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## **Properties of Dominator of an M-Semigroup**

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Abstract: In this paper we discuss few properties of a collection of a special type of element of an M-semigroup, namely dominator. In an M-semigroup a dominator may be empty or properly contained in it or equal to the semigroup itself. Keywords: M- semi group, dominator, idempotent elements, Rectangular Band.

#### I. INTRODUCTION

In this paper we find a position of a dominator D in an M-semigroup M. We also find a necessary and sufficient condition for the existence of the dominator, a necessary and sufficient condition for a dominator D = M. Further, we decompose the dominator D, in which case decomposed part is a semi-inflation of the dominator. We also discuss some properties of dominator of an M-semigroup.

#### **II. PRELIMINARIES**

- 1) A subset S'of a semigroup S is said to be a sub semi group of S if S' is a semigroup with the same binary operation of S.
- 2) A semigroup S is said to be right(left) singular if for all x,y in S, xy=y (xy=x) such a semigroup is also called a right(left) zero semigroup
- 3) Let X and Y be any two nonempty sets. Then the system (S=X x Y; \*) where (x,y) \* (x',y') = (x,y');For all x,x' in X and y,y' in y is a band. It is called a rectangular band on X x Y (3)
- 4) A decomposition of a semigroup S is meant a partition of S into union of disjoint subsemigroups  $S_i$  of S, where  $i \in A$ , an index set.

A decomposition as above is sometimes denoted by  $\bigcup_{i \in A} S_i$ ; it is also said that S is decomposed over A

5) Let  $S = \bigcup_{i \in A} S_i$  be a decomposition of a semigroup S into subsemigroups  $S_i$  over an index A. If for each (i,j) in AxA there exists

an element k of A such that  $S_i S_j \subseteq S_k$  then A becomes thereby a band. It is then said that S is the union of the band A of semigroups  $S_i$  ( $i \in A$ ); sometimes it is also said that, "S is a band A of semigroups  $S_i$ ,  $i \in A$ 

6) If a semigroup S is the union of a band A of semigroups  $S_i$  ( $i \in A$ ) then A is the homomorphic image of S under the homomorphism,

 $f: S \to A, xf = i$  for x in  $S_i$  ( $i \in A$ ) and the semigroups  $S_i$  ( $i \in A$ ) are the congruence classes of S induced by the homomorphism f.

7) If  $f: S \to A$  is a homomorphism of a semigroup S onto a band A then S is the union of the band A of semigroups  $S_i = (i) f^{-1}, i \in A$ 

8) If A is band of type  $\zeta$ , S is a band A of semigroups  $S_i$  ( $i \in A$ ), and each semigroup  $S_i$ 

 $(i \in A)$  is a semigroup of type  $\Im$ , then S is called as a  $\zeta$  - band A of  $\Im$  - semigroup.

The concept that is being defined now is due to Clifford and Preston (1). Let B be a semigroup. With each i of B, associate a set  $G_i$  consisting i (i in B) which are mutually disjoint. Let  $G = \bigcup G_i$  (i in B) and let the product in B be extended to a product in G by defining xy=ij if x is in Gi and y in Gj (i,j in B). Then G is a semigroup which is called an Inflation of B. The following result is also due to the above authors:

9) The definition of inflation as given below is due to Tamura (8).

Let B be a given semigroup. Let S be any semigroup. Then S is an inflation of B if and only if,

S contains B as a semigroup,

B contains a homomorphic image of S,



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10) An element d of a semigroup S is called a dominator element of S if dyd=d for all y in S. By D,the dominator of S, is meant the set of all dominator elements of S.

#### **III.DOMINATOR OF AN M-SEMIGROUP**

#### A. Definition

An element x of a semigroup S is said to be a dominator of S if and only if xyx = x for all  $y \in S$  [2]. The set D of all dominators of S is called the dominator of S denoted by D. The dominator of a semigroup may be empty.

1) *Examples:* The following are examples of M-semigroups in which the dominator  $D = \phi$ .

		e	f	a	b		
(i)	e	e	f	a	b		
	f	e	f	a	b		
	a	a	b	e	f		
	b	a	b	e	f		
(ii)		e	f	a	b	c	d
	e	e	f	a	b	c	d
	f	e	f	a	b	c	d
	a	a	b	a	b	c	d
	b	a	b	a	b	c	d
	c	c	d	c	d	a	b
	d	c	d	c	d	a	b

The following are two examples of M-semigroups which contains a proper dominator D.

		e	f	a	b
	e	e	f	a	b
(i)	f	e	f	a	b
	a	a	b	a	b
	b	a	b	a	b

Here  $D = \{a, b\}$ .

		e	f	a	b	c	d
(ii)	e	e	f	a	b	c	d
	f	e	f	a	b	c	d
	a	a	b	a	b	a	b
	b	a	b	a	b	a	b
	c	c	d	a	b	a	b
	d	c	d	a	b	a	b



Here  $D = \{a, b\}$ .

Examples of M-semigroups in which the dominator is itself is aright zero semigroup.

B. Lemma: A dominator  $D \neq \phi$  of an M-semigroup  $M \cong R \times S$  has the following properties:

(i) 
$$D \cap R = \phi$$
 or  $D = R = M$   
(ii)  $D \cap Me \cong D \cap Mf$ ;  $e, f \in R$   
(iii)  $D = \bigcup (D \cap Me)$ .  
 $e \in R$ 

1) *Proof:* (i) If  $D \cap R \neq \phi$ . Let e belongs to  $D \cap R$ . For any x belongs to M, exe = xe.

But, exe = e since  $e \in D$ . Therefore xe = e, for all x belongs to M. Therefore for any a belongs to M, ea = xea. That is, a = xa, for all x belongs to M. That is, every element of M is a right zero element and hence M is a right zero semigroup [2.2] which is a rectangular band. Therefore M = R = D.ie,  $R \cap D = \phi$  or R = M = D.

(ii) Follows from for any ideal I of an M-semigroup M  $I \cap Me \cong I \cap Mf$ , ;  $e, f \in R$ .[4]

A semigroup S contains a dominator if and only if it contains an ideal I which is a rectangular band. Then I ise dominator of S [2].

Each  $I \cap Me, e \in R$  is a left ideal of I[4].

### C. Lemma

Every ideal I of an M-semigroup  $M \cong R \times S$  is a disjoint union of subsemigroup  $I \cap Me, e \in R$ . That is,  $I = \bigcup_{e \in R} (I \cap Me)$ . [4]

### D. Lemma

A semigroup S contains a dominator if and only if it contains an ideal I which is a rectangular band. Then I is the dominator of S [1]. Follows from 3.3

The following lemmas gives the conditions for the existence of the dominator in an M-semigroup.

#### E. Lemma:

In an M-semigroup  $M \cong R \times S$ , if the dominator D exists then  $D \subset E \setminus R$  where E is the set of idempotents of M.

1) Proof: Since D is a rectangular band ideal,

 $D \subseteq E$  being a rectangular band and  $D \cap R = \phi$  being an ideal.

If R = E and D exists, then  $D \cap E = \phi$  and  $D \subseteq E$  implies  $D = \phi$ .



Hence the lemma.

#### F. Lemma

In a left cancellative M-semigroup  $M \cong R \times S$ ,  $D = \phi$  or D = M.

*Proof:* If M is a left cancellative then every idempotent of M is a left identity. That is, E = R. 1)

Therefore  $D \subset E = R$ .

From 3.2(i),  $D = \phi$  or D = M = R.

#### G. Lemma

In an M-semigroup  $M \cong R \times S$ , if the dominator D of M is equal to M then M is left cancellative.

1) Proof: D = M implies  $D \cap R \neq \phi$  and  $D \neq \phi$ implies D = R by 4.8(i) implies D = M = R. That is  $xyx = x = x^2$  for all x, y belongs to R = M. That is, if xy = xz, then y = z since x belongs to R. Hence M is left cancellative, and hence the lemma. From 3.6 and 3.7 we have:

#### H. Theorem

If an M-semigroup  $M \cong R \times S$  has a nonempty dominator D, then D = M if and only if M is left cancellative.

#### Lemma Ι.

In an M-semigroup  $M \cong R \times S$  if any one of the left identities e of R is primitive then  $D \neq M$  implies  $D = \phi$ .

1) Proof: Let a particular  $e \in R$  be primitive. For any idempotent g of Me, ge = g and eg = g.

Therefore, ge = eg = g.

That is, e = g, since e is primitive.

Hence, e is the only idempotent in Me.

Let  $D \neq M$ , if  $D \neq \phi$ , D being the kernel of M, D intersects all Me,  $e \in R$  and  $D \cap R = \phi$ . This implies, there are idempotent elements other than e in Me, for all e belongs to R. This contradicts the property that e is primitive. Hence the lemma.

Theorem

Л.

An M-semigroup  $M \cong R \times S$  contains a dominator D, if and only if S contains a rectangular band ideal.

1) Proof: Let S contains a rectangular band ideal De. That is Me,  $e \in R$  contains a rectangular band ideal De,  $e \in R$ . Consider

$$D = \bigcup_{e \in R} De.$$

Since.

For any x belongs to De, a  $Me \cong Mf$ ,  $De \cong Df$   $(e, f \in R)$  a belongs to Df,



$$ax = axx = (ax)x$$
  
= (element of Me) ×  $\in$  De.  
xa = xaa  $\in$  Df.  
xax = xaxx = x(ax)x = x.

Therefore D is a rectangular band. For any xe belongs to De, and af belongs to Mf,  $e, f \in R$  ,

 $xe \cdot af = xaf = xf \cdot af \in Df$ , since  $xf \in Df$ .  $af \cdot xe = axe \in De$ .

That is D is a rectangular band ideal and hence D is the dominator.

Conversely, let M contain a dominator D. D is a rectangular band ideal of M. That is,

$$D = \bigcup (D \cap Me) \text{, for a fixed } e \in D \cap Me \subseteq Me \text{.}$$

Since D is an ideal of M,  $D \cap Me$  is an ideal of Me.

Let xe, ye belong to  $D \cap Me$ .

Then  $xe \cdot xe = xe$ , since xe belongs to D.

 $xe \cdot ye \cdot xe = xe$ , since xe, ye belongs to D.

Therefore  $D \cap Me$  is a rectangular band for all  $e \in R$ .

Since  $S \cong Me$ , S contains a rectangular band ideal.

Since the dominator of an M-semigroup is an ideal, we have the following:

#### **IV. CONCLUSION**

This paper discussed the "Properties of Domination of an M semi group".

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