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L $(2,1)$ –EDGE Coloring of Some Graphs

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Abstract: ***$L'(2,1)$-edge coloring is a distance constrained edge labeling based on the edge distance. If $e_1 = (u_1, v_1)$ and $e_2 = (u_2, v_2)$ are two edges of G, then the edge distance of e_1 and e_2 is defined as $ed(e_1, e_2) = min\{d(u_1, u_2), d(u_1, v_2), d(v_1, u_2), d(v_1, v_2)\}$. If $ed(e_1, e_2) = 0$ then these edges are called neighbor edges. In this paper, we investigate the $L'(2,1)$- edge coloring of some graphs. The $L'(2,1)$-edge coloring of a graph G is an assignment of non-negative integers to the edges e_1 and e_2 of G such that $|c(e_1) - c(e_2)| \geq 2$ if $ed(e_1, e_2) = 0$ and $|c(e_1) - c(e_2)| \geq 1$ if $ed(e_1, e_2) = 1$. No restriction is placed on colors assigned to edges at distance 2 or more. We also define the $L'(2,1)$- edge coloring number, $\lambda'(G)$ of some graphs viz. cycles, complete graphs, wheel graphs, complete bipartite graphs, fan graphs and friendship graphs.***
Keywords: Edge distance, $L'(2,1)$- edge coloring number, cycles, complete graphs, wheel graphs, complete bipartite graphs, fan graphs, friendship graphs.

I. INTRODUCTION

We consider only finite simple undirected graphs. The vertex set and edge set of the graph G is denoted by V(G) and E(G) respectively. For basic notation and terminology, we refer to G. Chartrand and P. Zhang, "Introduction to Graph Theory" [1]. This notion of edge coloring is obtained from [2]. In [4], we have studied the $L'(2,1)$- edge coloring of stars, trees, path graphs and ladder graphs. In this paper, we have extended our study to few more classes of graphs, say, cycles, complete graphs, wheel graphs, complete bipartite graphs, fan graphs and friendship graphs.

Definition 1.1. A fan graph f_n; $n \geq 2$ is obtained by joining all vertices of P_n(Path on n vertices) to a further vertex called the center and contains $n + 1$ vertex and $2n - 1$ edges.That is, $f_n = P_n + K_1$. Fan graph f_4 is shown in the following figure.

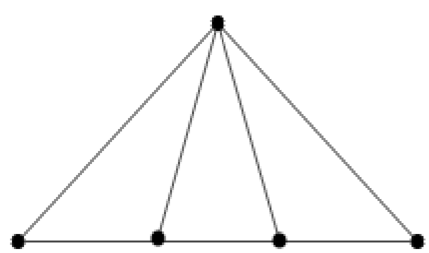

Fig: f_4

Definition1.2. A friendship graph F_n; $n \geq 2$ is a graph which consists of n triangles with a common vertex.Friendship graph F_4 is shown in the following figure.

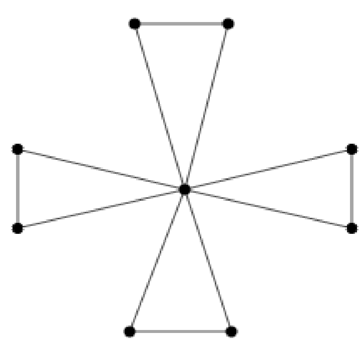

Fig: F_4

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Definition 1.3. If $e_1 = (u_1, v_1)$ and $e_2 = (u_2, v_2)$ are two edges of G, then the edge distance of e_1 and e_2 is defined as $ed(e_1,e_2) = \min\{d(u_1,u_2), d(u_1,v_2), d(v_1,u_2), d(v_1,v_2)\}$. If $ed(e_1,e_2) = 0$ then these edges are called neighbor edges.[3]

Definition 1.4. The $L'(2,1)$- edge coloring of a graph G is an assignment of non-negative integers to the edges e_1 and e_2 of G such that $|c(e_1) - c(e_2)| \geq 2$ if $ed(e_1,e_2) = 0$ and $|c(e_1) - c(e_2)| \geq 1$ if $ed(e_1,e_2) = 1$. No restriction is placed on colors assigned to edges at distance 2 or more [4].

Definition 1.5. The color span of an $L'(2,1)$- edge coloring of a graph G is $\lambda'(c) = \max\{|c(e_1) - c(e_2)| : e_1, e_2 \in E(G)\}$. As 0 is the least color used in this coloring, the color span is always the maximum color used in the $L'(2,1)$- edge coloring $'c'$ of a graph G.

Definition 1.6. By an $L'(2,1)$- edge coloring number[3], $\lambda'(G)$, we mean the smallest positive integer k such that there exists an $L'(2,1)$- edge coloring $c: E(G) \rightarrow \{0, 1, 2, \ldots, k\}$. That is, $\lambda'(G)$ is the smallest maximum color used among the $L'(2,1)$- edge coloring of G.

II. MAIN RESULTS

A. Theorem 1. Let C_n be a cycle on n vertices. Then the $L'(2,1)$- edge coloring number $\lambda'(C_n) = 4 \ \forall \ n$.

Proof: When n is least, all the three edges are at edge distance 0. Hence, by $L'(2,1)$- edge coloring, we see that number $\lambda'(C_n)$ is never less than 4. We now observe that the sequence (0,2,4) repeated $({}^{n}/_{3})$ times gives an optimal edge coloring for $n \equiv 0 \ (\mathrm{mod}\ 3)$. When $n \equiv 1 (\mathrm{mod}\ 3)$, coloring four of the edges consecutively using the colors {0,4,1,3} and the remaining edges with the help of the sequence $(0,4,2)$ repeated $\left(\frac{n-4}{3}\right)$ times gives an optimal edge coloring of C_n. Similarly, when $n \equiv 2 (\mathrm{mod}\ 3)$, coloring five of the edges consecutively using the colors {1,3,0,2,4} and the remaining edges with the help of the sequence $(0,2,4)$ repeated $\left(\frac{n-5}{3}\right)$ times gives an optimal edge coloring of C_n. Refer Figure 1. From the above three cases, we see that the $L'(2,1)$- edge coloring number $\lambda'(C_n) = 4 \ \forall \ n$.

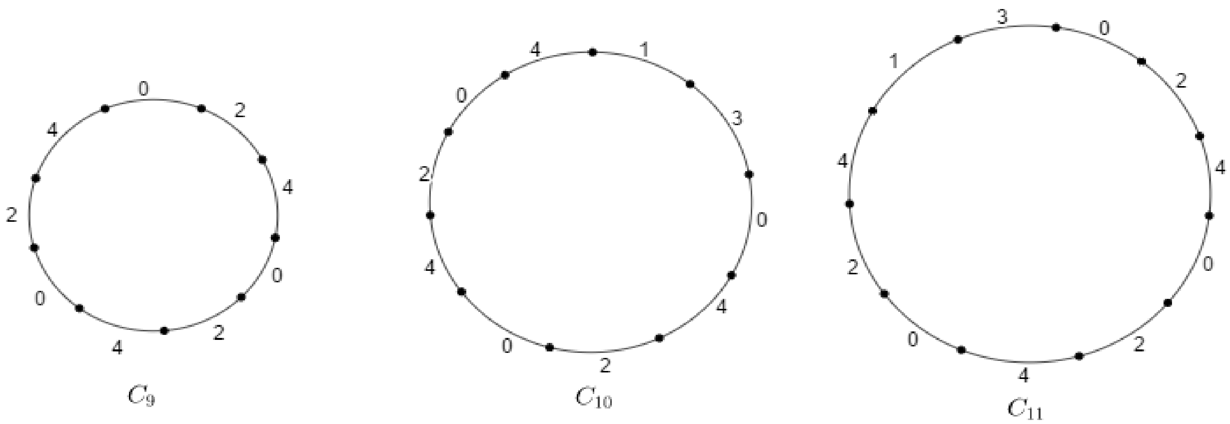

Figure 1: $L'(2,1)$- edge coloring of C_n

B. Theorem 2. Let K_n be a complete graph on n vertices. Then the $L'(2,1)$- edge coloring number $\lambda'(K_n) = n^2 - n - 2$.

Proof: In case of optimal coloring of , none of the edges can be colored using odd numbers. Refer figure 2. Hence, edges of any complete graph can be colored optimally using the colors $0, 2, \ldots, (2 \quad - 2)$; where m is the size of the graph. Clearly, $\lambda'(\quad) = 2 \quad - 2 = 2\binom{}{2} - 2 = {}^{2} - \quad - 2.$

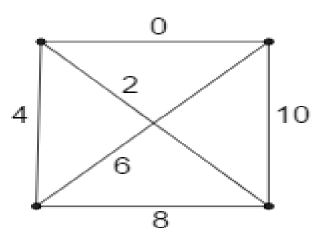

Figure 2: $L'(2,1)$- edge coloring of $_4$

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C. Theorem 3. The $L'(2,1)$- edge coloring number of a complete bipartite graph $_{,}$ is given by, $\lambda'(\; _{,} \;) = (\quad - 1)$.

Proof: As the diameter of $_{,}$ is 2, none of the edge colors are repeated. The edges at edge distance one can be colored consecutively implies that both even and odd numbers are used in coloring the edges. As the size of $_{,}$ is , we see that at least colors are used in the optimal coloring of $_{,}$. However, the presence of the color *0* indicates that $\lambda'(\; _{,} \;) = (\quad - 1)$.

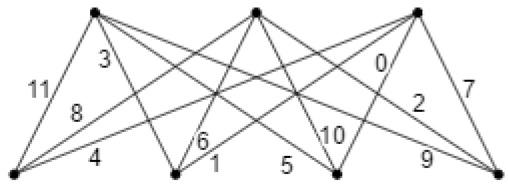

Figure 3: $L'(2,1)$- edge coloring of $_{3,4}$

D. Theorem 4. For a Fan graph $\;;\; \geq 2$, $'(\quad) = \begin{cases} 2 \; ; \; 2 \leq \quad \leq 3 \\ 7 \; ; \quad = 4 \\ 2 \quad - 2 \; ; \quad \geq 5 \end{cases}$.

1) *Proof:* It is obvious that $'(\; _2) = 4$. Consider the fan graph $_3$. Let be the edge of $_3$ which is at edge distance zero from the remaining edges. Let us color the edges of $_3$ using the color class C= $\{0,1,2,3,4,5\}$. Now, if $(\;) = \; ; \; \in$, then, none of the remaining edges of $_3$ can be given three of the colors from . That is, the remaining four edges need to be colored using three colors, which implies that one of the colors is repeated. Hence, the $L'(2,1)$- edge coloring number of $_3$ is at least 6 and from the figure 4 we see that $'(\; _3) = 6$.

Consider the fan graph $_4$. As the size of $_4$ is 7, assume that we can color the edges of $_4$ using the colors *0,1,2,3,4,5,6*. By theorem 2 of [4], color the edges of the subgraph $_{1,4}$ using the even numbers *0,2,4,6* and the edges of the path graph using the odd numbers *1,3,5*. Here, we arrive at a contradiction due to color 3. Hence one more color is required and the $L'(2,1)$- edge coloring number of $_4$ is at least 7. From the figure 5 it follows that $'(\; _4) = 7$.

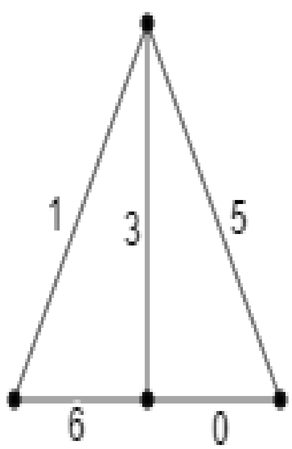

Figure 4: $_3$

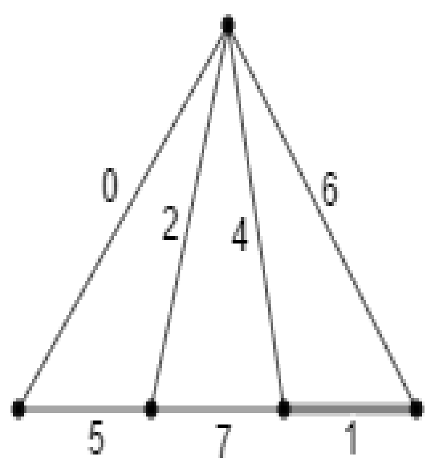

Figure 5: $_4$

Now let us consider the fan graph $\;;\; \geq 5$. Let Δ be the maximum degree of . Color the edges of the subgraph $_{1,\Delta}$ using the even numbers $0,2,4,\ldots,(2\Delta - 2)$ as seen in theorem 2 of [4]. Clearly none of the edges of can be colored using the same even numbers. An optimal coloring can be obtained by coloring the edges of using any of the $(\Delta - 1)$ odd numbers in between *0* to $(2\Delta - 2)$. The proof is complete by assuming $\Delta =$ and $'(\quad) = 2 \quad - 2; \quad \geq 5$.

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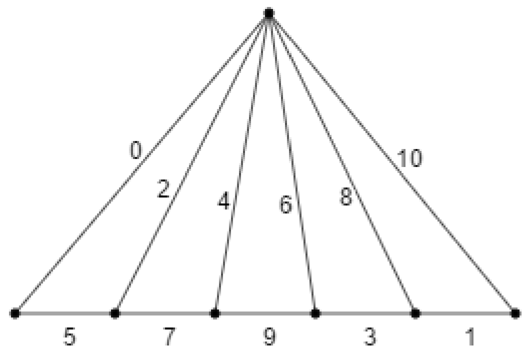

Figure 6: L′(2, 1)- edge coloring of $_{6}$

E. Theorem 5. For the Wheel graph $\ ;\ \geq 2,\ {}^{\prime}(\ ) = \begin{cases} 2 & ;\ 2 \leq \ \leq 3 \\ 2\ -1 & ;\ 4 \leq \ \leq 5 \\ 2\ -2 & ;\ \geq 6 \end{cases}$

1) Proof: For $= 2,3$, Figure 7gives an optimal edge coloring of $_2$ and $_3$. Hence, ${}^{\prime}(\ ) = 2$ for $= 2,3$.

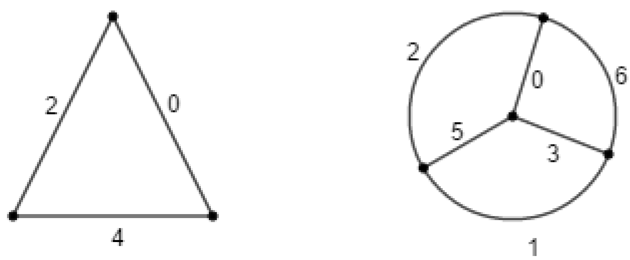

Figure 7:L′(2, 1)- edge coloring of $_2$ and $_3$

For $4 \leq \ \leq 5$, coloring the inner edges using even numbers $0,2, \dots , 2\ -2$ and the outer edges using the odd numbers we see that the outer edges can be colored using the color class $\{1,3, \dots , (2\ -2) + 1\}$.
Hence, ${}^{\prime}(\ ) = 2\ -2 + 1 = 2\ -1$.Refer Figure 8.

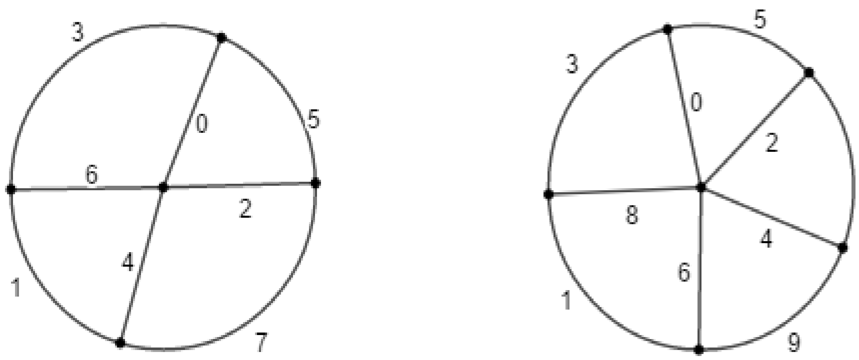

Figure 8: L′(2, 1)- edge coloring of $_4$ and $_5$

For ≥ 6, color the inner edges $;\ 1 \leq \ \leq$ using even numbers $\{0,2, \dots , 2\ -2\}$ and color the outer edges $_{+1}; 1 \leq \ \leq \ -1$ and $_1$ using the odd numbers between 0 to $2\ -2$. This indicates that (2n-2) is the largest color used in any optimal coloring of $;\ \geq 6$. Hence, ${}^{\prime}(\ ) = 2\ -2$ for ≥ 6.

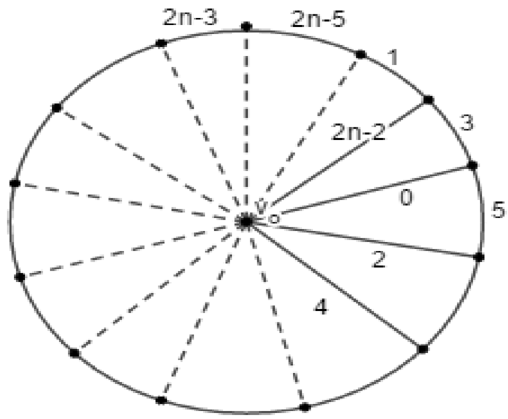

Figure 9:$L'(2,1)$- edge coloring of ; ≥ 6

F. *Theorem 6.* For a friendship graph $'(\quad) = 2(2\quad - 1)$; ≥ 2.

1) *Proof.* Let be the vertex of maximum degree Δ in . As seen in theorem 2 of [4], color all the edges incident to using $0, 2, \ldots, 2\Delta - 2$. We can now obtain an optimal coloring of by coloring the remaining edges (those not incident to) using the odd numbers between 0 to $2\Delta - 2$. Hence, $'(\quad) = 2\Delta - 2 = 2.2\quad - 2 = 2(2\quad - 1)$.

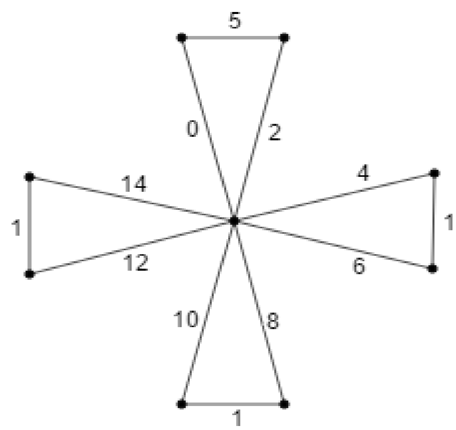

Figure 10:$L'(2,1)$- edge coloring of $_4$

III. CONCLUSIONS

Here we investigate the $L'(2,1)$- edge coloring number of cycles, complete graphs, wheel graphs, complete bipartite graphs, fan graphs and friendship graphs. Similar work can be carried out for other families also.

IV. ACKNOWLEDGMENT

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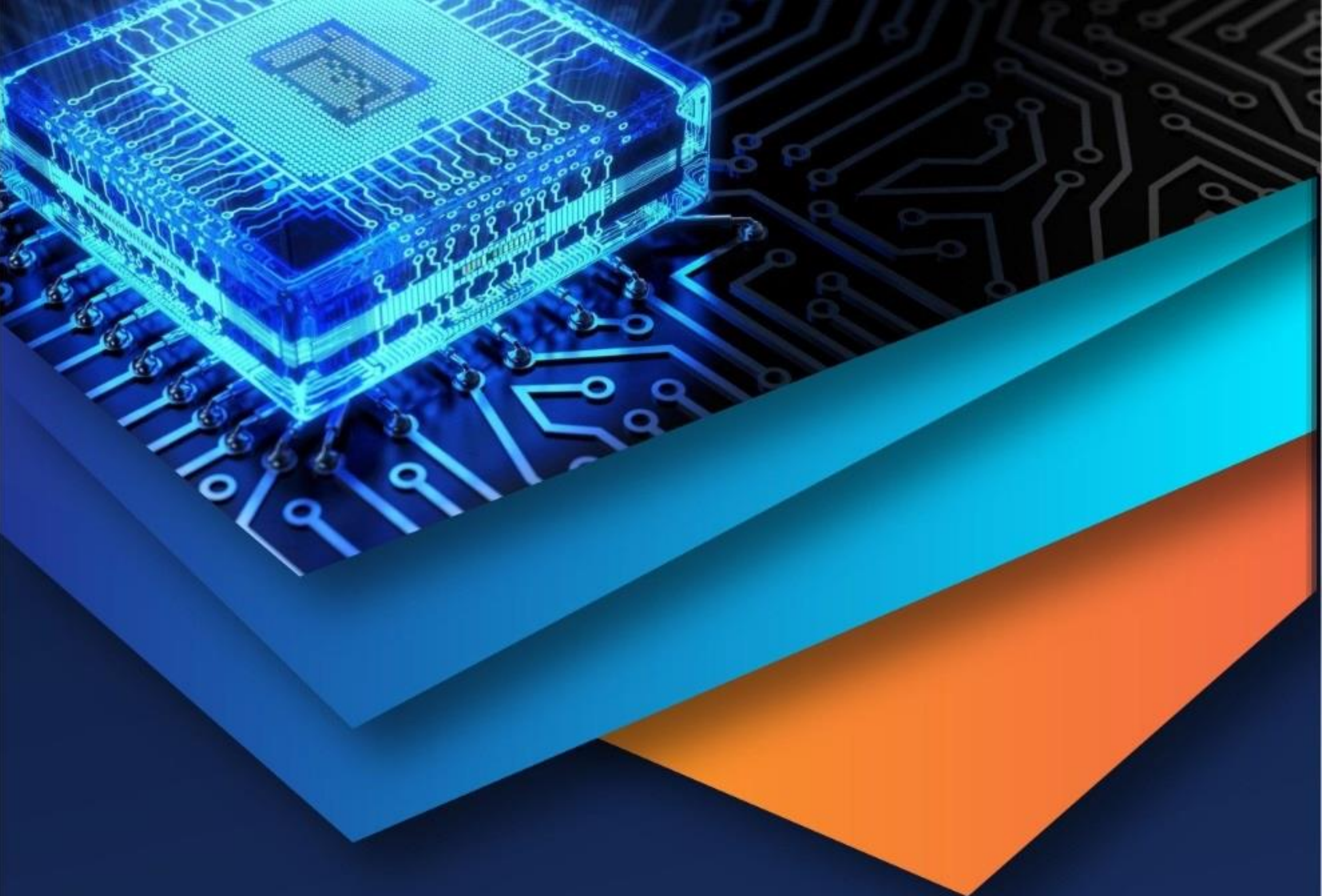
REFERENCES

[1] Chatrand and P. Zhang, “Introduction to Graph Theory”, Fourth Edition,Tata McGraw-Hill, 2006
[2] R. K. Yeh, Labelling Graphs with a Condition at Distance Two, Ph. D. Thesis,University of South Carolina, 1990
[3] M. A. Balci and P. D¨undar, Average Edge-Distance in Graphs, Selçuk Journal of Applied Mathematics, 11, pp. 63-70, 201
[4] D. Deepthy and Joseph Varghese Kureethara, $L'(2,1)$- Edge Coloring of Trees and Cartesian Product of Path Graphs, International Journal of Pure and Applied Mathematics,117, pp. 135-143, 2017.



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