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# Fuzzy Ideals and Anti Fuzzy Ideals of Near-Ring

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***Abstract: The aim of this paper is to extend the notion of a fuzzy subnear-ring, fuzzy ideals of a near ring, anti fuzzy ideals of near-ring and to give some properties of fuzzy ideals and anti fuzzy ideals of a near-ring.***
***Keywords: Near-ring, Near-subring, Ideals of near-ring, Fuzzy set, Fuzzy subring, Fuzzy ideals of near-ring,Anti fuzzy ideals***

## I. INTRODUCTION

The theory of fuzzy set was introduced by Zadeh[3], applying which Rosenfeld[6] in 1971 defined fuzzy subgroups. Salah Abou-Zaid[4]introduced the theory of a fuzzy subnear-ring and fuzzy ideals of a near-ring. Fuzzy ideals of a ring and a characterization of a regular ring studied by Lui[7]. The notion of fuzzy ideals of near rings with interval valued membership functions introduced by B. Davvaz[10] in 2001. In 2001, Kyung Ho Kim and Young BaeJun[11] in their paper entitled “Normal fuzzy R-subgroups in near-rings” introduced the concept of a normal fuzzy R-subgroup in near-rings and explored some related properties. In 2005, Syam Prasad Kuncham and Satyanarayana Bhavanari in their paper entitled “Fuzzy Prime ideal of a Gamma-near-ring” introduced fuzzy prime ideal in Γ-near-rings.The anti-fuzzy ideals of near-ring defined by F. A. Azam, A. A. Mamun and F. Nasrin. In this paper we study the concept of fuzzy ideals of a near ring and some difference properties of fuzzy ideals and anti-fuzzy ideals of a near-ring.

## II. PRELIMINARIES

For the sake of continuity we recall some basic definitions.

### *A. Definition 2.1*

A set *N* together with two binary operations + (called *addition*) and · (called *multiplication*) is called a (right) *near-ring* if:

1) *N* is a group (not necessarily abelian) under addition;
2) multiplication is associative (so *N* is a semi group under multiplication); and
3) multiplication distributes over addition on the *right*: for any $x, y, z \in N$ it holds that $(x + y).z = (x.z) + (y.z)$.This near-ring will be termed as right near-ring. If $z.(x + y) = z.x + z.y$ instead of condition (3), the set N satisfies, then we call N a left near-ring. Near-rings are generalised of a rings, addition needs not be commutative and (more important) only one distributive law is postulated.

### *B. Examples 2.2*

(1)$Z$ be the Set of positive and negative integers with 0. $(Z, +)$ is a group . Define ‘.’ on Z by $a.b = a$ for all $a, b \in Z$. Clearly $(Z, +, .)$ is a near ring. (2) Let $Z_{12} = \{0,1,2,3, \ldots ,11\}$. $(Z_{12},+)$ is a group under ‘+’ modulo 12. Define ‘.’ on $Z_{12}$ by $a.b = a$ for all $a, b \in Z$. Clearly $(Z_{12}, +, .)$ is a near ring. (3) Let $M_{2\times2} = \{(a_{ij})/ Z : Z$ is treated as a near ring$\}$. $M_{2\times2}$ under the operation of ‘+’ and matrix multiplication ‘.’ is defind by the following:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} . \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+b & a+b \\ c+d & c+d \end{bmatrix}$$

Because we use the multiplication in Z i.e. $a.b = a$. So

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} . \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix} = \begin{bmatrix} a+b & a+b \\ c+d & c+d \end{bmatrix}.$$

It is easily verified $M_{2\times2}$ is a near ring. We denote $xy$ instead of $.y$ . Note that $x.0 = 0$and $x(-y) = -xy$ but in general $0x \neq 0$ for some $x \in R$. An ideal *I* of a near-ring *R* is a subset of *R* such that
(1) $(I, +)$is a normal subgroup of $(R, +)$, (2) $RI \subseteq I$ (3) $(r + i)s - rs \in I$for any $i \in I$ and any $r, s \in R$.

## III. FUZZY IDEALS OF NEAR-RINGS

### A. *Definition 3.1*

Let $R$ be a near-ring and $\mu$ be a fuzzy subset of $R$. We say a fuzzy subnear-ring of $R$ if (1) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$, (2) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in R$.

### B. *Definition3.2*

Let $R$ be a near-ring and $\mu$ be a fuzzy subset of $R$. $\mu$ is called a fuzzy left ideal of R if $\mu$ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$.

1) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,
2) $\mu(y+x-y) \geq \mu(x)$,
3) $\mu(xy) \geq \mu(y)$ or $\mu(xy) \geq \mu(x)$

### C. *Definition3.3*

Let $R$ be a near-ring and $\mu$ be a fuzzy subset of $R$. $\mu$ is called a fuzzy right ideal of R if $\mu$ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$.(1) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$, (2) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, (3) $\mu(y+x-y) \geq \mu(x)$, (4) $\mu((x+i)y - xy) \geq \mu(i)$.

### D. *Example 3.4*

Let $R := \{a, b, c, d\}$ be a set with two binary operations as follows:

| + | a | b | c | d |
|---|---|---|---|---|
| a | a | b | c | d |
| b | b | a | d | c |
| c | c | d | b | a |
| d | d | c | a | b |

| . | a | b | c | d |
|---|---|---|---|---|
| a | a | a | a | a |
| b | a | a | a | a |
| c | a | a | a | a |
| d | a | a | b | b |

The we can easily see that $(R, +)$ is a group and $(R, .)$ is an semigroup and satisfies left distributive law. Hance $(R, +, .)$ is a left near-ring. Define a fuzzy subset $\mu : R \to [0,1]$ by $\mu(c) = \mu(d) < \mu(b) < \mu(a)$. Then $\mu$ is a left fuzzy ideal of R.

### E. *Example 3.5*

Let $R := \{a, b, c, d\}$ be a set with two binary operations as follows.

| + | a | b | c | d |
|---|---|---|---|---|
| a | a | b | c | d |
| b | b | a | d | c |
| c | c | d | b | a |
| d | d | c | a | b |

| . | a | b | c | d |
|---|---|---|---|---|
| a | a | a | a | a |
| b | a | a | a | a |
| c | a | a | a | a |
| d | a | b | c | b |

Then we can easily see that $(R, +, .)$ is a left near-ring. Define a fuzzy subset $\mu : R \to [0,1]$ by $\mu(c) = \mu(d) < \mu(b) < \mu(a)$. Then $\mu$ is a fuzzy left ideal of $R$, but not fuzzy right ideal of $R$, Since $\mu((c+d)d - cd) = \mu(d) < \mu(b)$.

### F. *Proposition 3.6*

If a fuzzy subset $\mu$ of $R$ satisfies the properties $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$ then

 

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1) $\mu(0_R) \geq \mu(x)$
2) $\mu(-x) = \mu(x)$, for all $x, y \in R$

*Proof.*(1) We have that for any $x \in R$

$$\mu(0_R) = \mu(x - x)$$
$$\geq \min\{\mu(x), \mu(x)\}$$
$$= \mu(x)$$

Hence $\mu(0_R) \geq \mu(x)$.

(2) By (1), we have that

$$\mu(-x) = \mu(0_R - x)$$
$$\geq \min\{\mu(0_R), \mu(x)\}$$
$$= \mu(x)$$

Hence $\mu(-x) = \mu(x)$. ■

G. *Proposition 3.7*

Let $\mu$ be a fuzzy ideal of R. If $\mu(x - y) = \mu(0_R)$ then $\mu(x) = \mu(y)$.

*Proof.* Assume that $\mu(x - y) = \mu(0_R)$ for all $x, y \in R$. Then

$$\mu(x) = \mu(x - y + y)$$
$$\geq \min\{\mu(x - y), \mu(y)\}$$
$$= \min\{\mu(0_R), \mu(x)\}$$
$$= \mu(y)$$

So, $\mu(x) \geq \mu(y)$ (1)

Also, $\mu(y) = \mu(y - x + x)$

$$\geq \min\{\mu(y - x), \mu(x)\}$$
$$= \min\{\mu(0_R), \mu(x)\}$$
$$= \mu(x)$$

So, $\mu(y) \geq \mu(x)$ (2)

From equation (1) and (2), Hence $\mu(x) = \mu(y)$. □

H. *Proposition 3.8*

If $\mu: R \to [0,1]$ is a fuzzy ideals of near-ring $R$ with multiplicative identity $1_R$. Then $\mu(0_R) \geq \mu(x) \geq \mu(1_R)\ \forall\ x \in R$.

*Proof*: We know that, $\mu(x) = \mu(-x)$

And now,

$$\mu(0_R) = \mu(x - x)$$

$= \mu(x + (-x))$

$\geq \min\{\mu(x), \mu(-x)\}$

$= \mu(x)$ (1)

Also $\mu(x) = \mu(x, 1_R)$

$\geq \min\{\mu(x), \mu(1_R)\}$

$\geq \mu(1_R)$ (2)

From equation (1) and (2),

$\mu(0_R) \geq \mu(x) \geq \mu(1_R)\ \forall\, x \in R.$ ■

## IV. ANTI-FUZZY IDEALS OF NEAR-RING

### A. *Definition 4.1*

Let $R$ be a near-ring and $\mu$ be a fuzzy subset of $R$. $\mu$ is called an anti-fuzzy left ideal of R if $\mu$ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$.

1) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$,

2) $\mu(y + x - y) \leq \mu(x)$,

3) $\mu(xy) \leq \mu(y)$ or $\mu(xy) \leq \mu(x)$

### B. *Definition4.2*

Let $R$ be a near-ring and $\mu$ be a fuzzy subset of $R$. $\mu$ is called a anti fuzzy right ideal of R if $\mu$ is a fuzzy subnear-ring of R and satisfies: for all $x, y \in R$,

1) $\mu(x - y) \leq \max\{\mu(x), \mu(y)\}$,

2) $\mu(xy) \leq \max\{\mu(x), \mu(y)\}$

3) $\mu(y + x - y) \leq \mu(x)$

4) $\mu((x + i)y - xy) \leq \mu(i)$.

*C. Proposition 4.3* For every anti fuzzy ideals $\mu$ of $R$,

1) $\mu(0_R) \leq \mu(x), \forall\, x \in R$.

2) $\mu(x) = \mu(-x), \forall\, x \in R$.

3) $\mu(x - y) = \mu(0_R) \Rightarrow \mu(x) = \mu(y), \forall\, x, y \in R$.

*Proof.*(1) $\mu(0_R) = \mu(x - x)$

$\leq \max\{\mu(x), \mu(x)\}$

$= \mu(x)$ .

(2) $\mu(-x) = \mu(0_R - x)$

$\leq \max\{\mu(0_R), \mu(x)\}$

$= \mu(x).$

For all $x \in R$. Since $x$ is arbitrary, we conclude that $\mu(-x) = \mu(x)$.

(3) Assume that $\mu(x - y) = \mu(0_R)$ for all $x, y \in R$. Then

$$\mu(x) = \mu(x - y + y)$$
$$\leq max\{\mu(x - y), \mu(y)\}$$
$$= \max\{\mu(0_R), \mu(x)\}$$
$$= \mu(y)$$

So, $\mu(x) \leq \mu(y)$ (1)

Also,

$$\mu(y) = \mu(y - x + x)$$
$$\leq max\{\mu(y - x), \mu(x)\}$$
$$= \max\{\mu(0_R), \mu(x)\}$$
$$= \mu(x)$$

So, $\mu(y) \leq \mu(x)$ (2)

From equation (1) and (2)

Hence $\mu(x) = \mu(y)$. ■

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