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Construction of the Diophantine Triple involving Pronic Number

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Abstract: We search for three distinct polynomials with integer coefficients such that the product of any two numbers increased by a non-zero integer (or polynomials with integer coefficients) is a perfect square.
Keywords: Diophantine triples, Pronic number, Perfect square.
Notations: Pronic Number of rank $n = n(n+1)$

I. INTRODUCTION

Let n be an integer. A set of positive integers $\{a_1, a_2, \ldots, a_m\}$ is said to have the property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$; such a set is called a Diophantine m-tuple of size m. The problem of construction of such set was studied by Diophantus. Many mathematicians considered the problem of the existence of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n [1] and also for any linear polynomials n. Further, various authors considered the connections of the problem of Diaphanous, Davenport and Fibonacci numbers in [2-14]. In this communication, we present three sections where in each of which we find the Diophantine triples from Pronic number with different ranks. A few interesting relations among the numbers in each of the above Diophantine triples are presented.

II. METHOD OF ANALYSIS

A. Section A

Let $a = n^2 - n$ and $b = n^2 + n$ be Pronic number of rank $n-1$ and n respectively such that $ab + (3n^2 + 1)$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac + (3n^2 + 1) = \beta^2 \tag{1}$$

$$bc + (3n^2 + 1) = \gamma^2 \tag{2}$$

Setting $\beta = a + \alpha$ and $\gamma = b + \alpha$, then subtracting (1) from (2), we get

$$\begin{aligned} c(b-a) &= \gamma^2 - \beta^2 = (\gamma+\beta)(\gamma-\beta) \\ &= (a+b+2\alpha)(b-a) \end{aligned}$$

Thus, we get $c = a + b + 2\alpha$

Similarly by choosing $\beta = a - \alpha$ and $\gamma = b - \alpha$, we obtain $c = a + b - 2\alpha$

Here we have $\alpha = n^2 + 1$ and thus two values of c are given by $c = 4n^2 + 2$ and $c = -2$.

Thus, we observe that $\{n^2 - n,\ n^2 + n,\ 4n^2 + 2\}$ and $\{n^2 - n,\ n^2 + n,\ -2\}$ are Diophantine triples with the property $D(3n^2 + 1)$.

Some numerical examples are given below in the following table.

Table 1

n	Diophantine Triples	$D(3n^2+1)$
1	(0,2,6) & (0,2,-2)	4
2	(2,6,18) & (2,6,-2)	13
3	(6,12,38) & (6,12,-2)	28

We present below, some of the Diophantine triples for Pronic number of rank mentioned above with suitable properties.

Table 2

a	b	c	$D(n)$
n^2-n	n^2+n	$4n^2+4$	$5n^2+4$
		-4	
n^2-n	n^2+n	$4n^2+6$	$7n^2+9$
		-6	
n^2-n	n^2+n	$4n^2+8$	$9n^2+16$
		-8	

B. Remarkable Observation

In general $\{n^2-n,\ n^2+n,\ 4n^2+2n\}$ and $\{n^2-n,\ n^2+n,\ -2n\}$ are Diophantine triples with the property $D\left((2n+1)n^2+n^2\right)$.

C. Section B

Let $a=n^2-3n+2$ and $b=n^2-n$ be Pronic number of rank $n-2$ and $n-1$ respectively such that $ab+\left(n^2-2n+1\right)$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+\left(n^2-2n+1\right)=\beta^2 \tag{3}$$

$$bc+\left(n^2-2n+1\right)=\gamma^2 \tag{4}$$

Applying the procedure as mentioned in section (A), we obtain

$$c=4n^2-8n+4 \text{ and } c=0$$

Thus, we observe that $\{n^2-3n+2,\ n^2-n,\ 4n^2-8n+4\}$ and $\{n^2-3n+2,\ n^2-n,\ 0\}$ are Diophantine triples with the property $D(n^2-2n+1)$.

Some numerical examples are given below in the following table.

Table 3

n	Diophantine Triples	$D(n^2-2n+1)$
2	(0,2,4) & (0,2,0)	1
3	(2,6,16) & (2,6,0)	4
4	(6,12,36) & (6,12,0)	9

We present below, some of the Diophantine triples for Pronic number of rank mentioned above with suitable properties.

Table 4

a	b	c	$D(n)$
n^2-3n+2	n^2-n	$4n^2-8n+6$ -2	$3n^2-6n+4$
n^2-3n+2	n^2-n	$4n^2-8n+8$ -4	$5n^2-10n+9$
n^2-3n+2	n^2-n	$4n^2-8n+10$ -6	$7n^2-14n+16$

D. Remarkable Observation

In general $\{n^2-3n+2, n^2-n, 4n^2-8n+2n\}$ and $\{n^2-3n+2, n^2-n, -2t\}$ are Diophantine triples with the property $D((2k+1)n^2-2pn+p^2)$ where $n=2,3,\dots$ $t=0,1,2,\dots$

$k=0,1,2,\dots$ $p=1,2,\dots$

E. Section C

Let $a=n^2-5n+6$ and $b=n^2-3n+2$ be Pronic number of rank $n-3$ and $n-2$ respectively such that $ab+(-5n^2+20n-11)$ is a perfect square say α^2.

Let c be any non-zero integer such that

$$ac+(-5n^2+20n-11)=\beta^2 \tag{5}$$

$$bc+(-5n^2+20n-11)=\gamma^2 \tag{6}$$

Applying the procedure as mentioned in section (A), we obtain

$$c=4n^2-16n+10 \text{ and } c=6$$

Thus, we observe that $\{n^2-5n+6,\ n^2-3n+2,\ 4n^2-16n+10\}$ and $\{n^2-5n+6,\ n^2-3n+2,\ 6\}$ are Diophantine triples with the property $D(-5n^2+20n-11)$.

Some numerical examples are given below in the following table.

Table 5

n	Diophantine Triples	$D(-5n^2+20n-11)$
1	(2,0,-2) & (2,0,6)	4
4	(2,6,10) & (2,6,6)	-11
5	(6,12,30) & (2,12,6)	-36

We present below, some of the Diophantine triples for Pronic number of rank mentioned above with suitable properties.

Table 6

a	b	c	$D(n)$
n^2-5n+6	n^2-3n+2	$4n^2-16n+12$ 4	$3n^2+12n-8$
n^2-5n+6	n^2-3n+2	$4n^2-16n+14$	$-n^2+4n-3$

		2	
n^2-5n+6	n^2-3n+2	$4n^2-16n+16$ 0	n^2-4n+4

III. CONCLUSION

In this paper we have presented a few examples of constructing a Diophantine triples for Pronic number of different rank with suitable properties. To conclude one may search for Diophantine triples for other special number with their corresponding suitable properties.

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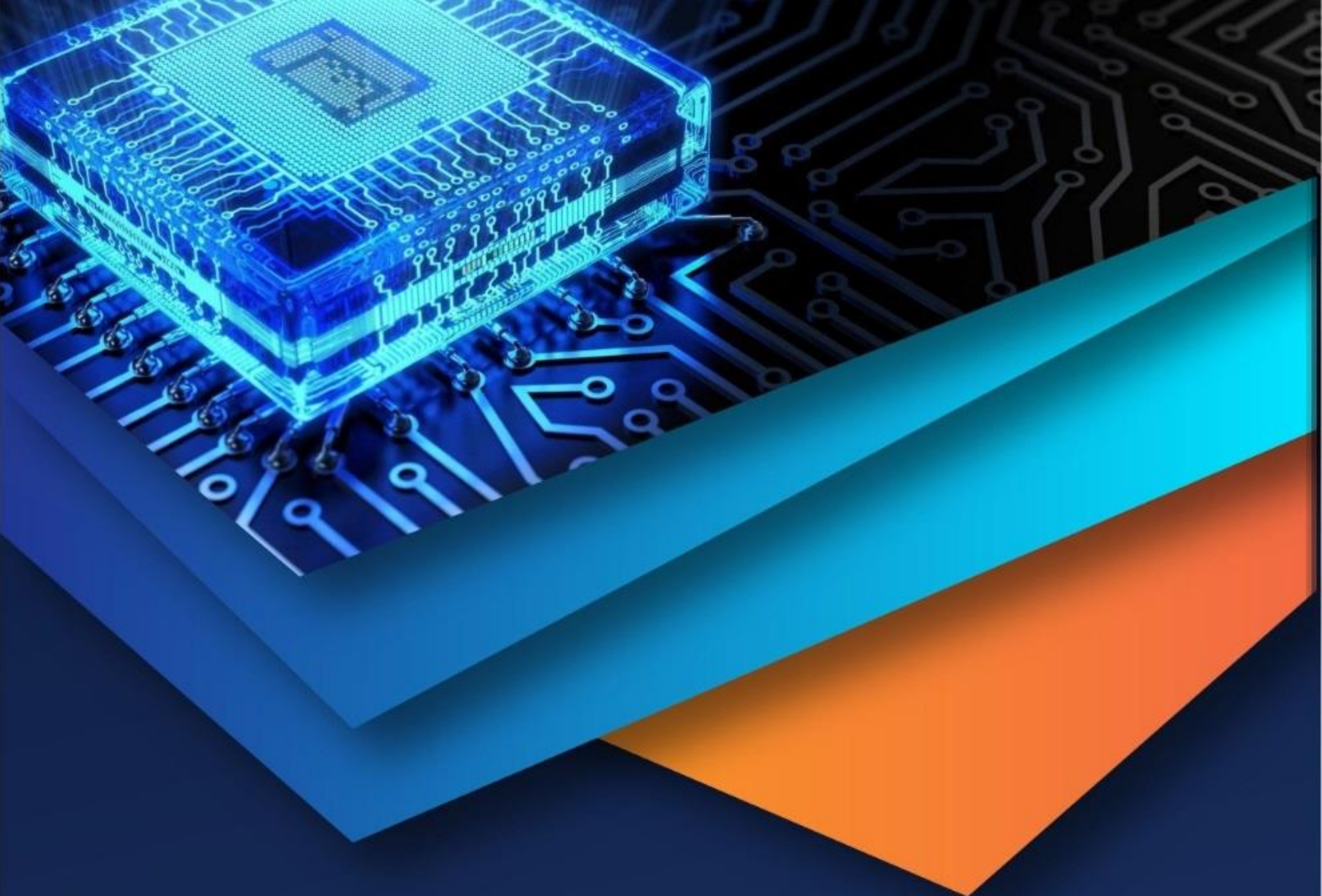

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