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# A Method for solving Intuitionistic Fuzzy Assignment Problem Using Yager's Method

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***Abstract: In this paper,Intuitionistic Fuzzy assignment problem with the Triangular Intuitionistic fuzzy numbers is introduced. Methods are proposed to find the solution of Intuitionistic fuzzy assignment problem. Yager's Ranking method based on the magnitude of membership function and non-membership function of a Intuitionistic Fuzzy Number is utilized to the Intuitionistic fuzzy numbers. Numerical example is provided to illustrate the methods.***

***Keywords: Intuitionistic Fuzzy Numbers,Triangular Intuitionistic Fuzzy Numbers(TrIFN),Yager's Ranking of Intuitionistic fuzzy numbers, Intuitionistic Fuzzy Assignment Problem(IFAP),Triangular Intuitionstic Fuzzy Assignment problem(TrFAP).***

## I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh[9] and it dealt with imprecision, vagueness in real life situations. Belmann &Zadeh[5] proposed the concept of decision making problems involving uncertainty and imprecision. Amitkumar and Anila Gupta[1] investigated Assignment and travelling salesman problem with coefficients as LR fuzzy parameters. Amitkumar et al.[2] gave Fuzzy linear programming approach for solving Fuzzy Transportation problem with Transshipment. Amitkumar et al.[3], discussed the method for solving Fully Assignment Problems Using Triangular Fuzzy Numbers.P.K.De and Bharti Yadav[6] given a general approach for solving Assignment problems involving with Fuzzy costs coefficients.The idea of Intuitionistic fuzzy set(IFS)introduced by Atanassov[4] is the generalization of Zadeh's[9] Fuzzy set.An IFS is characterized by membership degree as well as non-membership degree. since its introduced, the IFS theory has been studied and applied in different areas including decision making. In recent past, ranking intuitionistic fuzzy numbers(IFNs) draws the attention of several researchers. Nehi[7] ranked IFNs based on characteristic values of membership and non-membership functions of IFN. Assignment problem(AP) is used worldwide in solving real world problems. An assignment problem plays an important role in an assigning of persons to jobs, or classes to rooms, operators to machines, drivers to trucks to routes, or problems to research teams, etc. The assignment problem is a special type of linear programming problem(LPP)in which our objective is to assign n number of jobs to n number of machines(persons) at a minimum cost. This paper is organized as follows: Section 2 deals with some basic terminology and ranking of triangular intuitionistic fuzzy numbers. Section 3 describes the solution procedure of an intuitionistic fuzzy assignment problem, In section 4 to illustrate the proposed method a numerical example with results and discussion is discussed and followed by the conclusions are given in section 5.

## II. PRELIMINARIES

### A. Fuzzy set

Let A be a classical set, $\mu_A(x)$ be a function from A to [0,1]. A fuzzy set $A^*$ with the membership function $\mu_A(x)$ is defined by $A^*=\{(x, \mu_A(x) ; x\in A$ and $\mu_A(x)\in[0,1]$.

### B. Intuitionistic Fuzzy set

Let X be denote a universe of discourse, then an intuitionistic fuzzy set A in x is given by a set of ordered triple $\tilde{A}_I =\{<x, \mu_A(x),\boldsymbol{v}_A(x)>:x\in X\}$

Where $\mu_A(x)$, $\boldsymbol{v}_A (x):X\to[0,1]$,are functions such that $0\le \mu_A(x)+\boldsymbol{v}_A(x):x\le1$, $\forall x\in X$.each x the membership $\mu_A(x)$ and $\boldsymbol{v}_A(x)$ represent the degree of membership and the degree of non-membership of the element $x\in X$ to $A\subset X$ respectively.

### C. Fuzzy Number

The notion of fuzzy numbers was introduced by Dubois.D and Prade.H(1980). A fuzzy subset A of the real line R with membership function $\mu_A:R\to[0,1]$ is called a fuzzy number if

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1) A is normal, i.e.,there exists an element $x_0 \in A$ such that $\mu_A(x_0)=1$
2) A is fuzzy convex,
   i.e., $\mu_A(\lambda x_1+(1-\lambda)x_2 \geq \mu_A(x_1) \wedge \mu_A(x_2) \forall (x_1, x_2) \in R \; \& \forall \lambda \in [0,1]$
3) $\mu_A$ is upper semi continuousSupp A is bounded where Supp A={x ∈R: $\mu_A(x)>0$}

### D. Triangular Fuzzy Number

A fuzzy number A is defined to be a triangular fuzzy number if its membership functions $\mu_A(x):\mathbb{R}\rightarrow[0,1]$ is equal to.

$$\mu_A(x)=\begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } x \in [a_1, a_2] \\ \frac{a_3-x}{a_3-a_2} & \text{if } x \in [a_2, a_3] \\ 0 & \text{otherwise} \end{cases}$$

Where $a_1 \leq a_2 \leq a_3$.This fuzzy number is denoted by$(a_1, a_2, a_3)$

### E. Triangular Intuitionistic Fuzzy Number

A Triangular Intuitionistic Fuzzy Number $\tilde{A}^I$ is an Intuitionistic
Fuzzy set in R with the following membership funtion $\mu_A(x)$ and non membership
Function $\boldsymbol{v}_A(x)$

$$\mu_A(x)=\begin{cases} 0 & \text{for } x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ 1 & \text{for } x = a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 1 & \text{for } x > a_3 \end{cases}$$

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$$\nu_A(x)=\begin{cases} 0 & \text{for } x< a_1 \\ \frac{a_2-x}{a_2-a_1} & \text{for } a_1\le x\le a_2 \\ 0 & \text{for } x=a_2 \\ \frac{x-a_2}{a_3-a_2} & \text{for } a_2\le x\le a_3 \\ 1 & \text{for } x>a_3 \end{cases}$$

Where $a'_1\le a_1 \le a_2 \le a_3\le a'_3$ and $\mu_A(x)$, $\nu_A(x)\le 0.5$ for $\mu_A(x)= \nu_A(x)$ for all $x\in R$. this TrIFN is denoted by $\tilde{A}^I=(a_1,a_2,a_3)(a'_1,a_2,a'_3)$

*F. Ranking Function*

Let A I and B I be two TrIFNs.The ranking of $\tilde{A}^I$ and $\tilde{B}^I$ by the R(.) on E, the set of TrIFNs is defined as follows:

1) $R(\tilde{A}^I) > R(\tilde{B}^I)$ iff $\tilde{A}^I > \tilde{B}^I$
2) $R(\tilde{A}^I) < R(\tilde{B}^I)$ iff $\tilde{A}^I < \tilde{B}^I$
3) $R(\tilde{A}^I) = R(\tilde{B}^I)$ iff $\tilde{A}^I \approx \tilde{B}^I$
4) $R(\tilde{A}^I+\tilde{B}^I) = R(\tilde{A}^I) +R(\tilde{B}^I)$
5) $R(\tilde{A}^I-\tilde{B}^I) = R(\tilde{A}^I) - R(\tilde{B}^I)$

*G. Arithmetic operation of Triangular Intuitionistic Numbers*

The Arithmetic operations between two TIFNs defined on universal set of real numbers R, are reviewed as in Atanassov(1986,1989),Nehi(2010),Amitkumar(2013)

1) Let $A= (b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ and $B=( b'_1,a'_1,b'_2,a'_2,a'_3,b'_3,a'_4,b'_4)$ be two TrIFNs, then $A\oplus B=(b_1+b'_1,a_1+a'_1,b_2+b'_2,a_2+a'_2,a_3+a'_3,b_3+b'_3,a_4+a'_4,b_4+b'_4)$
2) Let $A= (b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ and $B=(b'_1,a'_1,b'_2,a'_2,a'_3,b'_3,a'_4,b'_4)$ be two TrIFNs, then $A\ominus B=(b_1-b'_4,a_1-a'_4,b_2-b'_3,a_2-a'_3,a_3-a'_2,b_3-b'_2,a_4-a'_1,b_4-b'_1)$
3) Let $A=( b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ be a TIFN and r be real number then
4) Let $A=( b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ and $B=( b'_1,a'_1,b'_2,a'_2,a'_3,b'_3,a'_4,b'_4)$ be two non- negative TIFNs, then $A\otimes B=(b_1b'_1,a_1a'_1,b_2b'_2,a_2a'_2,a_3,a'_3,b_3b'_3,a_4,a'_4,b_4b'_4)$

*H. Magnitute Measure*

Let $\tilde{A}$= (<l,m,n> <p,q,r>)

Then $R(\tilde{A}) = (x,y,z)$

Where $x=\frac{l+p}{2}$

$y=\frac{m+q}{2}$

$z=\frac{n+r}{2}$

*I. Intuitionistic Fuzzy Assignment Problem*

Intuitionistic Fuzzy Assignment Problem(IFAP) can be stated in the form of nxn fuzzy cost table $[\tilde{c}_{ij}]$ of real numbers as given in the following table:

Jobs

| | 1 | 2 | …. | j | … | n |
|---|---|---|---|---|---|---|
| 1 | $\tilde{C}_{11}$ | $\tilde{C}_{12}$ | …. | $\tilde{C}_{1j}$ | …. | $\tilde{C}_{1n}$ |
| 2 | $\tilde{C}_{21}$ | $\tilde{C}_{22}$ | …. | $\tilde{C}_{2j}$ | …. | $\tilde{C}_{2n}$ |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| i (persons) | $\tilde{C}_{i1}$ | $\tilde{C}_{i2}$ | …. | $\tilde{C}_{ij}$ | …. | $\tilde{C}_{in}$ |
| . | . | . | . | . | . | . |
| | . | . | . | . | . | . |
| n | $\tilde{C}_{n1}$ | $\tilde{C}_{n2}$ | …. | $\tilde{C}_{nj}$ | …. | $\tilde{C}_{nn}$ |

The costs or time $\tilde{c}_{ij}$ are Intuitionistic trapezoidal fuzzy matrix.

$\tilde{C}_{ij}$ = [ $C_{ij}^{(1)}$, $C_{ij}^{(2)}$, $C_{ij}^{(3)}$, $C_{ij}^{(4)}$, $C_{ij}^{(5)}$, $C_{ij}^{(6)}$, $C_{ij}^{(7)}$ ,$C_{ij}^{(8)}$ ] is the cost of assigning the

$j^{th}$ job to the $i^{th}$ person. The objective is to minimize the total cost of assigning all the jobs to the available persons (one job to one person.)

Mathematically assignment problem can be stated as:

Minimize Z= $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$ i=1,2………n j=1,2……..n

Subject to

$\sum_{j=1}^{n} x_{ij}=1$, i=1,2,………n …………(1)

$\sum_{i=1}^{n} x_{ij}=1$, j=1,2,………n $x_i$

$$x_{ij} = \begin{cases} 1, & \text{if the } i^{th} \text{ person is assigned the } j^{th} \text{ job} \\ 0, & \text{otherwise} \end{cases}$$

is the decision variable donoting the assignment of the poerson i to job j,$c_{ij}$ is the cost of assigning the $j^{th}$ job to the $i^{th}$ person. The objecticve is to minimize the total cost of assigning all the jobs to the available persons.(one job to one person). When the costs $\tilde{C}_{ij}$ are fuzzy numbers,then the fuzzy assignment problem becomes

$Y(\tilde{z})=\sum_{i=1}^{n}\sum_{j=1}^{n} Y(\tilde{c}_{ij})x_{ij}$ ……………(2)

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Subject to the same conditons(1).

We defuzzify the fuzzy cost coefficients into crisp ones by a fuzzy number ranking method . Yager's Ranking index [8] is defined by

$$Y(\tilde{c})=\int_0^1 0.5(c^L_\alpha + c^U_\alpha)$$ , where $(c^L_\alpha, c^U_\alpha)$ is the $\alpha$- level cut of the fuzzy number $\tilde{c}$.

## III. TRIANGULAR INTUITIONISTIC FUZZY APPROXIMATION METHOD

The proposed steps of Triangular Intuitionistic Fuzzy Approximation method to find a optimal assignment for TrIFAP follows.

A. *Step 1:* Test whether the given TrIFAP is balanced or not.
1) If it is a balanced one(i.e, the number of persons are equal to the number of jobs) then go to step 3.
2) If it is an unbalanced one(i.e, the number of persons are not equal to the number of jobs ) then go to step 2.

B. *Step 2:* Add dummy rows or dummy columns, so that the Intuitionistic fuzzy cost matrix becomes a Intuitionistic fuzzy square matrix. The Triangular Intuitionistic fuzzy cost entries of dummy rows or columns are always Intuitionistic fuzzy zero

C. *Step 3:* Determine the minimum and the next minimum Triangular Intuitionistic fuzzy costs in each row and column of TrIFAP using the ranking method.

D. *Step 4:* Determine the Triangular Intuitionistic fuzzy difference between the minimum and next minimum Triangular Intuitionistic fuzzy costs in each row and column and display them alongside the Triangular Intuitionistic fuzzy assignment matrix against the respective rows. . Similarly compute the differences for each column.

E. *Step 5:* Identify the row or column with the largest difference among all the rows and columns. If a tie occurs use arbitrary tie-breaking choice. Let the difference correspond to $i^{th}$ row and let $\tilde{c}_{ij}$ be the smallest Triangular Intuitionistic fuzzy cost in the $i^{th}$ row. Assign that Triangular Intuitionistic fuzzy entry and cross off the $i^{th}$ row and $j^{th}$ column.

F. *Step 6:*Recompute the column and row Triangular Intuitionistic fuzzy differences for the reduced TrIFAP and go to step 5. Repeat the procedure until each row and column has one Intuitionistic fuzzy assignment.

## IV. NUMERICAL EXAMPLES

Consider a Triangular Intuitionistic fuzzy assignment problem with rows representing four persons A,B,C,D and columns representing the four jobs, job 1, job 2, job 3 and job 4. The following cost matrix[$\tilde{c}_{ij}$ ]whose elements are Triangular Intuitionistic fuzzy numbers.

(<1,2,3>,<5,6,9>) (<3,4,5>,<5,6,13>) (<2,3,4>,<4,5,10>) (<3,4,6>,<7,8,10>)
(<3,5,6>,<7,9,10>) (<1,3,4>,<5,7,8>) (<3,5,8>,<7,9,10>) (<2,4,6>,<6,8,14>)
(<1,6,7>,<7,8,9>) (<1,4,5>,<5,8,9>) (<2,4,5>,<6,8,15>) (<3,4,8>,<9,10,14>)
(<4,5,6>,<8,11,16>) (<7,8,9>,<9,10,11>) (<4,5,8>,<8,9,14>) (<3,7,8>,<9,11,12>)

### A. *Solution*

The intuitionistic triangular fuzzy numbers are represented by Triangular fuzzy numbers

Now,

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | (3,4,6) | (4,5,9) | (3,4,7) | (5,6,8) |
| B | (5,7,8) | (3,5,6) | (5,7,9) | (4,6,10) |
| C | (4,7,8) | (3,6,7) | (4,8,10) | (6,7,11) |
| D | (6,8,11) | (8,9,10) | (6,7,11) | (6,9,10) |

…….(1)

We calculate y(3,4,6) by applying the Yager's Ranking method.

The membership function of the Triangular Fuzzy Number (3,4,6) is,

$\mu_A(x) = \frac{x-3}{4-3}$ ,$3 \le x \le 4$

$= \frac{x-6}{4-6}$ ,$4 \le x \le 6$

The α-cut of the fuzzy number(3,4,6) is $(c^L_\alpha, c^U_\alpha)$=( α+9-2 α) for which

$Y(\tilde{c}_{11}) = y(3,4,6) = \int_0^1 0.5(c^L_\alpha, c^U_\alpha)\, d\alpha$

$= \int_0^1 0.5(\alpha + 9 - 2\alpha)\, d\alpha$

=4.25

Proceeding similarly, the Yager's indices for the costs $\tilde{c}_{ij}$ are calculated as:

$Y(\tilde{c}_{12})$=5.75 , $y(\tilde{c}_{13})$=4.5 ,$y(\tilde{c}_{14})$=6.25 ,$y(\tilde{c}_{21})$=6.75 ,$y(\tilde{c}_{22})$=4.75 ,$y(\tilde{c}_{23})$=7 ,$y(\tilde{c}_{24})$=5.5 ,$y(\tilde{c}_{31})$=6.5 ,$y(\tilde{c}_{32})$=5.5 , $y(\tilde{c}_{33})$=7.5 , $y(\tilde{c}_{34})$=7.75 ,$y(\tilde{c}_{41})$=8.25 ,$y(\tilde{c}_{42})$=9 ,$y(\tilde{c}_{43})$=7.75 , $y(\tilde{c}_{44})$=7.5

We replace these values for their corresponding $\tilde{c}_{ij}$ in (1) and solve the resulting assignment problem by using Hungarian method.

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | 4.25 | 5.75 | 4.5 | 6.25 |
| B | 6.75 | 4.75 | 7 | 5.5 |
| C | 6.5 | 5.5 | 7.5 | 7.75 |
| D | 8.25 | 9 | 7.75 | 7.5 |

Performing row reductions

$$\begin{pmatrix} 0 & 1.5 & 0.25 & 2 \\ 2 & 0 & 2.25 & 0.75 \\ 1 & 0 & 2 & 2.25 \\ 0.75 & 1.5 & 0.25 & 0 \end{pmatrix}$$

Performing column reductions

$$\begin{pmatrix} 0 & 1.5 & 0 & 2 \\ 2 & 0 & 2 & 0.75 \\ 1 & 0 & 1.75 & 2.25 \\ 1.75 & 1.5 & 0 & 0 \end{pmatrix}$$

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By Hungarian method ,the assignment is

$$\begin{pmatrix} (0) & 1.5 & 0 & 2 \\ 2 & (0) & 2 & 0.75 \\ 1 & 0 & 1.75 & 2.25 \\ 0.75 & 1.5 & (0) & 0 \end{pmatrix}$$

$$\begin{pmatrix} (1) & 2.25 & 0 & 2 \\ 1.25 & (0) & 1.25 & 0 \\ 0.25 & 0 & 1 & 1.5 \\ 0.75 & 2.25 & (0) & 0 \end{pmatrix}$$

$$\begin{pmatrix} (1) & 2.25 & 0 & 2 \\ 1.25 & (0) & 1.25 & 0 \\ 0.25 & 0 & 1 & 1.5 \\ 1.75 & 2.25 & (0) & 0 \end{pmatrix}$$

$$\begin{pmatrix} (0) & 2.5 & 0 & 2 \\ 1 & (0) & 1 & 0 \\ 0 & 0 & 0.75 & 1.25 \\ 0.75 & 2.5 & (0) & 0 \end{pmatrix}$$

The optimal assignment matrix is

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| A | (0) | 2.5 | 0 | 1.25 |
| B | 1 | (0) | 0.25 | 0 |
| C | 0 | 0 | (0) | 0.5 |
| D | 1.5 | 3.25 | 0 | ( 0) |

The optimal assignment schedule is

A→1 ,B→ 2, C→3, D→ 4

The optimum cost is 24**.**

## V. CONCLUSION

In this paper an Intuitionistic Fuzzy Assignment Problem whose cost being taken as Triangular Intuitionistic Fuzzy Number is considered. The optimal solution is obtained by defuzzifying the costs into crisp values by a new measure and solving by usual Hungarian method. This method is systematic procedure, easy to apply and can be utilized for all type of intuitionistic fuzzy assignment problem whether maximize or minimize objective function.

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