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A New Ranking Approach on Fuzzy Complementarity Problem

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Abstract: In this paper, a new ranking procedure based on hexagonal fuzzy numbers, is applied to a linear complementarity problem (LCP) with fuzzy coefficients. By this ranking method, any linear complementarity problem can be converted into a crisp value problem to get an optimal solution. We solve the fuzzy linear complementarity problem with hexagonal fuzzy numbers is illustrated by means of an example.
Keywords: Fuzzy Set, Fuzzy Number, Hexagonal Fuzzy Numbers, Fuzzy Linear Complementarity Problem, Centroid points

I. INTRODUCTION

The fuzzy set theory has been applied in many fields such as operation research, control theory, management sciences, engineering, etc. Fuzzy sets have been introduced by Lotfi. A. Zadeh(1965)[1]. A fuzzy set is a class of objects with a continuum of grades of membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. Nagoorgani. A., Mohamed Assarudeen. S. N proposed by the fuzzy numbers and fuzzy values are widely used in engineering applications because of their suitability for representing uncertain information. In standard fuzzy arithmetic operations we have some problem in subtraction and division operation [2]. Cottle, R.W., Dantzig. G. B proposed by linear programming, quadratic programming and bimartix (two-person, non-zero sum) games lead to the consideration of the following fundamental problem with some constructive procedures for solving the fundamental problem under various assumptions on the data [5]. R. E. Bellman and L. A. Zadeh proposed by decision-making in a fuzzy environment is meant a decision process in which the goals and/or the constraints, but not necessarily the system under control are fuzzy in nature. This means that the goals and/or the constraints constitute classes of alternatives whose boundaries are not sharply defined [6].

Ludo Vander Heyden proposed by algorithm solves a sequence of sub problems of different dimensions, the sequence being possible non monotonic in the dimension of the sub problem solved. Every sub problem is the linear complementarity problem defined by a leading principal minor of the matrix M. Index-theoretic arguments characterizes the points at which non monotonic behaviour occurs [7]. In fuzzy environment ranking fuzzy numbers is a very important in decision making procedure.

Ranking fuzzy numbers were first proposed by Jain [8] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [9], and more recently by Chen and Hwang [10].Lee and Li [11] proposed the comparison of fuzzy numbers. Liou and Wang [12] presented ranking fuzzy numbers with interval values. In section 2, discusses the basic definition. In section 3, concepts of triangular, trapezoidal and hexagonal fuzzy numbers is reviewed. In section 4, the proposed ranking method is discussed. In section 5, solving the fuzzy linear complementarity problem and algorithm to solve the problem and algorithm to solve the problem is discussed. To solve this method, a numerical example is solved in section 6. Conclusion is discussed in section 7.

II. PRELIMINARIES

A. *Definition*

A fuzzy set $\tilde{A}$ is defined by $\widetilde{A} = \{(x, \mu_A(x)) : x \epsilon A, \mu_A(x) \epsilon [0,1]\}$. In this pair $(x, \mu_A(x))$,the first element x belongs to the classical set A and the second element $\mu_A(x)$ belongs to the interval [0,1] called membership function.

B. *Definition*

A fuzzy set $\tilde{A}$ of the real line R with membership function $\mu_{\tilde{A}}(x)$: R→ [0,1] is called fuzzy number if

1) $\tilde{A}$must be normal fuzzy set.
2) $\alpha_{\tilde{A}}$must be closed interval for every $\alpha \in [0,1]$.
3) The support of $\tilde{A}$ must be bounded.

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C. Definition

A fuzzy number $\tilde{A}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3) where a_1, a_2 and a_3 are real numbers and its membership function is given below

$$\mu_A(x) = \begin{cases} \frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ \frac{(a_3-x)}{(a_3-a_1)} & for\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

D. Definition

A fuzzy set $\tilde{A}$ =(a_1, a_2, a_3, a_4) is said to be trapezoidal fuzzy number where $a_1 \le a_2 \le a_3 \le a_4$if its membership function is given by

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \frac{(a_4-x)}{(a_4-a_3)} & for\ a_3 \le x \le a_4 \\ 0 & for\ x > a_4 \end{cases}$$

III. CONCEPT OF HEXAGONAL FUZZY NUMBERS

A. Definition

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by$\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$where $(a_1, a_2, a_3, a_4, a_5, a_6)$ are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given below

$$\mu_A(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)} & for\ a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\frac{(x-a_2)}{(a_3-a_2)} & for\ a_2 \le x \le a_3 \\ 1 & for\ a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\frac{(x-a_4)}{(a_5-a_4)} & for\ a_4 \le x \le a_5 \\ \frac{1}{2}\frac{(a_6-x)}{(a_6-a_5)} & for\ a_5 \le x \le a_6 \\ 0 & for\ x < a_1 \end{cases}$$

B. Remark

When $w = 1$, the hexagonal fuzzy number is a normal hexagonal fuzzy number.

IV. PROPOSED RANKING METHOD

The centroid of a hexagonal fuzzy number is considered to be the balancing point of the hexagon (figure.1)

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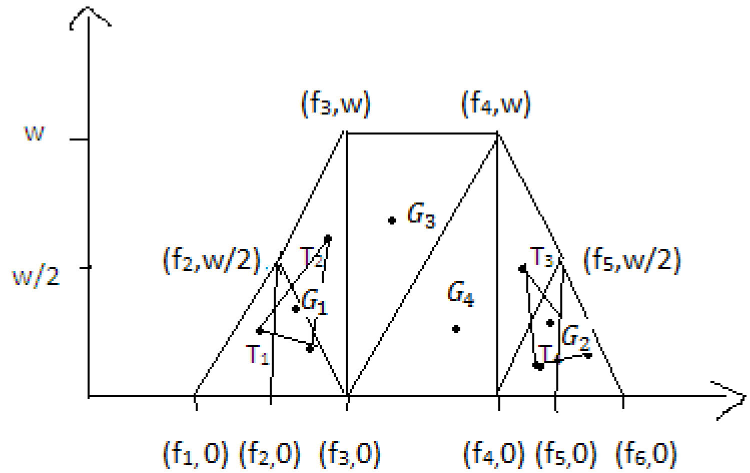

figure.1

Divide the hexagonal into eight triangles. By using the centroid formula of triangle, calculate the ranking as centroid of triangle T_1, T_2 and T_3 are as follows

$T_1 = \left(\frac{f_1+2f_2}{3}, \frac{w}{6}\right); T_2 = \left(\frac{2f_2+f_3}{3}, \frac{w}{6}\right); T_1 = \left(\frac{2f_3+f_2}{3}, \frac{w}{2}\right)$

Centroid of the centroid of T_1, T_2 and T_3 are $G_1 = \left(\frac{f_1+5f_2+3f_3}{9}, \frac{5w}{18}\right)$

Centroid of triangle T_4, T_5 and T_6 are as follows

$T_1 = \left(\frac{2f_4+f_5}{3}, \frac{w}{2}\right); T_2 = \left(\frac{f_4+2f_5}{3}, \frac{w}{6}\right); T_1 = \left(\frac{2f_5+f_6}{3}, \frac{w}{6}\right)$

Centroid of the centroid of T_4, T_5 and T_6 are $G_2 = \left(\frac{3f_4+5f_5+f_6}{9}, \frac{5w}{18}\right)$

Centroid of triangle G_3 and G_4 is

$G_3 = \left(\frac{2f_3+f_4}{3}, \frac{2w}{3}\right)andG_4 = \left(\frac{f_3+2f_4}{3}, \frac{w}{3}\right)$

The centroid of centroid of the triangles G_1, G_2, G_3 and G_4 are

$$R(\tilde{A}_H) = \left(\frac{f_1 + 5f_2 + 12f_3 + 12f_4 + 5f_5 + f_6}{36}, \frac{14w}{36}\right)$$

The ranking function of the generalized hexagonal fuzzy number

$\tilde{A}_H = (f_1, f_2, f_3, f_4, f_5, f_6; w)$ Which maps the set of all fuzzy numbers to a set of real numbers is defined by

$$R(\tilde{A}_H) = (\bar{x}_0, \bar{y}_0) = \left(\frac{f_1 + 5f_2 + 12f_3 + 12f_4 + 5f_5 + f_6}{36}\right)\left(\frac{14w}{36}\right)$$

V. LINEAR COMPLEMENTARITY PROBLEM (LCP)

Given a real $n \times n$ square matrix M and a $n \times 1$ real vector q, then the linear complementarity problem denoted by LCP (q, M) is to find real $n \times 1$ vector W, Z such that

$$W - MZ = q \qquad (1)$$

$$W_j \geq 0, Z_j \geq 0, for\ j = 1,2, \dots, n \qquad (2)$$

$$W_j Z_j = 0, for\ j = 1,2, \dots, n \qquad (3)$$

Here the pair (W_j, Z_j) is said to be a pair of complementarity variables.

A. *Fuzzy linear complementarity problem (flcp):*

Assume that all parameters in (1) – (3) are fuzzy and are described by triangular fuzzy numbers. Then the following fuzzy Linear Complementarity Problem can be obtained by replacing crisp parameters with triangular fuzzy numbers.

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$$\widetilde{w} - \widetilde{M}\tilde{z} = \tilde{q} \qquad (4)$$

$$\widetilde{w}_i, \tilde{z}_i \geq 0, for\ i = 1,2, \dots, n \qquad (5)$$

$$\widetilde{w}_i \tilde{z}_i = 0, for\ i = 1,2, \dots, n \qquad (6)$$

The pair $(\widetilde{w}_i, \tilde{z}_i)$ is said to be a pair of fuzzy complementarity variables.

If$\tilde{q} \geq 0$, then we have the solution satisfying (4) - (6), by letting $\widetilde{w} = \tilde{q}$ and $\tilde{z} = 0$.

If$\tilde{q} \leq 0$, we will consider the following system

$$\widetilde{w} - \widetilde{M}\tilde{z} - \tilde{e}\tilde{z}_0 = \tilde{q}(7)$$

$$\widetilde{w}_j \geq 0, \tilde{z}_j \geq 0, \tilde{z}_0 \geq 0, j = 1,2, \dots, n \qquad (8)$$

$$\widetilde{w}_j \tilde{z}_j = 0, j = 1,2, \dots, n \qquad (9)$$

Where $\tilde{z}_0$ is an artificial fuzzy variable and $\tilde{e}$ is an n-vector with all components equal to one.

B. Algorithm for fuzzy linear complementarity problem

Lemke suggested an algorithm for solving linear complementarity problems. Based on this idea, an algorithm for solving fuzzy linear complementarity problem is developed here.

1) Step:1: Introduce the fuzzy artificial variable $\tilde{z}_0$and consider the system (7) – (9).
If$\tilde{q} \geq 0$,stop;then $(\widetilde{w}, \tilde{z}) = (\tilde{q}, \tilde{0})$ is a fuzzy complementary basic feasible solution.
If$\tilde{q} \leq 0$, display the system (7), (8) as given in the simplex table.
Let -$q_s = maximum\ \{-q_i / 1 \leq i \leq n\}$, and update the table by pivoting at row s and the ~ $_0$ column. Thus the fuzzy basic variables ~ $_0$ ~ for $= 1,2, \dots,$ and $\neq$ are positive.
Let ~ = ~ and go to step: 2.

2) Step: 2
Let ~ be the updated column in the current table under the variable~ .

If~ ≤ 0, go to step: 5 otherwise determine the index r by the following minimum ratio test, = min$_{1\leq \ \leq}$ $\left\{ > 0 \right\}$ where is the updated right-hand side column denoting the values of the fuzzy basic variables. If the fuzzy basic variable at row r is~ $_0$, go to step: 4, otherwise, go to step: 3.

/ $\geq 0\}$is found such that every point in R satisfying (7), (8) and (9), where $(\ ,\tilde{Z},\ _0)$ is the almost fuzzy complementary basic feasible solution, and ~ is an extreme direction of the set defined by (7) and (8) having a ~ in the row corresponding to~ ,–~ in the rows of the current basic variables and zeros everywhere else.

VI. NUMERICAL EXAMPLE

Consider the following FLCP (~ , ~)where

$$\sim = \begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix} \sim = \begin{pmatrix} 3 \\ 5 \\ -9 \\ -5 \end{pmatrix}$$

A. Solution

The fuzzy linear complementarity problem is converted into hexagonal fuzzy linear complementarity problem as

$$\sim\ = \begin{pmatrix} (-4,-3,1,2,3,4;1) & (-4,-3,-2,-1,3,4;1) & (-4,-3,-2,-1,3,4;1) & (-4,-3,-2,-1,3,4;1) \\ (-4,-3,-2,-1,3,4;1) & (-4,-3,1,2,3,4;1) & (-4,-3,-2,-1,3,4;1) & (-4,-3,-2,-1,3,4;1) \\ (-4,-3,1,2,3,4;1) & (-4,-3,1,2,3,4;1) & (-8,-1,2,3,4,5;1) & (-3,-2,-1,1,2,3;1) \\ (-4,-3,1,2,3,4;1) & (-4,-3,1,2,3,4;1) & (-3,-2,-1,1,2,3;1) & (-8,-1,2,3,4,5;1) \end{pmatrix}$$

$$\sim\ = \begin{pmatrix} (-2,-1,3,4,5,;1) \\ (-5,2,5,6,7,8;1) \\ (-12,-11,-10,-9,-5,-4;1) \\ (-8,-7,-6,-5,-2,5;1) \end{pmatrix}$$

By using our proposed ranking method, we obtained

$$(-4,-3,1,2,3,4;1) = 0.389 \quad (-4,-3,-2,-1,3,4;1) = -0.389$$

$$(-8,-1,2,3,4,5;1) = 0.778 \quad (-3,-2,-1,1,2,3;1) = 0$$

$$(-2,-1,3,4,5,;1) = 1.167 \qquad (-5,2,5,6,7,8;1) = 1.945$$

$$(-8,-7,-6,-5,-2,5;1) = -1.945$$

Now, we have

$$\sim\ = \begin{pmatrix} 0.389 & -0.389 & -0.389 & -0.389 \\ -0.389 & 0.389 & -0.389 & -0.389 \\ 0.389 & 0.389 & 0.778 & 0 \\ 0.389 & 0.389 & 0 & 0.778 \end{pmatrix}$$

$$\sim\ = \begin{pmatrix} 1.167 \\ 1.945 \\ -3.501 \\ -1.945 \end{pmatrix}$$

The above problem can be written in the simplex table format and also introducing $\sim_0$

Basic variables	$\tilde{w}_1$	$\tilde{w}_2$	$\tilde{w}_3$	$\tilde{w}_4$	$\tilde{z}_1$	$\tilde{z}_2$	$\tilde{z}_3$	$\tilde{z}_4$	$\tilde{z}_0$	$\tilde{q}$
$\tilde{w}_1$	1	0	0	0	-0.389	0.389	0.389	0.389	-0.389	1.167
$\tilde{w}_2$	0	1	0	0	0.389	-0.389	0.389	0.389	-0.389	1.945
$\tilde{w}_3$	0	0	1	0	-0.389	-0.389	-0.778	0	-0.389	-3.501
$\tilde{w}_4$	0	0	0	1	-0.389	-0.389	0	-0.778	-0.389	-1.945

$\therefore$ $\sim_3$leaves from the basic variable.

Here bold letters denotes the pivot element and the corresponding column is the leaving variable.

Basic variables	$\sim_1$	$\sim_2$	$\sim_3$	$\sim_4$	$\sim_1$	$\sim_2$	$\sim_3$	$\sim_4$	$\sim_0$	$\sim$	Ratio
$\sim_1$	1	0	-1	0	0	0.778	1.167	0.389	0	4.668	4
$\sim_2$	0	1	-1	0	0.778	0	1.167	0.389	0	5.446	4.67
$\sim_0$	0	0	-2.571	0	1	1	2	0	1	9	4.5
$\sim_4$	0	0	-1	1	0	0	**0.778**	-0.778	0	1.556	2

$\therefore$ $\sim_4$leaves the basis,$\sim_3$ enter the basis.

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Basic variables	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{0}$	$\tilde{}$	Ratio
$\tilde{}_{1}$	1	0	0.5	-1.5	0	0.778	0	**1.556**	0	2.334	1.5
$\tilde{}_{2}$	0	0	0.5	-1.5	0.778	0	0	1.556	0	3.112	2
$\tilde{}_{0}$	0	0	0	-2.571	1	1	0	2	1	5	2.5
$\tilde{}_{3}$	0	0	-1.285	1.285	0	0	1	-1	0	2	-

∴ $\tilde{}_{1}$leaves the basis and $\tilde{}_{4}$ enter the basis.

Basic variables	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{0}$	$\tilde{}$	Ratio
$\tilde{}_{4}$	0.643	0	0.321	-0.964	0	0.5	0	1	0	1.5	-
$\tilde{}_{2}$	-1	0	0	0	**0.778**	-0.778	0	0	0	0.778	1
$\tilde{}_{0}$	-1.285	0	-0.642	-0.643	1	0	0	0	1	2	2
$\tilde{}_{3}$	0.643	0	-0.964	0.321	0	0.5	1	0	0	3.5	-

s

Basic variables	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{0}$	$\tilde{}$	Ratio
$\tilde{}_{4}$	0.643	0	0.321	-0.964	0	0.5	0	1	0	1.5	3
$\tilde{}_{1}$	-1.285	0	0	0	1	-1	0	0	0	1	-
$\tilde{}_{0}$	0	0	-0.642	-0.643	1	**1**	0	0	1	1	1
$\tilde{}_{3}$	0.643	0	-0.964	0.321	0	0.5	1	0	0	3.5	7

∴ $\tilde{}_{0}$leaves the basis and $\tilde{}_{2}$ enter the basis.

Basic variables	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{1}$	$\tilde{}_{2}$	$\tilde{}_{3}$	$\tilde{}_{4}$	$\tilde{}_{0}$	$\tilde{}$	Ratio
$\tilde{}_{4}$	0.643	0	0.642	-0.643	0	0	0	1	-0.5	**1**	
$\tilde{}_{1}$	-1.285	0	0	0	1	0	0	0	1	**2**	
$\tilde{}_{2}$	0	0	-0.642	-0.643	0	1	0	0	1	**1**	
$\tilde{}_{3}$	0.643	0	-0.642	0.642	0	0	1	0	-0.5	**3**	

∴ The corresponding solution of the FLCP is $= 0, (\ _{1}, \ _{2}, \ _{3}, \ _{4}) = (2,1,3,1)$.

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VII. CONCLUSION

In this paper, a hexagonal fuzzy ranking method is proposed by using centroid of centroid of a triangle. An algorithm is also used to solve the fuzzy linear complementarity problem by using proposed hexagonal ranking method. It has been explained with numerical examples. This ranking procedure can be applied in various decision making problems.

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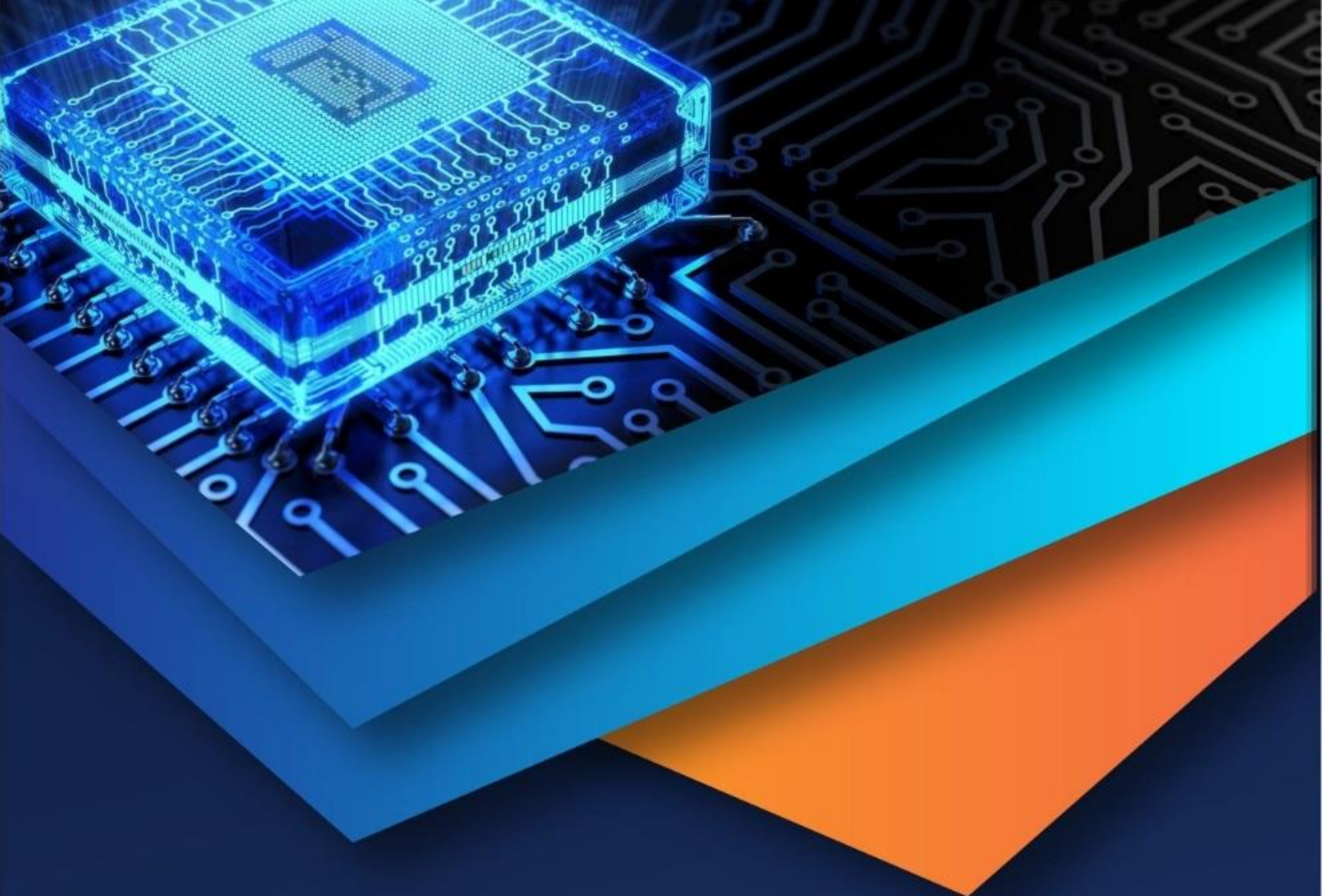

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