International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

Fuzzy Matrix with Application in Automata

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Abstract: A sequential machine is a dynamic system operating in discrete time that transforms sequence of input states received at the input of the system to sequence of output states produced at the output of the system. The produce large sequence of fuzzy internal states and output states for any given sequences of fuzzy input states of a fuzzy automation.
Keywords: Fuzzy internal states, Fuzzy automation, Transition relation, Fuzzy input and output state

I. INTRODUCTION

The sequences may be finite (or) count ably infinite. The transformation is accomplished by the concept of the dynamically changing interval state. At the same time, a new internal state is determined, which replaces its predecessor. The new internal state is stored in the system to be used subsequently. A finite state machine is called fuzzy automata when its states are characterized by fuzzy sets, the production of responses and next states is facilitated by suitable fuzzy relations.

II. PRELIMINARIES

A. Definition 3.1 Fuzzy Matrices

Let F_{mn} denot the set of all $m \times n$ matrices over F . If $m = n$, we write F_n. Elements of F_{mn} are called as membership value matrices binary fuzzy relation matrices (or) Fuzzy Matrices. Boolean matrices over the Boolean algebra $\{0,1\}$ are special types of fuzzy matrices.

B. Definition 3.2 Multiplication on Fuzzy Matrices

Let $A = (a_{ij}) \in F_{mp}$ and $B = (b_{ij}) \in F_{pn}$.
The max - min product

$$AB = \left(\sup_{k} \left\{ inf\{a_{ik}, b_{kj}\} \right\} \right) \in F_{mn}$$

The product AB is defined if and only if the number of column of A is the same as the number of rows of B; A and B are said to be conformable for Multiplication.

1) Example

Let $A = \begin{bmatrix} 0.8 & 0.1 \\ 0.2 & 0.7 \end{bmatrix}$ $and\ B = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.3 \end{bmatrix}$

Find AB.

2) Solution

$$AB = \begin{bmatrix} [0.8\ 0.1]\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} & [0.8\ 0.1]\begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix} \\ [0.2\ 0.7]\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix} & [0.2\ 0.7]\begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} Sup\{inf\{0.8,0.6\}\}, inf\{0.1,0.4\} & Sup\{inf\{0.8,0.5\}\}, inf\{0.1,0.3\} \\ Sup\{inf\{0.2,0.6\}\}, inf\{0.7,0.4\} & Sup\{inf\{0.2,0.5\}\}, inf\{0.7,0.3\} \end{bmatrix}$$

$$= \begin{bmatrix} Sup\{0.6,0.1\} & Sup\{0.5,0.1\} \\ Sup\{0.2,0.4\} & Sup\{0.2,0.3\} \end{bmatrix}$$

$$AB = \begin{bmatrix} 0.6 & 0.5 \\ 0.4 & 0.3 \end{bmatrix}$$

C. Definition 3.3Fuzzy Automata

A finite fuzzy automation A is a fuzzy relational system defined by the quintuple $A =< X, Y, Z, R, S >$
where
X is a non- empty finite set of input states
Y is a non- empty finite set of output states
Z is a non- empty finite set of internal states
R is a fuzzy relation on $X \times Y$
is a fuzzy relation on $X \times Y \times Z$

III. DERIVATION

Let us consider $X = \{x_1, x_2, \dots \dots \dots . x_{n-1}, x_n\}$,$Y = \{y_1, y_2, \dots \dots \dots . y_{n-1}, y_n\}$ and $Z = \{z_1, z_2, \dots \dots \dots . z_{n-1}, z_n\}$as the set of input states , output states and internal states respectively .Let A_t, B_t, C_t and E_t denote the fuzzy sets that characterize respectively, the stimulus response, response, current internal state and emerging internal state (next state) of the automation at time t.Given A_t and C_t at some time t, then by using fuzzy relations R and S, we can determine B_t and E_t. Clearly $A_t \in$ F (X), $B_t \in$ F (Y) and $C_t, E_t \in$ F (Z)Where F (.) is the set of fuzzy sets on the set (.) A fuzzy set C_1 which characterizes the initial state must be given to make fuzzy automation operate. Then $C_t = E_{t-1}$ for each time $t \in N - \{1\}$.The equation $C_t = E_{t-1}$ is assumed to be implemented by the block called storage. It's role is to store the produced fuzzy set E_t at time t and release it the next time under the label C_t.Given a sequence A_1,A_2,…… and an initial characterization C_1 of the internal state, fuzzy response relation R and state transition relation δ allow us to generate the corresponding sequences B_1,B_2,….. and C_2=E_1, C_3=E_2 .Now let us describe the operator of a fuzzy automation as follows For any given fuzzy input state A_t, the ternary state – transition relation δ is converted into a binary relation δ_{A_t} on z× z by the formula

$$\delta_{A_t}(z_i, z_j) = \max_{k \epsilon N_n}\left(min\left[A_t(x_k), \delta_{x_k}(z_i, z_j)\right]\right) \quad \dots (1)$$

Then assuming the present fuzzy state C_t is given . The fuzzy next state Et and the fuzzy output state B_t are determined by the max-min compositions

$$E_t = C_t \text{and } B_t = C_t\, R \quad \dots (2)$$

Equations (1) and (2) are sufficient to handle the sequences of fuzzy states. For instance a sequence A_1,A_2,…… A_r of r- fuzzy input state applied to a given initial fuzzy state C_1 , the fuzzy automata produces the sequence of fuzzy internal states.

$$E_1 = C_1\, \delta_{A_1}, E_2 = E_1\, \delta_{A_2} \text{ and } E_r = E_{r-1}\, \delta_{A_r}$$

Thus$E_r = C_1\, \delta_{A_1}, \dots, \delta_{A_r}$. The corresponding sequence of fuzzy output states

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$B_1 = C_1 R$, $B_2 = E_1 R$ and $B_r = E_{r-1} R$

Thus $B_r = C_1 \delta_{A_1}, \delta_{A_2}, \dots \dots . \delta_{A_{r-1}} \cdot R$.

IV. ILLUSTRATIVE EXAMPLE

Consider a fuzzy automation with $x = \{x_1, x_2\}$,$y = \{y_1, y_2, y_3\}$ and $z = \{z_1, z_2, z_3, z_4\}$ whose output relation

$$R = \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 0.3 \end{bmatrix} \end{array}$$

State transition relation δ are defined by the following matrices respectively for the input states x_1 and x_2..

$$\delta_{A1}(z_i, z_j) = \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0 & 0.4 & 0.2 & 1 \\ 0.3 & 1 & 0 & 0.2 \\ 0.5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\delta_{A2}(z_i, z_j) = \begin{array}{c} \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{array} \begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.3 & 0 & 0.6 \end{bmatrix} \end{array}$$

Generate sequence of two fuzzy initial states land output states under the following conditions:
The initial fuzzy states is $C_1 = [1,0.8,0.6,0.4]$ the input fuzzy states are $A_1 = [1,0.4]$ and $A_2 = [0,1]$.
Solution: Let us assume that the initial state of the automation is $C_1 = [1,0.8,0.6,0.4]$and fuzzy input $A_1 = [1,0.4]$ is given.
By using the equation

$$\delta_{At}(z_i, z_j) = \max_{k\epsilon\{1,2\}}\left(min[A_t(x_k), \delta_k(z_i, z_j)]\right)$$

Let us compute the equation δ_{A1}

$$\delta_{A1}(z_1, z_1) = max\{min[A_1(x_1), \delta_{x1}(z_1, z_1)], min[A_1(x_2), \delta_{x2}(z_1, z_1)]\}$$
$$= max\{\min(1,0), \min(0.4,0)\}$$
$$= 0$$

$$\delta_{A1}(z_1, z_2) = max\{min[A_1(x_1), \delta_{x1}(z_1, z_2)], min[A_1(x_2), \delta_{x2}(z_1, z_2)]\}$$
$$= max\{\min(1,0.4), \min(0.4,0)\}$$
$$= 0.4$$

$$\delta_{A1}(z_1, z_3) = max\{min[A_1(x_1), \delta_{x1}(z_1, z_3)], min[A_1(x_2), \delta_{x2}(z_1, z_3)]\}$$
$$= max\{\min(1,0.2), \min(0.4,1)\}$$
$$= 0.4$$

$$\delta_{A1}(z_1, z_4) = max\{min[A_1(x_1), \delta_{x1}(z_1, z_4)], min[A_1(x_2), \delta_{x2}(z_1, z_4)]\}$$
$$= max\{\min(1,1), \min(0.4,0)\}$$
$$= 1$$

Thus the First row of δ_{A1} is [0 0.4 0.4 1]

$$\delta_{A1}(z_2, z_1) = max\{min[A_1(x_1), \delta_{x1}(z_2, z_1)], min[A_1(x_2), \delta_{x2}(z_2, z_1)]\}$$
$$= max\{\min(1,0.3), \min(0.4,0.2)\}$$
$$= 0.3$$

$$\delta_{A1}(z_2, z_2) = max\{min[A_1(x_1), \delta_{x1}(z_2, z_2)], min[A_1(x_2), \delta_{x2}(z_2, z_2)]\}$$
$$= max\{\min(1,1), \min(0.4,0)\}$$
$$= 1$$

$$\delta_{A1}(z_2, z_3) = max\{min[A_1(x_1), \delta_{x1}(z_2, z_3)], min[A_1(x_2), \delta_{x2}(z_2, z_3)]\}$$
$$= max\{\min(1,0), \min(0.4,0)\}$$

$= 0$

$\delta_{A1}(z_2, z_4) = max\{min[A_1(x_1), \delta_{x1}(z_2, z_4)], min[A_1(x_2), \delta_{x2}(z_2, z_4)]\}$
$= max\{\text{min }(1,0.2), \text{min }(0.4,1)\}$
$= 0.4$

Thus the Second row of δ_{A1} is [0.3 1 0 0.4]

$\delta_{A1}(z_3, z_1) = max\{min[A_1(x_1), \delta_{x1}(z_3, z_1)], min[A_1(x_2), \delta_{x2}(z_3, z_1)]\}$
$= max\{\text{min }(1,0.5), \text{min }(0.4,0)\}$
$= 0.5$

$\delta_{A1}(z_3, z_2) = max\{min[A_1(x_1), \delta_{x1}(z_3, z_2)], min[A_1(x_2), \delta_{x2}(z_3, z_2)]\}$
$= max\{\text{min }(1,0), \text{min }(0.4,0)\}$
$= 0$

$\delta_{A1}(z_3, z_3) = max\{min[A_1(x_1), \delta_{x1}(z_3, z_3)], min[A_1(x_2), \delta_{x2}(z_3, z_3)]\}$
$= max\{\text{min }(1,0), \text{min }(0.4,0)\}$
$= 0$

$\delta_{A1}(z_3, z_4) = max\{min[A_1(x_1), \delta_{x1}(z_3, z_4)], min[A_1(x_2), \delta_{x2}(z_3, z_4)]\}$
$= max\{\text{min }(1,1), \text{min }(0.4,1)\}$
$= 1$

Thus the Third row of δ_{A1} is [0.5 0 0 1]

$\delta_{A1}(z_4, z_1) = max\{min[A_1(x_1), \delta_{x1}(z_4, z_1)], min[A_1(x_2), \delta_{x2}(z_4, z_1)]\}$
$= max\{\text{min }(1,0), \text{min }(0.4,1)\}$
$= 0.4$

$\delta_{A1}(z_4, z_2) = max\{min[A_1(x_1), \delta_{x1}(z_4, z_2)], min[A_1(x_2), \delta_{x2}(z_4, z_2)]\}$
$= max\{\text{min }(1,0), \text{min }(0.4,0.3)\}$
$= 0.3$

$\delta_{A1}(z_4, z_3) = max\{min[A_1(x_1), \delta_{x1}(z_4, z_3)], min[A_1(x_2), \delta_{x2}(z_4, z_3)]\}$
$= max\{\text{min }(1,0), \text{min }(0.4,0)\}$
$= 0$

$\delta_{A1}(z_4, z_4) = max\{min[A_1(x_1), \delta_{x1}(z_4, z_4)], min[A_1(x_2), \delta_{x2}(z_4, z_4)]\}$
$= max\{\text{min }(1,1), \text{min }(0.4,0.6)\}$
$= 1$

Thus the Last row of δ_{A1} is [0.4 0.3 0 1]

The matrix δ_{A1} is

$$\delta_{A1} = \begin{bmatrix} 0 & 0.4 & 0.4 & 1 \\ 0.3 & 1 & 0 & 0.4 \\ 0.5 & 0 & 0 & 1 \\ 0.4 & 0.3 & 0 & 1 \end{bmatrix}$$

To calculate the fuzzy next set E_1 and the fuzzy output state B_1 of the automation

$E_1 = C_1 . \delta_{A1}$

$$= [1 \quad 0.8 \quad 0.6 \quad 0.4] \begin{bmatrix} 0 & 0.4 & 0.4 & 1 \\ 0.3 & 1 & 0 & 0.4 \\ 0.5 & 0 & 0 & 1 \\ 0.4 & 0.3 & 0 & 1 \end{bmatrix}$$

$E_1 =$ [max(0,0.3,0.5,0.4) max(0.4,0.8,0,0.3) max(0.4,0,0,0) max (1,0.4,0.6,0.4)]

$E_1 =$ [0.5 0.8 0.4 1]

$B_1 = C_1 . R$

$$= [1 \quad 0.8 \quad 0.6 \quad 0.4] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0.5 & 1 & 0.3 \end{bmatrix}$$

= [max(1,0,0,0.4) max(0,0.8,0,0.4) max (0,0,0.6 ,0.3)]

International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor :6.887
Volume 6 Issue I, January 2018- Available at www.ijraset.com

B_1 = [1 0.8 0.6]

Assume that the next fuzzy input state i A_2 = [0,1] is given.

By using the equation

$\delta_{A2}(z_i, z_j) = \max_{k\epsilon\{1,2\}}(min[A_2(x_k), \delta_{xk}(z_i, z_j)])$

Let us compute the equation

$\delta_{A2}(z_1, z_1) = max\{min[A_2(x_1), \delta_{x1}(z_1, z_1)], min[A_2(x_2), \delta_{x2}(z_1, z_1)]\}$
$= max\{\text{min }(0,0), \text{min }(1,0)\}$
$= 0$

$\delta_{A2}(z_1, z_2) = max\{min[A_2(x_1), \delta_{x1}(z_1, z_2)], min[A_2(x_2), \delta_{x2}(z_1, z_2)]\}$
$= max\{\text{min }(0,0.4), \text{min }(1,0)\}$
$= max\{0,0\}$
$= 0$

$\delta_{A2}(z_1, z_3) = max\{min[A_2(x_1), \delta_{x1}(z_1, z_3)], min[A_2(x_2), \delta_{x2}(z_1, z_3)]\}$
$= max\{\text{min }(0,0.2), \text{min }(1,1)\}$
$= 1$

$\delta_{A2}(z_1, z_4) = max\{min[A_2(x_1), \delta_{x1}(z_1, z_4)], min[A_2(x_2), \delta_{x2}(z_1, z_4)]\}$
$= max\{\text{min }(0,1), \text{min }(1,0)\}$
$= 0$

Thus the First row of δ_{A2} is [0 0 1 0]

$\delta_{A2}(z_2, z_1) = max\{min[A_2(x_1), \delta_{x1}(z_2, z_1)], min[A_2(x_2), \delta_{x2}(z_2, z_1)]\}$
$= max\{\text{min }(0,0.3), \text{min }(1,0.2)\}$
$= 0.2$

$\delta_{A2}(z_2, z_2) = max\{min[A_2(x_1), \delta_{x1}(z_2, z_2)], min[A_2(x_2), \delta_{x2}(z_2, z_2)]\}$
$= max\{\text{min }(0,1), \text{min }(1,0)\}$
$= 0$

$\delta_{A2}(z_2, z_3) = max\{min[A_2(x_1), \delta_{x1}(z_2, z_3)], min[A_2(x_2), \delta_{x2}(z_2, z_3)]\}$
$= max\{\text{min }(0,0), \text{min }(1,0)\}$
$= 0$

$\delta_{A2}(z_2, z_4) = max\{min[A_2(x_1), \delta_{x1}(z_2, z_4)], min[A_2(x_2), \delta_{x2}(z_2, z_4)]\}$
$= max\{\text{min }(0,0.2), \text{min }(1,1)\}$
$= 1$

Thus the Second row of δ_{A2} is [0.2 0 0 1]

$\delta_{A2}(z_3, z_1) = max\{min[A_2(x_1), \delta_{x1}(z_3, z_1)], min[A_2(x_2), \delta_{x2}(z_3, z_1)]\}$
$= max\{\text{min }(0,0.5), \text{min }(1,0)\}$
$= 0$

$\delta_{A2}(z_3, z_2) = max\{min[A_2(x_1), \delta_{x1}(z_3, z_2)], min[A_2(x_2), \delta_{x2}(z_3, z_2)]\}$
$= max\{\text{min }(0,0), \text{min }(1,0)\}$
$= 0$

$\delta_{A2}(z_3, z_3) = max\{min[A_2(x_1), \delta_{x1}(z_3, z_3)], min[A_2(x_2), \delta_{x2}(z_3, z_3)]\}$
$= max\{\text{min }(0,0), \text{min }(1,0)\}$
$= 0$

$\delta_{A2}(z_3, z_4) = max\{min[A_2(x_1), \delta_{x1}(z_3, z_4)], min[A_2(x_2), \delta_{x2}(z_3, z_4)]\}$
$= max\{\text{min }(0,1), \text{min }(1,1)\}$
$= 1$

Thus the Third row of δ_{A2} is [0 0 0 1]

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Volume 6 Issue I, January 2018- Available at www.ijraset.com

$$\delta_{A2}(z_4, z_1) = max\{min[A_2(x_1), \delta_{x1}(z_4, z_1)], min[A_2(x_2), \delta_{x2}(z_4, z_1)]\}$$
$$= max\{\text{min }(0,0), \text{min }(1,1)\}$$
$$= 1$$
$$\delta_{A2}(z_4, z_2) = max\{min[A_2(x_1), \delta_{x1}(z_4, z_2)], min[A_2(x_2), \delta_{x2}(z_4, z_2)]\}$$
$$= max\{\text{min }(0,0), \text{min }(1,0.3)\}$$
$$= 0.3$$
$$\delta_{A2}(z_4, z_3) = max\{min[A_2(x_1), \delta_{x1}(z_4, z_3)], min[A_2(x_2), \delta_{x2}(z_4, z_3)]\}$$
$$= max\{\text{min }(0,0), \text{min }(1,0)\}$$
$$= 0$$
$$\delta_{A2}(z_4, z_4) = max\{min[A_2(x_1), \delta_{x1}(z_4, z_4)], min[A_2(x_2), \delta_{x2}(z_4 , z_4)]\}$$
$$= max\{\text{min }(0,1), \text{min }(1,0.6)\}$$
$$= 0.6$$

Thus the Last row of δ_{A2} is [1 0.3 0 0.6]

The matrix δ_{A2} is

$$\delta_{A2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.3 & 0 & 0.6 \end{bmatrix}$$

To calculate the fuzzy next set E_2 and the fuzzy output state B_2 of the automation

$$E_2 = C_2.\delta_{A2} = E_1.\delta_{A2}$$
$$= [0.5 \quad 0.8 \quad 0.4 \quad 1] \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0.2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0.3 & 0 & 0.6 \end{bmatrix}$$

$$E_2 = [\max(0,0.2,1)\ \max(0,0,0\ ,0.3)\ \max(0.5,0,0,0)\ \max\ (0,0.8,0.4,0.6)]$$
$$E_2 = [1\ \ 0.3\ \ 0.5\ \ 0.8\ ]$$
$$B_2 = E_1.R$$
$$= [0.5 \quad 0.8 \quad 0.4 \quad 1] \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0.5 & 1 & 0.3 \end{bmatrix}$$
$$= [\max(0.5\ , 0, 0\ , 0.5)\ \max(0, 0.8, 0, 1)\ \max\ (0, 0, 0.4\ \ ,0.3)]$$
$$B_2 = [\ 0.5\ \ 1\ \ 0.4\ ]$$

Similarly we can produce large sequence of fuzzy internal states and output states for any given sequences of fuzzy input states of a fuzzy automation.

V. CONCLUSION

The response of the system depends on the basis of the received stimulus and the internal state of the system, which in-turn results in a new internal state. The fuzzy automaton when characterized by fuzzy sets, the production of responses and next states are facilitated by suitable fuzzy relations.

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