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Performance Analysis of a Stable third order System using Fractional Order $PI^{\alpha}D^{\beta}$ Controller

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Abstract: The aim of the paper is to compare the performance of Fractional order $PI^{\alpha}D^{\beta}$ (FOPID) controller to that of a traditional PID controller. In this work a stable 3rd order plant is taken and controlled by both controllers and the performance of both the controllers are analysed using MATLAB/Simulink. Here, Ziegler-Nichols method is used for the tuning of PID controller and a stable 3rd order system is obtained by using the concept of Routh's stability criterion. It has been found that by varying the order of controller; a better performance can be obtained compared to the traditional PID controller.
Keywords: Fractional order $PI^{\alpha}D^{\beta}$ controller, PID controller, Fractional order system, Fractional capacitor

I. INTRODUCTION

Controller is a unit which generates a control signal to the final element, based on a measured deviation of the controlled variable from the set point [1]. There are two types of controller modes- Continuous and Discontinuous. A proportional–integral–derivative (PID) controller, is a continuous controller which is also known as three term controller, is a control loop feedback mechanism used for applications requiring continuously modulated control such as industrial control systems and a variety of other applications. The popularity of PID controllers in industrial applications are because of their simplicity, robustness, near-optimal performance and a wide range of applicability [1].

A PID controller continuously calculates an error value as the difference between a desired set point and a measured process variable and applies a correction based on proportional, integral and derivative terms (denoted as P, I and D respectively) which give their name to the controller [1].

A fractional order system can be defined as a dynamical system which can be modelled by a fractional differential equation which contains derivatives of non-integer order. The fractional order system can also be defined as an area where biochemistry, medicine and electrical engineering overlap which gives rise to a number of new potential applications [2, 3].

A generalization of PID controller is given by the fractional order PID controller presented as $PI^{\alpha}D^{\beta}$ [4].

PID controller is designed with the aim to achieve high performance which includes small settling time and low percentage overshoot. Improvisation in terms of performance of PID controllers can further be achieved by appropriate settings of fractional-I and fractional-D actions, which is termed as fractional order PID controller [5].

Fractional order PID controller is realized by replacing the traditional capacitor of PID controller with fractional capacitor.

Fractional capacitor (FC) is a passive circuit element that gives phase angle between 0 to -90 degree and remains constant with frequency [6, 7].

The impedance of a fractional capacitor (FC) is expressed as

$$Z(S) = \frac{1}{Cs^{\alpha}} \qquad (1)$$

Where C is the fractional capacitance and α ($0 < \alpha \leq 1$) is its order.

α(Known as fractional operator) is used to interpret the voltage-current relationship of a fractional capacitor. The phase difference is $\frac{-\pi\alpha}{2}$ between the voltage across its two terminals and current entering these terminals.

In [4] it was realized that the fractional order systems can be controlled in a better way by using fractional order PID controller instead of traditional PID controller. In this literature, different methods for the design of a FOPID controller have also been proposed. In [8] a FOPID is designed for a First Order plus Dead Time (FOPDT) system using minimization of Integral Square Error (ISE) method. Then the response of FOPID controller is compared with the traditional integer order PID controller and with integer order PI controller and it was observed that FOPID works more efficiently. In [9] 'Particle Swarm Optimization Technique' was used to design FOPID. A Tuning method for PID controller is described in [10] and also based on this a FOPID-toolbox for MATLAB is presented.

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Here a stable third order plant is included whose stability is confirmed by Routh's stability criterion. The system is controlled both by PID controller and fractional order $PI^{\alpha}D^{\beta}$ controller and the performance of both the controllers is compared by using Simulink. The paper is organized as follows. In section II, background of fractional order system has been discussed. In section III, details of fractional order PID controller has been presented. The plant which is controlled by using PID and FOPID has been discussed in section IV. In section V simulation results are presented and in section VI, the conclusions are drawn.

II. FRACTIONAL ORDER SYSTEM

The recent years have seen a considerable progress in the field of fractional order circuits and systems. The subject of fractional order circuits was approached by theorists back in 1960. In 1964, the concept of fractional capacitor came in to existence by [15] and thus a new era of fractional order system started.

A fractional order system can be defined as a dynamical system which can be modelled by a fractional differential equation which contains derivatives of non-integer order [2]. To study the behaviour of dynamical systems in physics, biology, electrochemistry, viscoelasticity and chaotic system, fractional order systems are very much useful [2].

The fractional order system can also be defined as an area where biochemistry, medicine and electrical engineering overlap which give rise to a number of new potential applications [2].

Fractional-calculus deals with derivatives and integrals of an arbitrary order, for example they can be of order 0.5 or 1.3 or of order π. Therefore classical first-order, second-order, or third-order derivatives and integrals are special cases where a derivative or integral are of arbitrary order. So, by using fractional order calculus the circuits and systems design are no more restricted to the integer-order domain [2].

A general dynamical system of fractional order can be written in the form

$$H(D^{\alpha_1,\alpha_2,\dots\dots\alpha_m})(y_1, y_2, \dots\dots\dots . y_l) = G\left(D^{\beta_1,\beta_2,\dots\dots\beta_n}\right)(u_1, u_2, \dots\dots\dots\dots u_k) \quad (2)$$

Where H and G are functions of the fractional derivative operator of orders $\alpha_1, \alpha_2, \dots\dots . \alpha_m$ and $\beta_1, \beta_2, \dots\dots\dots . \beta_n$, y_i and u_j are functions of time.

A common special case of this is the linear time invariant (LTI) system in one variable.

$$\left(\sum_{k=0}^{m} a_k D^{\alpha_k}\right) y(t) = \left(\sum_{k=0}^{n} b_k D^{\beta_k}\right) u(t) \quad (3)$$

The orders α_k and β_k are in general complex quantities.

By applying Laplace transform to the LTI system above (i.e. to equation 3), the transfer function becomes (assuming initial conditions as zero).

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\sum_{k=0}^{n} b_k s^{\beta_k}}{\sum_{k=0}^{m} a_k s^{\alpha_k}} \quad (4)$$

Simulation of fractional-order systems is crucial for any application. There are so many issues for the numerical simulations of fractional- order systems in time domain.

In the frequency domain, it is necessary to simulate the Laplace operator $s^{\alpha} = (jw)^{\alpha}$

$$s^{\alpha} = (jw)^{\alpha} = w^{\alpha}\left(cos\frac{\pi\alpha}{2} + j\, sin\frac{\pi\alpha}{2}\right) \quad (5)$$

III. FRACTIONAL ORDER $PI^{\alpha}D^{\beta}$ CONTROLLER

Among all the continuous controllers, PID controller is one of the most used controller type in control-loops. This is being used by control theorists since decades. Many researchers have studied about the design and tuning of this controller and the research is still going on.

The analytic expression of PID controller is

$$P(t) = K_p e_p + K_I \int_0^t e(t)dt + K_D \frac{d[e(t)]}{dt} + P_0 \quad (6)$$

PID controller can be written as

$$PID = K_P + \frac{K_I}{s} + K_D s \quad (7)$$

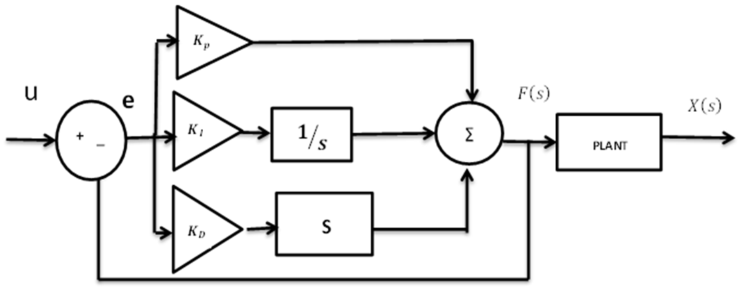

Fig. 1: Block diagram of PID controller

In the year 1999, the concept of fractional order PID controller came in to existence. Fractional order PID controller (FOPID) is a general form of PID controller and it is presented as $PI^{\alpha}D^{\beta}$.
Fractional order PID controller can be written as

$$PI^{\alpha}D^{\beta} = K_P + \frac{K_I}{s^{\alpha}} + K_D s^{\beta} \quad (8)$$

Where, α and β can take any fractional values like 0.01, 0.8, 0.3 etc.
When we put α = 1 and β = 1 then the FOPID works as a traditional or general PID controller.
PID controller is designed with the aim to achieve high performance which includes small settling time and low percentage overshoot. So, to improve the performance of PID controllers appropriate settings of fractional-I and fractional-D actions are done [5].
When fractional order PID controller is well tuned, much better performance than PID controller is achieved. It is possible due to the additional elements i.e. α and β other than K_P, K_I and K_D.

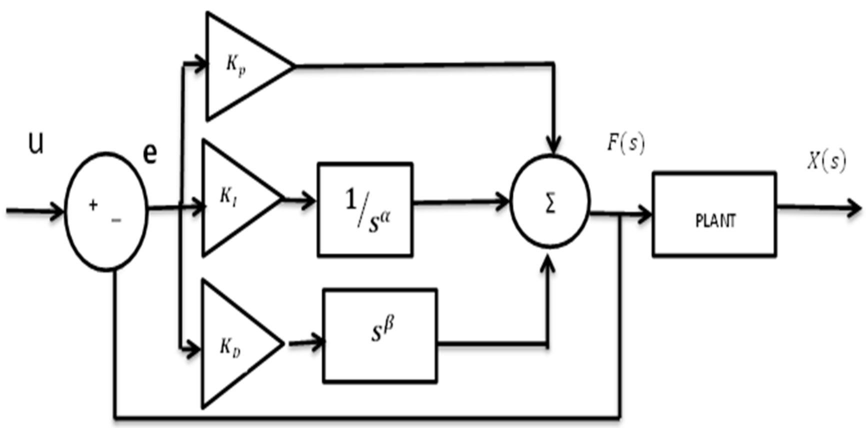

Fig. 2: Block diagram of FOPID controller

IV. A STABLE THIRD ORDER PLANT

The general form of a "third order plant" is

$$\frac{K}{aS^3+bS^2+cS+d} \quad (9)$$

The value of K, a, b, c and d should be taken in such a way that the plant would be stable.
A system is said to be stable if "every bounded input yields a bounded output" which is known as BIBO stability.
Also we can say a closed loop plant is stable if all the poles of it lay in the left half of s-plane i.e. poles have negative real parts.
For stable third order system, the value of the components are taken as K=1, a=1, b=6, c=8 and d=0. It is seen that a stable response is obtained by applying Routh's array principle. This can be substantiated by observing the step response of the plant whose transfer function is given in equation (5). Here the step response converge to steady state value as in Fig.4 and hence a stable system.

$$G(s) = \frac{1}{s^3+6s^2+8s} \quad (10)$$

And $H(s) = 1$

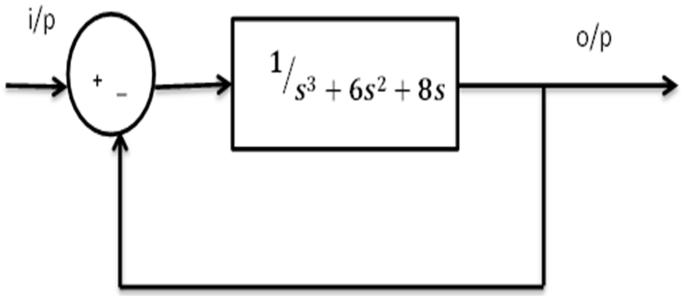

Fig. 3: Block diagram of third order plant

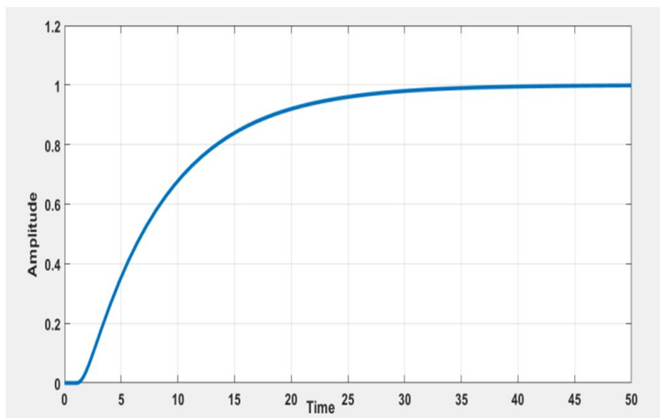

Fig. 4: Step response of third order plant

V. SIMULATION RESULTS

The third order plant is controlled with PID and FOPID controller and the results are compared. For the tuning of PID controller, Ziegler-Nichols method is used. According to [16] the integral, derivative and proportional gain are set to zero and proportional gain then increased until the close-loop system becomes critically stable. At this point the ultimate gain K_u is recorded which is same as the value of proportional gain for which the system has become critically stable. Also the corresponding period of oscillation T_u (known as the ultimate period) is recorded from the graph. It is found that $K_u = 48$ and $T_u = 2.22$

TABLE I

Ziegler-Nichols Method

Control Type	K_p	T_i	T_d	K_I	K_D
PID	$0.6\, K_u = 28.8$	$T_i = \frac{T_u}{2} = 1.11$	$T_d = \frac{T_u}{8} = 0.2775$	$K_I = \frac{K_P}{T_i} = 25.944$	$K_D = K_P T_d = 7.992$

The settling time (t_s), rise time (t_r) and percentage overshoot (M_p%) of the third order plant without controller, with PID controller and with a FOPID controller having unit integral order and derivative order are recorded and compared.

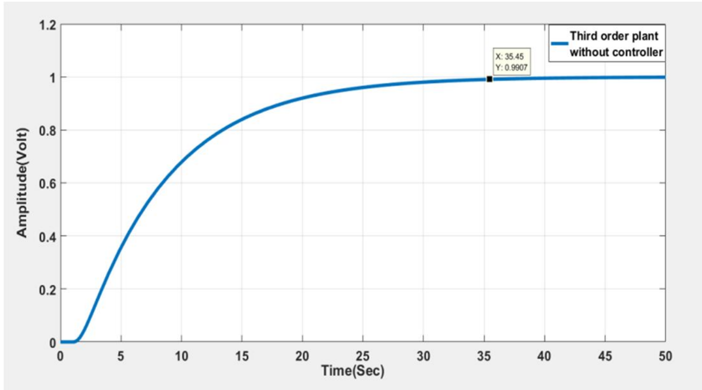

Fig. 5: Response of third order plant without any controller

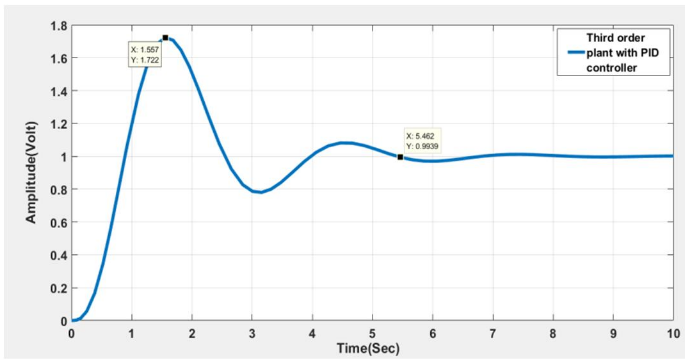

Fig. 6: Response of third order plant with PID controller

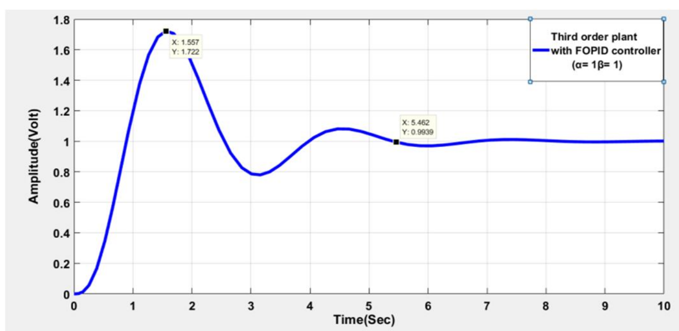

Fig. 7: Response of third order plant with FOPID controller (α=1 and β = 1)

TABLE II

Settling time, rise time and percentage overshoot of a third order plant

Third order Plant	SettlingTime(t_s) In sec	Rise Time(t_r) In sec	Percentage Overshoot (M_p%)
Without Controller	35.45	22.3	NA
With PID Controller	5.462	0.518	72.2
With FOPID Controller (α=1, β=1)	5.462	0.518	72.2

Now the simulation is done by varying α and β from 0.1 to 0.9 and the responses are compared with that of PID and then conclusion is drawn.
Figure 8 and Figure 9 shows the simulation results of FOPID controller with different values of α and β.

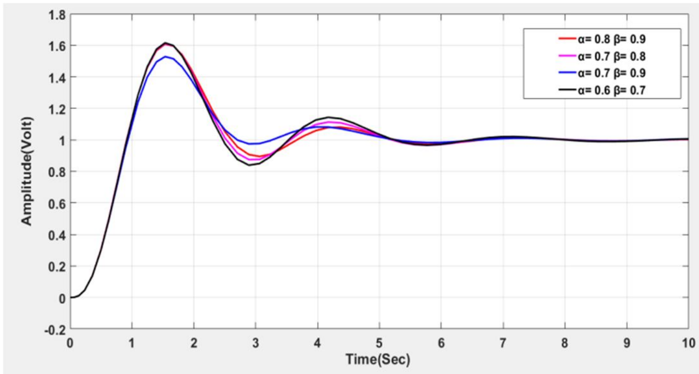

Fig. 8: Response of third order plant with FOPID by varying α and β

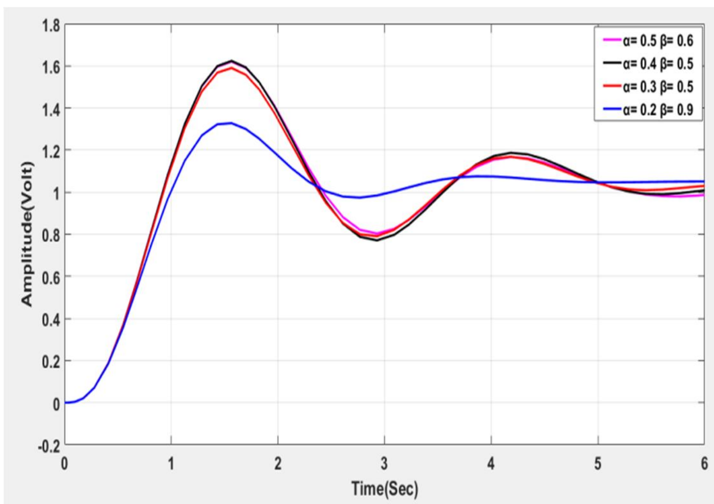

Fig. 9: Response of third order plant with FOPID by varying α and β

TABLE III
Settling time, rise time and percentage overshoot of a third order plant for different values of α and β

Third order Plant With FOPID Controller	SettlingTime(t_s) In sec	Rise Time (t_r) In sec	Percentage Overshoot (M_p%)
α=0.8 and β=0.9	4.936	0.56	60.2
α=0.7 and β=0.8	4.953	0.536	52.4
α=0.7 and β=0.9	4.649	0.563	61.1
α=0.6 and β=0.7	4.977	0.563	61.2
α=0.5 and β=0.6	5.011	0.563	60.8
α=0.4 and β=0.5	5.144	0.563	56.8
α=0.3 and β=0.5	5.084	0.563	51.93
α=0.2 and β=0.9	4.71	0.63	26.72

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VI. CONCLUSION

This paper presents the control of a stable 3rd order system using both by PID controller and fractional order $PI^{\alpha}D^{\beta}$ controller by using MATLAB R2016b Simulink and the results are compared. It is found from the simulation results that when $\alpha$and β values of FOPID are taken as one, then the FOPID is working as traditional PID controller (From Table II).

Then the values of $\alpha$and β are varied from 0.1 to 0.9 and the responses are compared with that of PID and then the settling time (t_s), rise time (t_r) and percentage overshoot ($M_p\%$) are found from the responses and are compared. It is found that for the values of $\alpha$and β given in table III the response of fractional order PID controller is better than PID controller.

It is clearly visible that in case of fractional order FOPID controller, for α=0.2 and β=0.9, the system's response is much better than that of traditional PID controller as the settling time and percentage overshoot of the plant with fractional order FOPID controller is much less than that of integer order PID controller.

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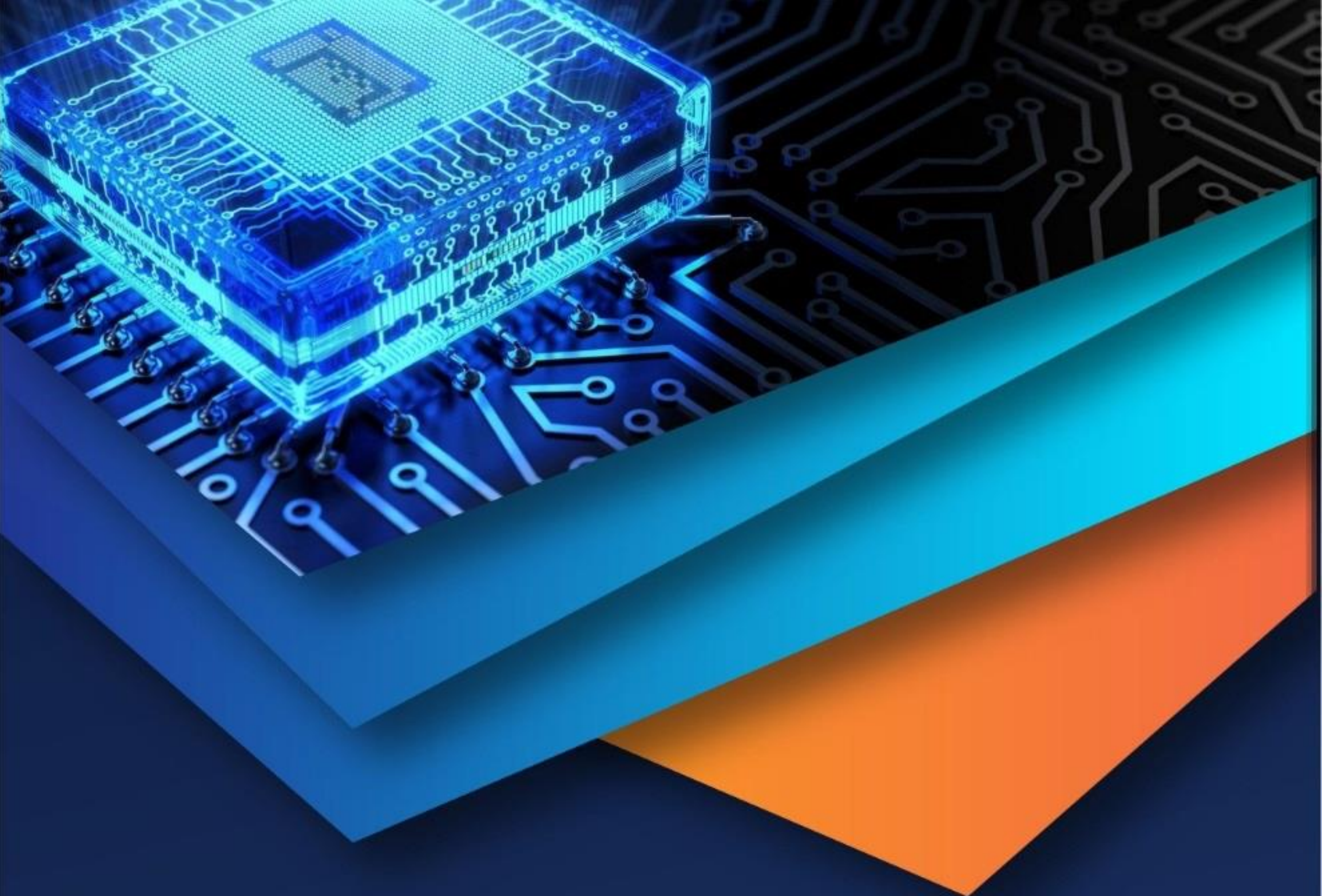

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