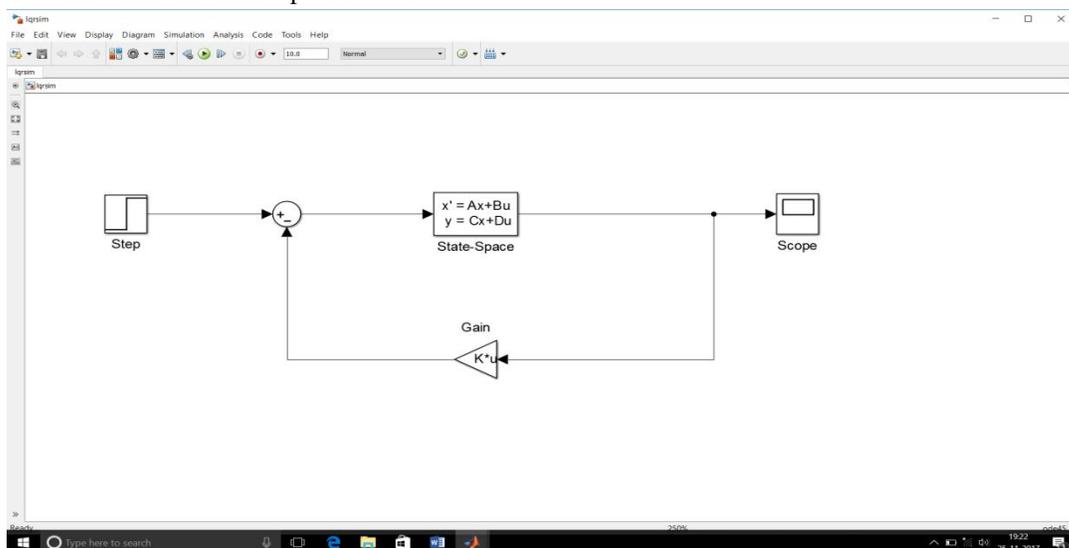
Closed loop state space model of the system is given as

$$\dot{x} = (A - BK)x$$

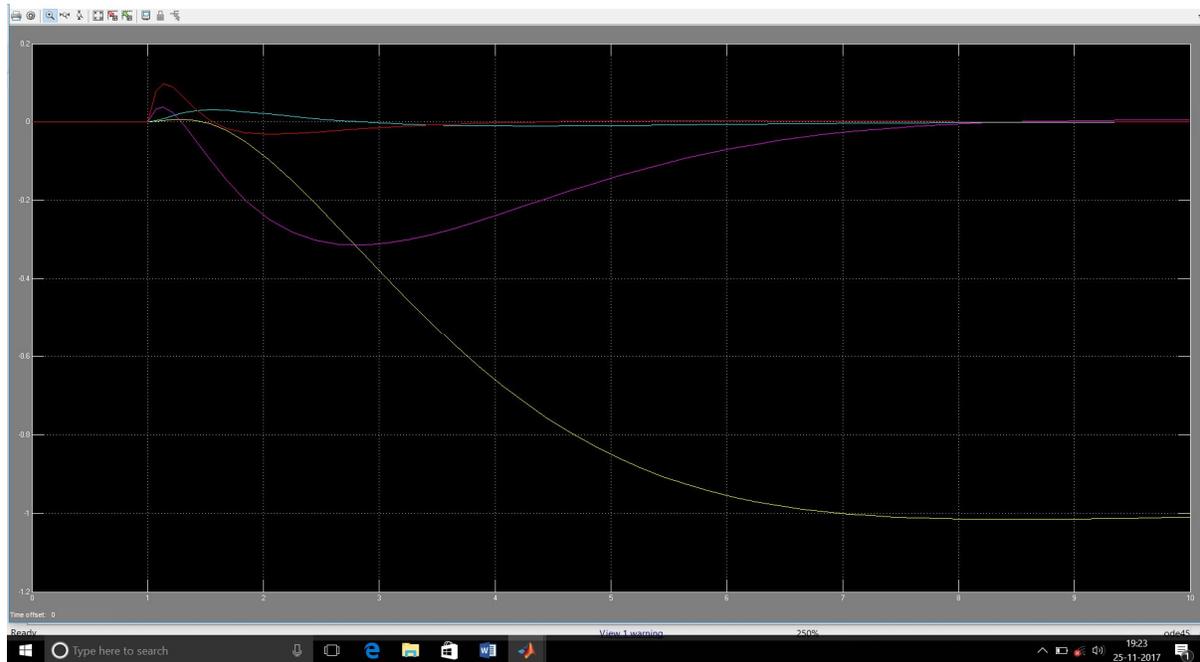
Aim of LQR controller is to locate each eigen values of the matrix (A-BK) in the left half of the plane s. such that the dynamic system becomes stable.

$$J = \frac{1}{2} \int (x^T(t)Qx(t) + u^T(t)Ru(t))dt$$

Where Q = positive semidefinite matrix R = positive value



Simulation Of Lqr



Lqr Output

## VI. CONCLUSION

The system becomes unstable without the controller. It is necessary to control the system such that by controlling we attain a stable state, which does not make the vehicle fall down. There are many controllers to control the system. But the stability also plays a major role in the system. Such that the controller should be selected in a way that provides a good stability. In this paper the stability of the vehicle is determined by state space equation and it is shown that the output of LQR is more stable than PID and fuzzy.

## REFERENCES

- [1] A Comparison of Controllers for Balancing Two Wheeled Inverted Pendulum Robot by Amir A. Bature, Salinda Buyamin, Mohamed. N. Ahmad, Mustapha Muhammad62.
- [2] Servo State Feedback Control of the Self Balancing Robot using MATLAB by V.Kongratana, S. Gulphanich, V. Tipsuwanporn and P. Huantham.
- [3] Simulation and Control of a Two-wheeled Self-Balancing Robot by Wei An and Yangmi Li.
- [4] Design and Simulation of LQR Controller with the Linear Inverted Pendulum by Haobin Dong, Lianghua He, Yongle Shi, YuanZhang.
- [5] Design of LQR and PID Controllers for the Self Balancing Unicycle Robot by Zeyan Hu, Lei guo, Shimin Wei, Qizheng Liao.



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