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# Weak and Strong forms of ğ-semi-irresolute Functions

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*Abstract: The purpose of this paper is to give two new types of irresolute functions called completely ğ-semi-irresolute functions and weakly ğ-semi- irresolute functions. We obtain their characterizations and basic properties. 2010 Mathematics Subject Classification: 54C10, 54C08, 54C05.*

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## I. INTRODUCTION AND PRELIMINARIES

Functions and of course irresolute functions stand among the most important and most researched points in the whole of mathematical science. In 1972, Crossley and Hildebrand [2] introduced the notion of irresoluteness. Many different forms of irresolute functions have been introduced over the years. Various interesting problems arise when one considers irresoluteness. Its importance is significant in various areas of mathematics and related sciences. Recently, as generalization of closed sets, the notion of ğ-semi-closed sets were introduced and studied by Veerakumar [16]. In this paper, we introduce and characterize the concepts of completely ğ-semi-irresolute and weakly ğ-semi-irresolute functions.

*A. Definition 1.1* A subset A of a space X is called

*1)* regular open if A = int(clA);

*2)* semi-open if A $\subseteq$ cl(intA).

The complement of regular open (resp. semi-open ) is called regular closed (resp. semi-closed).

*B. Definition 1.2*

*1)* A subset A of a space X is called:

*2)* ĝ-closed if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X;

*3)* *g-closed if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is ĝ-open in X

*4)* #g-semi-closed if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is *g-open in X

*5)* ğ-semi-closed if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is #g-semi-open in X. The complement of ğ-semi-closed ( resp. ĝ-closed, *g-closed and #g-semi-closed) is called ğ-semi-open( resp. ĝ-open, *g-open and #g-semi-open).

*B. Definition 1.3* A function f: X $\rightarrow$ Y is called:

*1)* strongly continuous if $f^{-1}$(V) is both open and closed in X for each subset V of Y;

*2)* completely continuous if $f^{-1}$(V) is regular open in X for each open subset V of Y;

*3)* ğ-semi-irresolute if $f^{-1}$(V) is ğ-semi-closed in X for each ğ-semi-closed subset V of Y

*4)* pre ğ-semi-closed if f(V) is ğ-semi-closed in Y for each ğ-semi-closed subset V of X.

## II. COMPLETELY ğ-SEMI-IRRESOLUTE FUNCTIONS

*A. Definition 2.1* A function f: X $\rightarrow$ Y is called completely ğ-semi-irresolute if the inverse image of each ğ-semi-open subset of Y is regular open in X.

Clearly, every strongly continuous function is completely ğ-semi-irresolute and every completely ğ-semi-irresolute function is ğ-semi-irresolute.

B.  *Remark 2.2*    The converses of the above implications are not true in general as seen from the following examples.

C.  *Example 2.3*  Let X = Y = { a, b, c}, $\tau = \sigma$ = {$\phi$, X = Y, {a}, {b, c}}. Then the identity function f: (X,$\tau$) $\rightarrow$ (Y,$\sigma$) is completely ğ-semi-irresolute but not strongly continuous.

D.  *Example 2.4* Let X = Y = { a, b, c},  $\tau$ = {$\phi$, X, {a}, {a, b}} and $\sigma$ = {$\phi$, Y, {a}}. Then the identity function f: (X,$\tau$) $\rightarrow$ (Y,$\sigma$) is ğ-semi-irresolute but not completely ğ-semi-irresolute.

E.  *Theorem 2.5*    The following statements are equivalent for a function f: X $\rightarrow$ Y.
1)  f is completely ğ-semi-irresolute.
2)  f: (X,$\tau$) $\rightarrow$ (Y, ĞSO(X)) is completely continuous.
3)  $f^{-1}$(F) is regular closed in X for each ğ-semi-closed set F of Y.
4)  Proof$\Leftrightarrow$ (ii) :  It follows from the definitions.
$\Rightarrow$ (iii) :  Let F be any ğ-semi-closed set of Y.  Then Y\F $\in$ ĞSO(Y).  By (i), $f^{-1}$(Y\F) = X\$f^{-1}$(F) $\in$ RO(X).  We have $f^{-1}$(F) $\in$ RC(X).  The converse is similar.

F.  *Definition 2.6*   A space X is said to be almost connected  (resp. ğ-semi-connected ) if there does not exist disjoint regular open (resp. ğ-semi-open)  sets A and B such that A U B = X.

G.  *Theorem 2.7*    If f: X $\rightarrow$ Y is completely ğ-semi-irresolute surjective function and X is almost connected, then Y is ğ-semi-connected.
1)  *Proof*    Suppose that Y is not ğ-semi-connected.  Then there exist disjoint ğ-semi-open sets A and B of Y such that A U B = X. Since f is completely ğ-semi-irresolute surjective, $f^{-1}$(A) and $f^{-1}$(B) are regular open sets in X. Moreover,  $f^{-1}$(A) U $f^{-1}$(B) = X, f$^{-1}$(A) $\neq$ $\phi$ and $f^{-1}$(B) $\neq$ $\phi$.  This shows that X is not almost connected, which is a contradiction to the assumption that X is almost connected.  By contradiction, Y is ğ-semi-connected.

H.  *Definition 2.8*   A space (X,$\tau$) is said to be ğ-semi-T₁  (resp. r-T₁ ) if for each pair of distinct points x and y of  X, there exist ğ-semi-open (resp. regular open) sets  U1 and U2  such that x $\in$ U1 and y $\in$ U2,  x $\notin$ U2 and y $\notin$ U1.

I.  *Theorem 2.9*   If f: (X,$\tau$) $\rightarrow$ (Y,$\sigma$) is completely ğ-semi-irresolute injective function and Y is ğ-semi-T₁, then X is r-T₁.
1)  *Proof*    Suppose that Y is ğ-semi-T₁.  For any two distinct points x and y of X, there exist ğ-semi-open sets F1 and F2 in Y such that f(x) $\in$ F1, f(y) $\in$ F2, f(x) $\notin$ F2 and f(y) $\notin$ F1.  Since f is injective completely ğ-semi-irresolute function, we have X is r-T₁.

J.  *Definition 2.10*    space (X,$\tau$) is said to be ğ-semi-T₂ if for each pair of distinct points x and y in X, there exist disjoint ğ-semi-open sets A and B in X such that x $\in$ A and y $\in$ B and A $\cap$ B = $\phi$.

### III.                                  WEAKLY Ğ-SEMI-IRRESOLUTE FUNCTIONS
A.  *Definition 3.1*    A function f: X $\rightarrow$ Y is said to be weakly ğ-semi-irresolute if for each point x $\in$ X  and each V $\in$ ĞSO(Y, f(x)), there exists a U $\in$ ĞSO(X, x) such that f(U) $\subset$ ğ-semi-cl(V).
It is evident that every ğ-semi-irresolute function is weakly ğ-semi-irresolute but the converse is not true.

B.  *Example 3.2*  Let X = Y = { a, b, c}, $\tau$ = {$\phi$, X, {a, b}} and  $\sigma$ = {$\phi$, Y, {a}, {a, b}}. Then the identity  function f: (X,$\tau$) $\rightarrow$ (Y,$\sigma$) is weakly ğ-semi-irresolute but not ğ-semi-irresolute.

C.  *Theorem 3.3*    For a function f: X $\rightarrow$ Y, the following statements are equivalent:
1)  f is weakly ğ-semi-irresolute.
2)  $f^{-1}$(V) $\subset$ ğ-semi-int ($f^{-1}$(ğ-semi-cl(V))) for every V $\in$ ĞSO(Y).

*3)* ğ-semi-cl($f^{-1}(V)$) $\subset$ $f^{-1}$(ğ-semi-cl(V)) for every V $\in$ ǦSO(Y).

(i) $\Rightarrow$ (ii) : Suppose that V $\in$ ǦSO(Y) and let x $\in$ $f^{-1}(V)$. It follows from (i) that f(U) $\subset$ ğ-semi-cl(V) for some U $\in$ ǦSO(X, x). Therefore, we have U $\subset$ $f^{-1}$(ğ-semi-cl(V)) and x $\in$ U $\subset$ ğ-semi-int ($f^{-1}$(ğ-semi-cl(V))). This shows that $f^{-1}(V)$ $\subset$ ğ-semi-int ($f^{-1}$(ğ-semi-cl(V))).

(ii) $\Rightarrow$ (iii) : Suppose that V $\in$ ǦSO(Y) and x $\notin$ $f^{-1}$(ğ-semi-cl(V)). Then f(x) $\notin$ ğ-semi-cl(V). There exists G $\in$ ǦSO(Y, f(x)) such that G $\cap$ V= $\phi$. Since V $\in$ ǦSO(Y) , we have ğ-semi-cl(G) $\cap$ V = $\phi$ and hence ğ-semi-int ($f^{-1}$(ğ-semi-cl(G))) $\cap$ $f^{-1}(V)$ = $\phi$. By (ii), we have x $\in$ $f^{-1}(G)$ $\subset$ ğ-semi-int ($f^{-1}$(ğ-semi-cl(G))) $\in$ ǦSO(X). Therefore, we obtain x $\notin$ ğ-semi-cl($f^{-1}(V)$). This shows that ğ-semi-cl($f^{-1}(V)$) $\subset$ $f^{-1}$(ğ-semi-cl(V)).

*4)* $\Rightarrow$ (i): Let x $\in$ X and V $\in$ ǦSO(Y, f(x)). Then x $\notin$ $f^{-1}$(ğ-semi-cl(Y \ ğ-semi-cl(V))). Since

*5)* Y \ ğ-semi-cl(V) $\in$ ǦSO(Y), by (iii), we have x $\notin$ ğ-semi-cl($f^{-1}$(Y \ ğ-semi-cl(V))). Hence there exists U $\in$ ǦSO(X, x) such that U $\cap$ $f^{-1}$(Y \ ğ-semi-cl(V)) = $\phi$. Therefore, we obtain f(U) $\cap$ (Y \ ğ-semi-cl(V)) = $\phi$ and hence f(U) $\subset$ ğ-semi-cl(V).This shows that f is weakly ğ-semi-irresolute.

D. *Theorem 3.4* A space X is ğ-semi-$T_2$ if and only if for any pair of distinct points x, y of X there exist ğ-semi-open sets U and V such that x $\in$ U and y $\in$ V and ğ-semi-cl(U) $\cap$ ğ-semi-cl(V) = $\phi$.

E. *Theorem 3.5* If Y is a ğ-semi-$T_2$ space and f: X $\rightarrow$ Y is a weakly ğ-semi-irresolute injection, then X is ğ-semi-$T_2$.

*1) Proof* Let x and y be any two distinct points of X. Since f is injective , we have f(x) $\neq$ f(y). Since Y is ğ-semi-$T_2$, by Theorem 3.4 there exist V $\in$ ǦSO(Y, f(x)) and W $\in$ ǦSO(Y, f(y)) such that ğ-semi-cl(V) $\cap$ ğ-semi-cl(W) = $\phi$. Since f is weakly ğ-semi-irresolute, there exist G $\in$ ǦSO(X, x) and H $\in$ ǦSO(X, y) such that f(G) $\subset$ ğ-semi-cl(V) and f(H) $\subset$ ğ-semi-cl(W). Hence we obtain G $\cap$ H = $\phi$. This shows that X is ğ-semi-$T_2$.

F. *Definition 3.6* A function f: X $\rightarrow$ Y is said to have a strongly ğ-semi-closed graph if for each (x, y) $\in$ (X x Y)\G(f), there exist U $\in$ ǦSO(X, x) and V $\in$ǦSO(Y, y) such that (ğ-semi-cl(U) x ğ- semi-cl(V)) $\cap$ G(f) = $\phi$.

G. *Theorem 3.7* If Y is a ğ-semi-$T_2$ space and f: X $\rightarrow$ Y is weakly ğ-semi-irresolute, then G(f) is strongly ğ-semi-closed.Let (x, y) $\in$ (X x Y)\G(f). Then y $\neq$ f(x) and by Theorem 3.4 there exist V $\in$ ǦSO(Y, f(x)) and W $\in$ ǦSO(Y, y) such that ğ-semi-cl(V) $\cap$ ğ-semi-cl(W) = $\phi$. Since f is weakly ğ-semi-irresolute, there exists U $\in$ ǦSO(X, x) such that f(ğ-semi-cl(U)) $\subset$ ğ-semi-cl(V). Therefore, we obtain f(ğ-semi-cl(U)) $\cap$ ğ-semi-cl(W) = $\phi$ and hence (ğ-semi-cl(U) x ğ-semi-cl(W)) $\cap$ G(f) = $\phi$. This shows that G(f) is strongly ğ-semi-closed in X x Y.

H. *Theorem 3.8* If a function f: X $\rightarrow$ Y is weakly ğ-semi-irresolute, injective and G(f) is strongly ğ-semi-closed, then X is ğ-semi-$T_2$.

*1) Proof* Let x and y be a pair of distinct points of X. Since f is injective, f(x) $\neq$ f(y) and (x, f(y)) $\notin$ G(f). Since G(f) is strongly ğ-semi-closed, there exist G $\in$ ǦSO(X, x) and V $\in$ ǦSO(Y, f(y)) such that f(ğ-semi-cl(G)) $\cap$ ğ-semi-cl(V) = $\phi$. Since f is weakly ğ-semi-irresolute, there exists H $\in$ ǦSO(X, y) such that f(H) $\subset$ ğ-semi-cl(V). Hence we have f(ğ-semi-cl(G)) $\cap$ f(H) = $\phi$; hence G $\cap$ H = $\phi$. This shows that X is ğ-semi-$T_2$.

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