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ON s-k –Tripotent Matrices

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Abstract: The concept of s-k-tripotent is introduced. Some basic results in s-k-tripotent matrices and their properties are given.
Keywords: Tripotent matrices, k-tripotent matrices.

I. INTRODUCTION

A Matrix 'A' that satisfies A^3=A is called a tripotent matrix. In this paper basic concept of s-k- tripotent matrices are introduced.A Theory of k-real and k-Hermitian matrices as a generalization of secondary real and secondary Hermitian Matrices was developed by hill and waters[2]. The secondary symmetric and secondary orthogonal matrices have been analysed by Ann lee [1] in the year 1976. The secondary k-Hermitian matrices are introduced in 2009 by Meenakshi, krishnamoorthy and Ramesh[3].S. Krishnamoorthy and P.S Meenakshi have studied the basic concepts of k-tripotent matrices as generalization of k-tripotent matrices. The sum and products of k- tripotent matrices are introduced in 2013 by S. Krishnamoorthy and P.S. Meenakshi[4]. Let' K' be the associated permutation matrix of K and let V be the permutation matrix with unit in the secondary diagonal. Clearly K &V satisfies the following properties:

$$K = K^T = \overline{K} = K^*, K^2 = I$$
$$V = V^T = V^{-1} = V^* \text{ and } V^2 = I$$

II. S-K-TRIPOTENT MATRICES

A. Definition

A matrix A=$\mathbf{a_{ij}}$in C_{nxn} is said to be s-k-tripotent if $\sum_{t=1}^{n} a_{n-k(i)t}\left[\sum_{m=1}^{n} a_{tm}\, a_{m(n-k)t}\right] = a_{ij}$

This equivalent to KVA^3VK=A.
It is easy to see that KVA^3VK=A
KVAVK= A^3
VKA^3KV= A
VKAKV= A^3

1) Example

Let A= $\begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$, K= $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$& V= $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$KVA^3VK=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & -1/2 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

KVA^3VK = A

Therefore A is s-k-tripotent matrix.

2) Theorem

If $A \in C^{nxn}$ be a s-k-tripotent matrix then

a) A^T, $\bar{A}$, A^* and A^{-1} are also s-k-tripotent

b) A^n is s-k-tripotent for all positive integers 'n'

c) A is periodic with period 9

d) A^3 is s-k-tripotent

3) Proof:

a) $A^T = \left(KVA^3VK\right)^T$

$= KV\, A^T A^T\, A^T VK$

$A^T = KV(A^T)^3VK$

$\therefore A^T$ is s-k-tripotent

$\overline{KVA^3VK} = KV(\bar{A}^3)VK$

$= KV(\bar{A})^3VK$

$= \bar{A}$

$\therefore \bar{A}$ is s-k- tripotent

$KVA^3VK = A^*$

$=KVA^{3*}VK$

$=KVA^{*3}VK$

$=A^*$

$\therefore A^*$ is s-k-tripotent.

$$\left(KV\,A^3VK\right)^{-1} = KV\,\left(A^3\right)^{-1}VK$$

$= KV(A^{-1})^3\, VK$

$= A^{-1}$

$\therefore A^{-1}$ is s-k-tripotent.

b) $A^n = (KVA^3VK)^n$

$=KVA^3VK\;\; KVA^3VK$…………n times

$= KVA^{3n}VK$

$= KV(A^n)^3VK$

$\therefore A^n$ is s-k-tripotent.

c) $A^9 = A^3A^3A^3$

$= (KVAVK)\;\; (KVAVK)\;\; (KVAVK)$

$= KVAAAVK$

$= KVA^3VK$

$A^9 = A$

$\therefore$ A^9 is period with 9.

d) Theorem

If $A \in C^{nxn}$ be a s-k-tripotent matrix then –A is s-k-tripotent.

a) Proof: Let s-k- tripotentmatrix $KVA^3VK=A$

$-(KVA^3VK)$

$\Rightarrow (KV-(A^3)VK)$

$\Rightarrow (KV(-A^3)VK)$

$-(KVA^3VK) = -A$

Hence $-A$ is s-k-tripotent.

e) Theorem

Let $A,B \in C^{nxn}$ be two s-k-tripotent matrices, A+B is s-k-tripotent matrices if and only if A = -B.

a) Proof: Let A and B are two s-k-tripotent matrices

$\therefore KVA^3VK=A$ and $KVB^3VK=B$

Assume A= -B

$A+B = KVA^3VK+KVB^3VK$

$= KV(A^3+B^3)VK$

$= KV(A+B)^3VK$

$\therefore A+B = KV(A+B)^3VK$

i.e.,A+B is s-k-tripotent.

Conversely,

A+B is s-k-tripotent.

$A+B = KV(A+B)^3VK$

$= KV(A^3+B^3+3AB^2+3A^2B)\ VK$

$= KV\ (A^3+B^3+3AB[A+B])\ VK$

$= KVA^3VK+KVB^3VK+KV(\ 3AB[A+B]\ )\ VK$

$= A+B+KV\ (\ 3AB\ [A+B])\ VK$

Hence $KV\ (\ 3AB[A+B])\ VK = 0$ if A= -B.

f) Theorem

If $A^9= A^*$ then Areduces to orthogonal projection.

$AB = BA$

$AB = KVA^3VK+KVB^3VK$

$= KVAAAVK\ KVBBBVK$

$= KV\ AAA\ BBB\ VK$

$= KVAABABBVK$

$= KVABABABVK$

$\therefore AB = KV(\ AB\ )^3\ VK$

Hence AB is s-k-tripotent.

B. Generalization

If $A_1\ A_2 \ldots\ldots A_n$ are s-k-tripotent belongs to a commuting family of matrices then $\prod_{i=1}^{n} A_i$ is s-k-tripotent.

1) Proof

Let $A_1a_2\ldots\ldots A_n$ s-k-tripotent.

$$KV\left(\prod_{i=1}^{n} A_i\right)^3 VK= KV\ [\ A_1\ A_2 \ldots\ldots A_n]\ VK$$

$= KV\ [\ A_1\ A_2 \ldots\ldots A_n]\ VK$

$= KV[\ A_1^3\ A_2^3\ A_3^3 \ldots\ldots A_n^3\ ]\ VK$

$= KVA_1^3A_2^3A_3^3\ldots\ldots A_n^3]\ VK$

$= A_1\ A_2 \ldots\ldots A_n$

$$= \prod_{i=1}^{n} A_i$$

Therefore s-k-tripotent.

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C. Theorem

If $A \in C^{nxn}$ then any two of the following statements implies the other one.

1) A is s-k-tripotent
2) A is s-k-symmetric
3) A is cube symmetric
4) *Proof:*

a) And (b) implies that (c)

$KVA^3VK = A$ and $KVA^TVK = A$

$KVA^3VK = KVA^TVK$

$A^3 = A^T$

$\therefore$ A is cube symmetric.

b) And (c) implies that (a)

$KV A^T VK = A$ and $A^3 = A^T$ substitute $A^3 = A^T$ in $KVA^T VK = KVA^3 VK = A$.

$\therefore$ A is s-k-tripotent.

c) And (a) implies that (b)

$A^3 = A^T$ $KV A^3 VK = A$ substitute $A^3 = A^T$ in $KVA^3 VK = A$.

$KVA^TVK = A$.

$\therefore$ A is s-k-symmetric.

D. Theorem

If $A \in C^{nxn}$ then any two of the following statements implies the other one.

1) A is s-k-tripotent
2) A is symmetric
3) A is s-k cube symmetric
4) *Proof*:

a) And (b) implies that (c)

$A = A^T$ and $KVA^3VK = A$ implies that $KVA^3VK = A^T$

Therefore A Isk-cube symmetric.

b) And (c) implies that (a)

$A = A^T$ and $KVA^3VK = A^T$ implies that $KVA^3VK = A$

Therefore A is s-k-tripotent.

c) And (a) implies that (b)

$KVA^3VK = A^T$ $KVA^3VK = A \Rightarrow A^T = A$

Therefore A is symmetric.

E. Theorem

If $A \in C^{nxn}$ then any two of the following statements implies the other one.

1) A is s-k-tripotent
2) A is s-k-Hermitian
3) A is s-k-cube Hermitia
4) *Proof*

a) And (b) implies that (c)

$KVA^3VK = A$ and $KVA^*VK = A$

$KVA^3VK = KVA^*VK$
$A^3 = A$
A is cube Hermitian.

b) And (c) implies that (a)

$KVA^*VK = A$ and $A^3 = A^*$
Substitute $A^3 = A^*$ in $KVA^*VK = A$
$\Rightarrow KVA^3VK = A$
Hence A is s-k-tripotent.

c) And (a) implies that (b)
$A^3 = A$ and $KVA^3VK = A$
Substitute $A^3 = A^*$ in $KVA^3VK = A \Rightarrow KVA^*VK = A$. Hence A is s-k-tripotent

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