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Some Weak and Strong Form of Fuzzy Super Closed Set

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Abstract: ***In the present paper we extend the concept of fuzzy closed sets in fuzzy topology and introduce new class of fuzzy weak and strong form super closed set and its characterization in fuzzy topology***

Key Words: ***fuzzy super closure, fuzzy super interior, fuzzy super closed, fuzzy super open set, fuzzy continuity, fuzzy super continuity.***

Aberrations: Fuzzy semi super open (FSSO), fuzzy semi super closed (FSSC), Fuzzy pre super open (FPSO), fuzzy pre super closed (FPSC), fuzzy α-super open (FαSO), fuzzy α-super closed (FαSO),fuzzy generalized continuous (FG-continuous), fuzzy semi generalized continuous (FSG-continuous), fuzzy generalized semi generalized super neighborhood (FGSGS–nhd) , fuzzy generalized semi pre continuous (FGSP-continuous),fuzzy super closed (FSC),fuzzy semi generalized super closed (FSGSC) ,fuzzy generalized semi pre super closed(FGSPSC), fuzzy generalized super closed (FGSC), fuzzy semi generalized super open (FSGSO),fuzzy semi super open (FSSO), fuzzy semi super closed (FSSC), fuzzy weak super closed (FωSC), fuzzy generalized weak super closed (FGαSC) etc..

I. INTRODUCTION

Several generalization of Fuzzy Super open and super closed sets Let X be a nonempty set and I =[0,1]. A fuzzy set on X is a mapping from X to 1. The null fuzzy set 0 on X into I which assumes only the values 0 and the whole fuzzy set 1 is a mapping from X on to [0, 1] which takes the values 1 only .The union (resp. intersection) of family $\{A_\alpha : \alpha \in \wedge\}$ of fuzzy set of X is defined to be the mapping sup A_α (resp. inf A_α).A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \le B(x)$ for each $x \in X$. A fuzzy point x_β in X is a fuzzy set defined by $x_\beta(y) = \beta$ for y = x and x(y) = 0 for $y \neq x$, $\beta \in [0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi coincident with a fuzzy set B is denoted by A_qB if and only if there exists a point $x \in X$ such that $A(x)+B(x)>1$. $A \le B$ if and only if $\rceil A_qB^c$ .A family τ of fuzzy set of X is called the fuzzy topology on X if 0 and 1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection . The member of τ are called fuzzy open sets and their compliment are fuzzy closed sets. For a fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed super set of A and the interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A.

II. PRELIMINARIES

A. *Defination2.1:* A subset A of a fuzzy topological space (X,τ) is called

1) Fuzzy Super closure $scl(A)=\{x \in X: cl(U) \cap A \neq \phi\}$
2) Fuzzy Super interior $sint(A) =\{x \in X: cl(U) \le A \neq \phi\}$
3) Fuzzy super closed (FSC) if $scl(A) \le A$.
4) Fuzzy super open (FSO) set if 1-A is fuzzy super closed sint(A)=A
5) Fuzzy pre super open set (FPSO) if $A \le int(cl(A))$ and fuzzy pre-Super closed (FPSC) set if $cl(int(A)) \le A$.
6) Fuzzy semi super open (FSSO)set if $A \le cl(int(A))$ and fuzzy semi super closed (FSSC) set if $int(cl(A)) \le A$.

B. *Definition 2.3:* A fuzzy set A of (X,τ) is called:

1) FSSO if $A \le cl(int(A))$ and a FSSC if $int(cl(A)) \le A$.
2) FPSO if $A \le int(cl(A))$ and a FPSC if $cl(int(A)) \le A$.
3) $F_\alpha SO)$ if $A \le int\ (cl(Int(A)))$ and a FαSC) if $cl\ (int(cl(A))) \le A$.
4) FSPSO if $A \le cl\ (int(cl(A)))$ and a FSPSC) if $int(cl(int(A))) \le A$.

C. *Lemma 2.1:* Let A be a fuzzy set in a fuzzy topological space (X, τ). Then
1) spcl(A) $\leq$ scl(A) $\leq$ αcl(A) $\leq$ cl(A) $\leq$ r cl(A)
2) spcl(A) $\leq$ pcl(A) $\leq$ α cl(A)

D. *Definition 2.4:* A fuzzy set A of (X,τ) is called:
1) FGSC if cl(A) $\leq$ H, whenever A $\leq$ H and H is FSO in X;
2) FSGSC) if cl(A) $\leq$ H, whenever A $\leq$ H and H is FSSO in X.
3) FGSSC) if cl(A) $\leq$ H, whenever A $\leq$ H and H is FSO set in X;
4) F_αGSC) if α -cl(A) $\leq$ H, whenever A $\leq$ H and H is F_αSO set in X;
5) F_αGSC) if α-cl(A) $\leq$ H, whenever A $\leq$ H and H is FSO set in X;
6) FGSPSC if spcl(A) $\leq$H, whenever A $\leq$ H and H is FSO set in X;
7) FGPSC) if pcl(A) $\leq$ H, whenever A $\leq$ H and H is FSO set in X;
8) F_ωSC if cl(A) $\leq$ H, whenever A $\leq$ H and H is FSSO set in X.

E. *Definition 2.5:* A fuzzy topological space (X,τ) is called a
1) Fuzzy $T_{1/2}$ space if every FGSC set is FSC.
2) Fuzzy T_ω space if every F_ωSC set is FSC.
3) Fuzzy T_b space if every FGSSC set is fuzzy super closed.

F. *Definition 2.6 :* A mapping f: (X, τ) $\rightarrow$(Y, σ) is said to be:
1) FG-continuous if f^{-1}(V) is FSC set in X, for every FSC set V in Y .
2) FSG-continuous if f^{-1}(V) is FSGSC in X, for each FSC set V in Y;
3) FGSP-continuous if f^{-1}(V) is FGSPSC in X, for every FSC set V in Y ;

G. *Definition 2.7:* A fuzzy set A of (X,τ) is called a FGSGSC set if cl(A) $\leq$ H whenever A $\leq$ H and H is FSGSO in X.
H. *Lemma 2.1***.** : Every FSC set is FGSGSC.
I. *Proof:* Let A be FSC set and H be any FSGSO set such that A $\leq$ H. Since A is FSC , cl(A) = A $\leq$ H. Hence A is FGSGSC.
J. *Lemma 2.2:* Every FGSGSC set is FGSC.
K. *Proof:* Let A be any FGSGSC set and H be any FSO set such that A $\leq$ H. Since every FSO set is FSGSO and A is FGSGSC, we have cl(A)$\leq$ H. Hence A is FGSC.
L. *Lemma 2.3:* Every FGSGSC set is F_ωSC.
M. *Proof:* Let A be any FGSGSC set and H be any FSSO set such that A$\leq$H. Since every FSSO set is FSGSO and A is FGSGSC, we have cl(A)$\leq$ H. Hence A is F_ωSC.

$\leq$ H. Hence A is FGSPSC and

III. CHARACTERIZATION OF FGSGSC SETS AND FGSGSO SETS

In this section we study several interesting characterizations of FGSGSC sets and FGSGSO.

1) *Definition 3.1:* A fuzzy set A in (X,τ) is called FGSGS -nhd of a fuzzy point x_λ if there exists a FGSGSO set B such that $x_\lambda \in$ B $\leq$ A. A FGSG-nhd , A is said to be FGSGSO-nhd (resp. FGSGSC-nhd) if and only if A is FGSGSO (resp. FGSGSC).
2) *Definition 3.2:* A fuzzy set A in (X,τ) is called FGSG-q-nhd of a fuzzy point x_λ (resp. fuzzy set B), if there exists a FGSGSO set U in (X,τ) such that x_λqU $\leq$A (resp. BqU $\leq$ A).
3) *Theorem 3.1:* If A and B are FGSGSC sets in (X,τ) then A$\cup$B is FGSGSC.
   Let A and B be two fuzzy FGSGSC sets in (X,τ) and let H be any FSGSO set such that A $\leq$ H and B $\leq$ H. Therefore we have cl(A) $\leq$ H and cl(B) $\leq$ H. Since A $\leq$ H and B $\leq$ H, we have A $\cup$ B $\leq$ H. Now cl(A $\cup$ B) = cl(A) $\cup$cl(B) $\leq$ H. Hence A $\cup$B is FGSGSC.
4) *Theorem3.2:* If A and B are FGSGSO sets in (X,τ) thenA$\cap$B is FGSGSO.
a) *Proof***.** Let A and B be two fuzzy FGSGSO sets in (X,τ). Then 1$-$A and 1$-$B are FGSGSC. By above Theorem (1$-$A)$\cap$ (1$-$B) is FGSGSC. Since (1$-$A) $\cup$ (1$-$B) = 1$-$(A $\cap$B). Hence A $\cap$ B is FGSGSO. *E. Theorem 3.3.:* If a fuzzy set A is FGSGSC in (X,τ) and cl(A) (1 $-$ cl(A)) = 0 then cl(A)$-$ A does not contain any non-zero FSGSC set in (X,τ). Let A be FGSGSC in (X ,τ) and cl(A) $\cap$ (1$-$cl(A)) = 0. We prove the result by contradiction. Let B be any

FSGSC in (X,τ) such that $B \leq Cl(A) - A \cap B = 0$. This gives $B \leq cl(A)$ and $B \leq 1-A$. We have $A \leq 1- B$, which is FSGSO. Since A is FGSGSC, we have $cl(A) \leq 1-B$. This implies $B \leq 1-cl(A)$. Therefore $B \leq cl(A) \cap 1-cl(A) = 0$. That is $B = 0$, which is a contradiction. Hence $cl(A)- A$ does not contain any non-zero FSGSC set in (X,τ).

5) *Theorem 3.4:* If a fuzzy set A is FGSGSC in (X,τ) and $cl(A)\cap (1-cl(A)) = 0$ then $cl(A)-A$ does not contain any non-zero FSC set in (X,τ).
6) *Proof:* It follows from the above theorem and the fact that every FSC set is FSGSC.
F. Theorem 3.5: If A is FGSGSC set in (X,τ) and $A \leq B \leq cl(A)$ then B is FGSGSC in (X,τ).
1) Proof: Let H be FSGSO set such that $B \leq H$. Since $A \leq B$, we have $A \leq H$. Since A is FGSGSC set, $cl(A) \leq H$. But $B \leq cl(A)$ implies $cl(B) \leq cl(cl(A)) = cl(A) \leq H$. Hence B is FGSGSC.
7) *Theorem 3.6:* If A is FGSGSO set in (X,τ) and $int(A) \leq B \leq A$, then B is FGSGSO in (X,τ).
1) Proof: Let A is FGSGSO set in (X,τ) and $int(A) \leq B \leq A$. Then $1-A$ is FGSGSC and $1 - A \leq 1 - cl(A) \leq cl(1-A)$. Then $1-B$ is FGSGSC. Hence B is FGSGSO.

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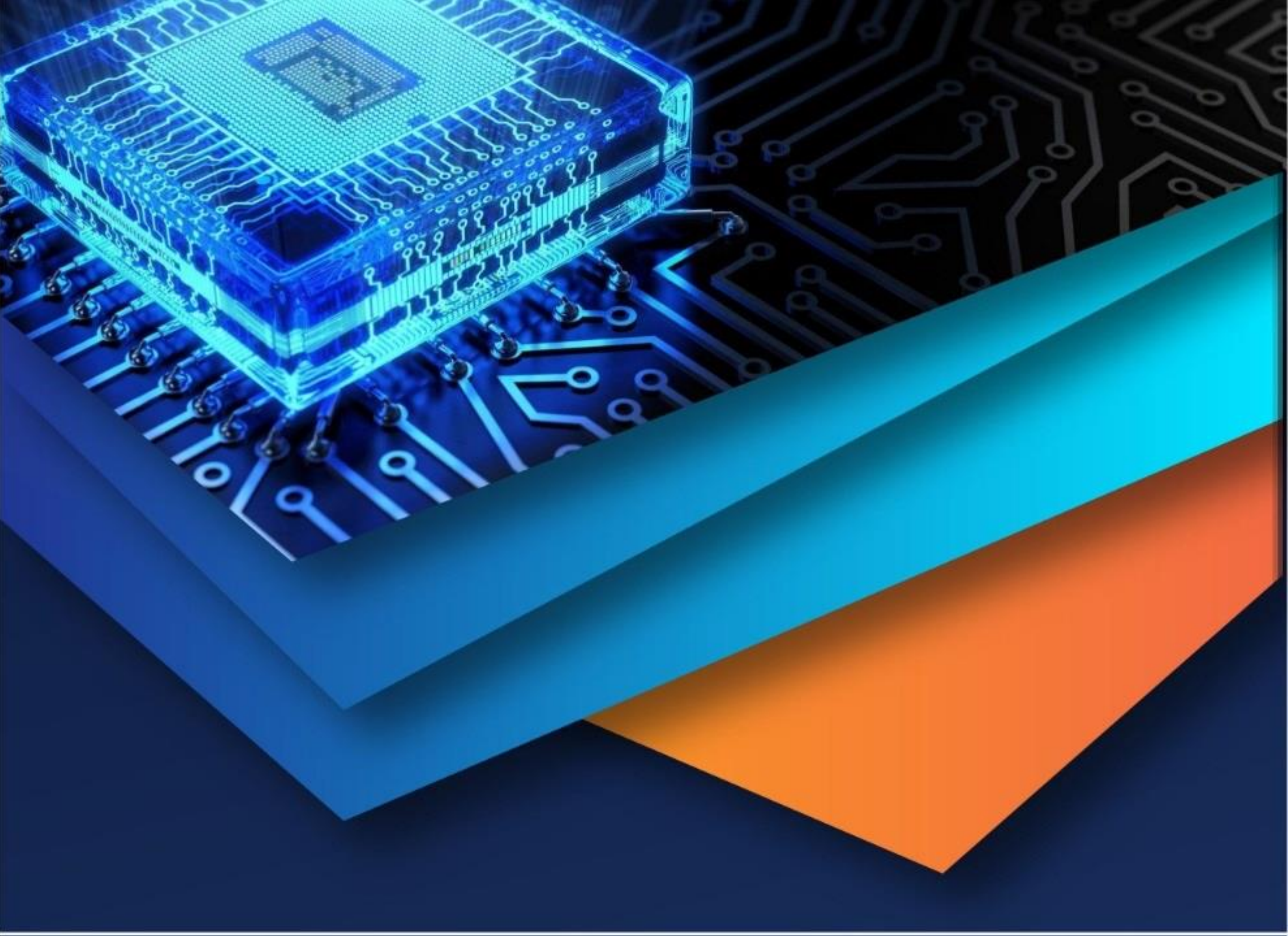

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