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Behaviour of Sequence of Function and Continuous Function on Unbounded Interval

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Abstract: In this paper, we give an example of sequence of function on an unbounded interval which converges uniformly to a function but their Riemann integral sequence is not convergent. We also give an example of a continuous unbounded function on an unbounded domain whose integral is finite
Keywords: Continuous, Uniformly convergent, Riemann Integral, Pointwise convergent, Unbounded

I. INTRODUCTION

In this article, we give two examples which are very important in understanding the things in bounded and unbounded domain. Our first example is related to the uniform convergence of function in an unbounded interval and second example is related to the integral of an unbounded continuous function on an unbounded domain.

II. PRELIMINARIES

A. Definition 2.1

Pointwise convergence: Let g_n be a real valued sequence of functions defined on a set $G \subset R$, *then* g_n converges point wise to a real valued function g on G, if

$$\lim_{n\to\infty} g_n(x) = g(x), x \in S$$

Uniform Convergence- Let g_n be a real valued sequence of function defined on a set $G \subset R$, then g_n converges uniformly to a real valued function g on G, if for every $\varepsilon > 0$, there exist a positive integer M such that

$|g_n(t) - g(t)| < \epsilon \; \forall \; t \in S \text{ and all } n > M.$

Theorem: Let g_n be a real valued sequence of functions having domain [a,b] and further let $g_n \to g$ converges uniformly on [a,b]. Then

$$\lim_{n\to\infty} \int_a^b g_n(t)dt = \int_a^b g(t)dt$$

Proof. For proof, [cf. [1], theorem 25.2]

III.DIFFERENCE IN THE BEHAVIOUR FOR AN UNBOUNDED DOMAIN

From theorem 2.1, it is a very obvious question that why only finite interval? Does the proof of theorem 2.1 hold good for infinite interval also? Answer is no. It holds good for only bounded interval. For this, we are giving an example where above theorem 2.1 do not hold. Thus we conclude that behaviour of sequence of functions is quite different on bounded and unbounded domains.

A. Example 3.1

In this example, we will take a sequence $\{g_n\}$ of functions, which converges uniformly to a function g on $[0.\infty)$, but

$$\lim_{n\to\infty} \int_0^\infty g_n(t)dt \neq \int_0^\infty g(t)dt$$

Take the sequence $g_n: [0,\infty) \to R$ defined by

$$g_n(x) = \begin{cases} \frac{c}{n}, 0 \le x \le 2n \\ 0, otherwise \end{cases}$$

Where c is any arbitrary non zero constant. Its limit function is g(x) = 0, as we can easily see that for each x, and for any

$\epsilon > 0, |g_n(x) - 0| = \left|\frac{c}{n}\right| < \epsilon$

for large n. Moreover, note that above convergence is independent of x. Thus $g_n \to g$ uniformly.

Now we compute the infinite integral:

$$\lim_{n\to\infty} \int_0^\infty g_n(t)dt = \lim_{n\to\infty} \int_0^{2n} \frac{c}{n} dt = 2c \neq 0$$ Also,

$$\int_0^\infty g(t)dt = \int_0^\infty 0dt = 0$$

Thus above two integrals clearly implies that

$$2c = \lim_{n\to\infty} \int_0^\infty g_n(t)dt \neq \int_0^\infty g(t)dt = 0$$

Thus theorem 2.1 does not hold good for finite intervals.

IV. AREA UNDER AN UNBOUNDED CONTINUOUS FUNCTION CAN BE BOUNDED

Now we will give an example of continuous function $g(t): [0,\infty) \to R$ which is unbounded, but $\int_0^\infty g(t)dt$ is finite.

For any positive integer $n \geq 2$, Let $g(t): [0,\infty) \to R$ defined as

$$g(t)=\begin{cases} n+n^a(t-n), when\ t \in [n-\frac{1}{n^a}, n) \\ n, when\ t = n \\ n+n^a(n-t), when\ t \in (n, n+\frac{1}{n^a}] \end{cases}$$

and $g(t) = 0, otherwise$

Where a>2 be any positive real number.

First of all note that g is an unbounded continuous function as g(n) = n for each $n \in N$. Now we see how the graph of function g looks. Note that in the interval $[n-\frac{1}{n^a}, n+\frac{1}{n^a}]$, graph of g is a triangle whose vertices are $\left(n-\frac{1}{n^a}, 0\right), (n,n), (n+\frac{1}{n^a}, 0)$

From $t = n+\frac{1}{n^a}$ to $t = (n+1)-\frac{1}{(n-1)^a}$, g(t) = 0.

Thus on the basis of this information, we can calculate the integral now. Also notice that area of the triangle formed in the interval $[n-\frac{1}{n^a}, n+\frac{1}{n^a}]$ is $\frac{1}{n^{(a-1)}}$.

Finally,

$$\int_0^\infty g(t)dt = \sum_{n=2}^\infty \int_{n-n^{-a}}^{n+n^a} g(t)dt + 0 = \sum_{n=2}^\infty (a_n)$$

Where a_n is area of the triangle formed by g in the interval $\left[n-\frac{1}{n^a}, n+\frac{1}{n^a}\right]$. $Further$

$$\int_0^\infty g(t)dt = \sum_{n=2}^\infty \frac{1}{n^{(a-1)}} < \infty$$

As we know for $a > 2$, above series is convergent and has finite sum.

V. CONCLUSION

From above two examples we can conclude that behaviour of a sequence of function or that of continuous function is very unpredictable on infinite domain. Thus all the result for uniform convergence on sequence of function for bounded domains cannot be generalized to unbounded domains.

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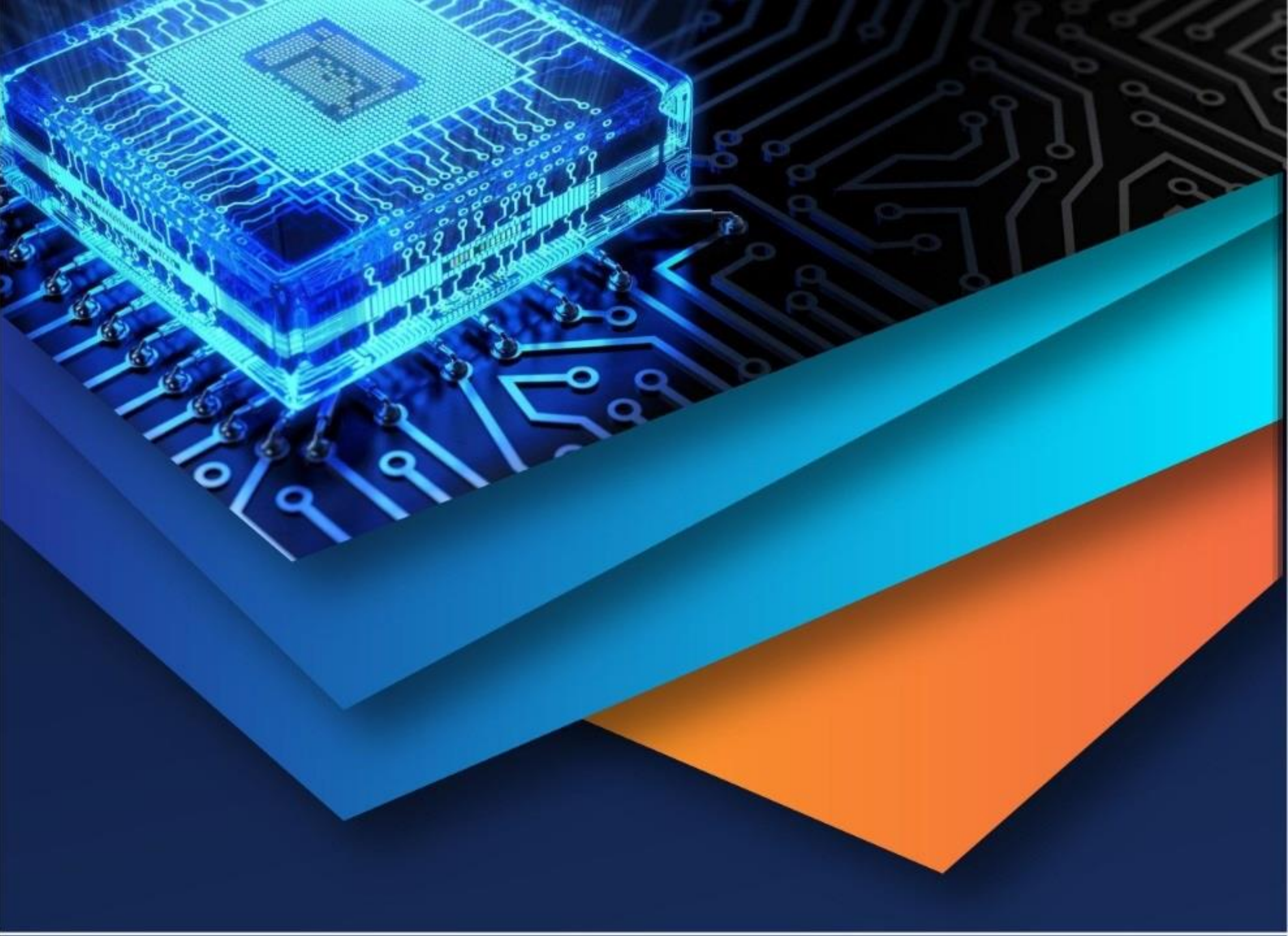

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