iJRASET
International Journal For Research in
Applied Science and Engineering Technology

INTERNATIONAL JOURNAL FOR RESEARCH

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

Volume: 6 Issue: I Month of publication: January 2018

DOI: http://doi.org/10.22214/ijraset.2018.1421

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

Fuzzy D- Super Continuous Mappings

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Abstract: The purpose of this is paper is to study and explore the concept of fuzzy D- super continuous mapping in fuzzy topological spaces some generalization in fuzzy topological spaces.
Keywords: Fuzzy topology, fuzzy super closure, fuzzy super interior ,fuzzy super open set, fuzzy super closed set, fuzzy super continuous mapping, fuzzy D- super continuous mapping*.*

I. INTRODUCTION

The fundamental concept of a fuzzy set introduced by Zadeh [9] in 1965, provides a natural foundation for building new branches of fuzzy mathematics. In 1968 Chang [2] introduced the concept of fuzzy topological spaces. Since then many topologists have contributed to the theory of fuzzy topological spaces. Today fuzzy topology has firmly established as one of the basic disciplines of fuzzy mathematics. In the present paper we introduce and study the concept of generalized super continuous mappings in fuzzy topological spaces. Let X be a nonempty set and I=[0,1] A fuzzy set in X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value 0 and the whole fuzzy set 1 is a mapping from X in to I which takes value I only. The Union $\cup A_\alpha$ (resp. intersection $\cap A_\alpha$) of a family $\{ A_\alpha : \propto \in A\}$ of fuzzy sets of X is defined to be the mapping Sup A_α (resp. Inf. A_α). A fuzzy set A of X is contained in a fuzzy set B of X denoted by A B if and only if $A(x) \leq B(x)$ for each $x \in X$. The complement A^c or 1-A of a fuzzy set A is defined by 1-A(x) for each x E X. A fuzzy point x in X is a fuzzy set defined by

$$x_\beta(y) = \begin{cases} \beta & \beta \in (0,1] \text{ for } y = x, \\ & y \in X \\ 0 & \text{otherwise} \end{cases}$$

Where x and β are respectively called the support and value of x. A fuzzy point $x_\beta \in A$ if and only if $\beta \leq A(x)$. A fuzzy set A is the union of all fuzzy points which belongs to A. A fuzzy point $x_\beta \in A$ is said to be quasi-coincident with the fuzzy set A denoted by x_β qA if and only if B + A(x) > 1. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by AqB if and only if there exist $x \in X$ such that A(x)+B(x) > 1. $A \leq B$ if and only if $\rceil(AqB^c)$.Let $f : X \to Y$ be a mapping. If A is a fuzzy set of X, then f(A) is a fuzzy set of X defined by

$$f(A)(y) = {}_{x \in f\text{-}1} \begin{cases} \text{Sup } A(x) \text{ if } f^{-1}(y) \neq \phi \\ 0 \quad \text{otherwise} \end{cases}$$

If B is a fuzzy set of Y, then $f^{-1}(B)$ is a fuzzy set of X defined by $f^{-1}(B)(x) = B(f(x))$, for each $x \in X$.A family τ of fuzzy sets of X is called a fuzzy topology [2] on X .if 0 and I belongs to τ and τ is super closed with respect to arbitrary union and finite intersection. The members of τ are called fuzzy open sets and their complements are fuzzy closed sets. For a fuzzy set A, the closure of A (denoted by cl(A)) is the intersection of all fuzzy closed super sets of A and the interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A. A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy generalized closed (fuzzy g-super closed) if $cl(A) \leq G$ whenever $A \leq G$ and G is fuzzy open [7].The complement of a fuzzy g- closed set is called fuzzy g- open ii A^c is fuzzy g- closed . Every fuzzy closed (resp. fuzzy open) set is fuzzy g- closed (resp. fuzzy g- open) but its converse may not be true.

A. Defination1.1[6,7,8]:- Let (X,τ) fuzzy topological space and $A \subseteq X$ then

1) Fuzzy Super closure $scl(A)=\{x \in X: cl(U) \cap A \neq \phi\}$

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2) Fuzzy Super interior sint(A) ={x∈X:cl(U)≤A≠ϕ}

B. Definition 1.2[6,7,8]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:
1) Fuzzy super closed if scl(A) ≤ A.
2) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

A fuzzy mapping f from a fuzzy topological space X into a fuzzy topological space Y is said fuzzy supper continuous if the inverse image of every fuzzy super closed subset of Y is fuzzy supper closed in X or equivalently, the inverse image of every fuzzy supper open subset of Y is fuzzy supper open in X. .

II. BASIC PROPERTIES OF FUZZY D- SUPER CONTINUOUS FUNCTIONS.

A. *Definition2.1:-* A fuzzy function f: X → Y from a fuzzy topological space X to fuzzy topological space Y is said to be fuzzy D-super continuous if for each fuzzy point x∈X and each fuzzy super open set V containing f(x) there is an fuzzy super open set U containing fuzzy point x such that f(U) ≤V.

B. *Definition2.2*:- fuzzy function f: X → Y to be fuzzy Z-super continuous if for each x∈X and each V containing f(x) there is an fuzzy super open set U containing x such that f(U) ≤ V. it is immediate From definitions that every fuzzy super continuous function is fuzzy D-super continuous. However the reverse implications do not hold in general.

C. *Theorem 2.1:-* Let a f: X → Y be a fuzzy function from a fuzzy topological space X into a fuzzy topological Y. Then following statements are equivalent.

1) f is fuzzy D- super continuous.
2) If V is a fuzzy super open subset of Y, then $f^{-1}(V)$ is a fuzzy super open subset of fuzzy topological space X.
3) If F is a fuzzy super closed B subset of Y, and then $f^{-1}(F)$ is fuzzy super closed in fuzzy topological space X.
4) *Proof:-* (a)⟹ (b): If V is an fuzzy super open F_σ-subset of Y, then for each x∈ $f^{-1}(V)$, V is a fuzzy neighborhood of f(x) and there is a neighborhood U of fuzzy point x such that f(U) ≤V. Thus being a fuzzy neibourhood of each of its points is super open.

(b)⟹ (a): Let x∈X and let V be an fuzzy super open set containing f(x). Then $f^{-1}(V)$ is an fuzzy super open set containing x and $f(f^{-1}(V))\leq V$.

(b)⟹ (c): Let F be a fuzzy super closed G_δ fuzzy subset of Y. Then Y-F is an fuzzy super open set F_σ and so $f^{-1}(F`)= X-f^{-1}(F)$ is fuzzy super open. Hence $f^{-1}(F)$ is fuzzy super closed in X.

(c)⟹ (b): Let V be an fuzzy super open set F_σ . Then Y-V is a fuzzy super closed set G_δ and so $f^{-1}(cl(V))=X-f^{-1}(V)$ is fuzzy super closed. Hence $f^{-1}(V)$ is super open in X.

Let a fuzzy function f: X → Y where X and Y are fuzzy Topological space then f is fuzzy D- super continuous fuzzy super closed function from a fuzzy normal space onto a fuzzy space Y such that each Singleton in Y is a fuzzy G_δ set. If either of the fuzzy topological space spaces X and Y, T_1, then Y is fuzzy Hausdorff. Proof: Case I: The fuzzy topological space Y is T_1. Let fuzzy point's y_1 and Y_2 be any two distinct points in….. Then {Y_1} and {Y_2} are fuzzy super closed G_δ fuzzy subsets of Y, and so by Theorem 2.1, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are fuzzy super closed subsets of X. By normality of X, there are fuzzy disjoint fuzzy super open sets U_1 and U_2 containing $f^{-1}(y_1)$ and $f^{-1}(y_2)$, respectively. Since f is fuzzy super closed, the set V_1= y-f(U`$_1$)And $V_2$ =Y-f(U`$_2$) are fuzzy super open in Y. It is easily verified that V_1 and V_2 are disjoint and contain y_1 and y_2, respectively. Thus Y is Hausdorff.

Case II. The fuzzy X is T_1. Let f(x) $\in$Y be any point. Since the fuzzy singleton {x} fuzzy super closed in X, {f(x)} is a fuzzy super closed subset of Y. So Y is T_1 and the proof is complete in view of case I.

D. *Theorem.2.4:* Let a fuzzy function f: X → Y where X and Y be topological space is fuzzy super continuous and g :Y→Z where Z is Fuzzy topological space is fuzzy D-super continuous, then gof is fuzzy D-super continuous.

1) *Proof:* Let F be a fuzzy super closed G_δ subset of Z. Then $g^{-1}(F)$ is fuzzy super closed, and since f is fuzzy Super continuous, $(gof)^{-1}(F) = f^{-1}(g^{-1}(F))$ is fuzzy super closed.

E. *Theorem 2.5:* Let f: X → Y where X and Y be topological space be a fuzzy quotient map. Then a function g : Y → Z is fuzzy D-super continuous If and only if gof is fuzzy D-super continuous.

1) *Proof:* Necessity is immediate from above theorem. To prove sufficiency, let V be a fuzzy super open F_σ-subset of Z. Then $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is fuzzy super open in X. Since f is a fuzzy quotient map, $g^{-1}(V)$ is fuzzy super open in Y and so g is fuzzy D-super continuous.

F. *Theorem2.6:* Let a fuzzy mapping f: X → Y where X and Y be topological space be either an fuzzy super open or fuzzy super closed surjection and let g : Y → Z be z be where topological space. Any fuzzy function such that gof is fuzzy D-super continuous. Then g is fuzzy D-super continuous.

1) *Proof:* Suppose f is fuzzy super open (respectively, fuzzy super closed), and let V≤Z be a fuzzy super open F_σ-set (respectively, V⊂Z be a fuzzy super closed G_δ set). Since gof is fuzzy D-super continuous, $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$ is fuzzy super open(respectively, fuzzy super closed). Since f is fuzzy super open (respectively, fuzzy super closed) and onto, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is fuzzy super open (respectively, fuzzy super closed), and consequently, g is fuzzy D-super continuous.

G. *Theorem 2.7:* The set of all fuzzy points of X at which f: X → Y where X and Y be topological space is not fuzzy D- super continuous Identical with the union of the boundaries of the inverse images of fuzzy super open F_σ subsets of Y.

1) *Proof:* Suppose f is not fuzzy D-super continuous at a point x∈X. Then there exists a fuzzy super open set F_σ set V containing f(x) such that for every fuzzy super open set U containing x, $f(U) \cap (Y - V) \neq \phi$ .Thus , for every fuzzy super open set U containing x, $U \cap (X - f^{-1}(V)) \neq \phi$. Therefore, x cannot be a fuzzy interior point of $f^{-1}(V)$. Hence x is a fuzzy boundary point of $f^{-1}(V)$.Now, let x belong to the boundary of $f^{-1}(V)$ for some fuzzy super open F_σ-subset V of Y. Then f(x) ∈ V. If f is fuzzy D-super continuous at x, then there is an fuzzy super open set U containing x such that f(U)≤V. Thus $x \in U \le f^{-1}f(U) \le f^{-1}(V)$, and so x is an fuzzy interior point of $f^{-1}(V)$, contradiction. Hence f is not fuzzy D-super continuous at x.

H. *Theorem 2.8:* A fuzzy D –super continuous image of a fuzzy connected space is fuzzy connected.

1) Proof: Let f: X → Y be a fuzzy D –super continuous surjection from a fuzzy connected space X onto Space Y. Suppose Y is not fuzzy connected, and let Y=A∪B be a partition of Y. Then both A and B are fuzzy super open subsets of Y. Since f is fuzzy D-super continuous, by theorem 2.1 both $f^{-1}(A)$ and $f^{-1}(B)$ are fuzzy super open subsets of X , $f^{-1}(A)$ and $f^{-1}(B)$, $X = f^{-1}(A) \cup f^{-1}(B)$ is a partition of X. This contradiction to the fuzzy connectedness of X establishes the theorem. A function f: X → Y is said to be fuzzy connected if f(C) is fuzzy connected for every fuzzy connected set C≤ X.

I. *Theorem.2.9* Let f and g be fuzzy D-super continuous functions from a space X into a fuzzy D-Hausdorff Space Y. Then the set A={x: f(x)=g(x) } is fuzzy super closed in X.

1) Proof: Let x∈X-A . Then f(x) ≠ g(x) , and so by hypothesis on Y, there are disjoint fuzzy super open F_σ sets U and V containing f(x) and g(x), respectively. Since f and g are fuzzy D-super continuous, then the sets $f^{-1}(U)$ *and* $g^{-1}(V)$ are fuzzy super open and contain x. Let $G = f^{-1}(U) \cap g^{-1}(V)$. Then G is a fuzzy super open set containing x and G∩A= ϕ .Thus A is fuzzy super closed in X.

J. *Theorem 2.10:* Let a fuzzy function f : X → Y where X and Y be topological space be a fuzzy D-super continuous injection into a fuzzy D-Hausdorff space Y. Then X is fuzzy Hausdorff.

1) *Proof:* Let x, y be any two distinct fuzzy points in X. Then f(x) ≠ f(y) , since Y is fuzzy D Hausdorff, there are disjoint fuzzy super open F_σ -sets u and V containing f(x) and f(y), respectively. By Theorem 2.1 $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint fuzzy super open sets containing x and y , respectively. Thus X is fuzzy Hausdorff.

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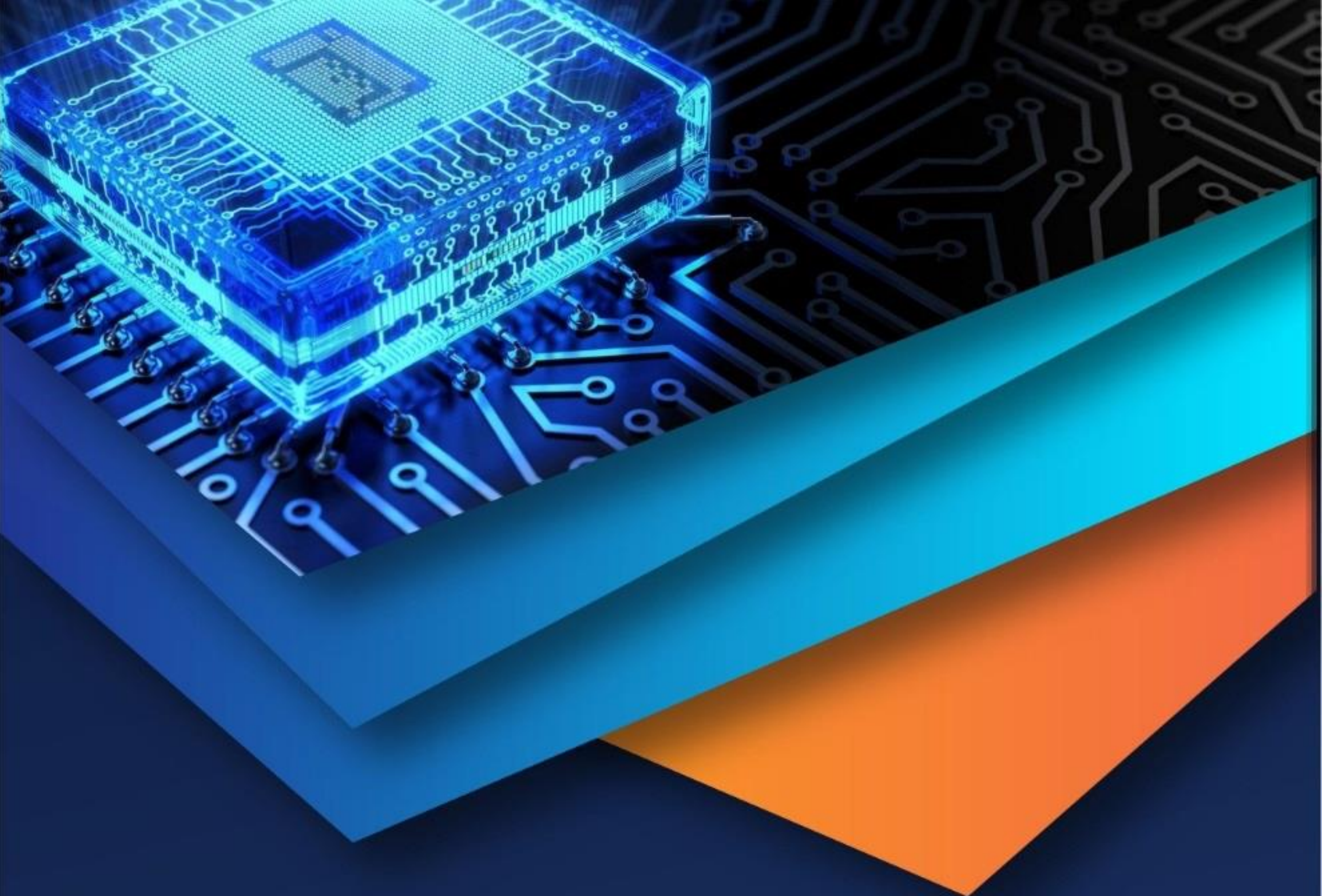

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