International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue III, March 2018- Available at www.ijraset.com

Properties of Dominator of an M-Semigroup

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Abstract: In this paper we discuss few properties of a collection of a special type of element of an M-semigroup, namely dominator. In an M-semigroup a dominator may be empty or properly contained in it or equal to the semigroup itself.
Keywords: M- semi group, dominator, idempotent elements, Rectangular Band.

I. INTRODUCTION

In this paper we find a position of a dominator D in an M-semigroup M. We also find a necessary and sufficient condition for the existence of the dominator, a necessary and sufficient condition for a dominator D = M. Further, we decompose the dominator D, in which case decomposed part is a semi-inflation of the dominator. We also discuss some properties of dominator of an M-semigroup.

II. PRELIMINARIES

1) A subset S'of a semigroup S is said to be a sub semi group of S if S' is a semigroup with the same binary operation of S.
2) A semigroup S is said to be right(left) singular if for all x,y in S, xy=y (xy=x) such a semigroup is also called a right(left) zero semigroup
3) Let X and Y be any two nonempty sets. Then the system (S=X x Y; *) where (x,y) * (x',y') = (x,y');For all x,x' in X and y,y' in y is a band. It is called a rectangular band on X x Y (3)
4) A decomposition of a semigroup S is meant a partition of S into union of disjoint subsemigroups S_i of S, where i$\in A$, an index set.

   A decomposition as above is sometimes denoted by $\bigcup_{i\in A} S_i$; it is also said that S is decomposed over A
5) Let S=$\bigcup_{i\in A} S_i$ be a decomposition of a semigroup S into subsemigroups S_i over an index A. If for each (i,j)in AxA there exists an element k of A such that $S_i S_j \subseteq S_k$ then A becomes thereby a band. It is then said that S is the union of the band A of semigroups S_i (i$\in A$); sometimes it is also said that, " S is a band A of semigroups S_i , $i \in A'$
6) If a semigroup S is the union of a band A of semigroups S_i (i$\in A$) then A is the homomorphic image of S under the homomorphism,

$f: S \to A, xf = i$ for x in S_i (i$\in A$) and the semigroups S_i (i$\in A$) are the congruence classes of S induced by the homomorphism f.

7) If $f: S \to A$ is a homomorphism of a semigroup S onto a band A then S is the union of the band A of semigroups $S_i = (i) f^{-1}, i \in A$
8) If A is band of type ζ , S is a band A of semigroups S_i ($i \in A$), and each semigroup S_i

($i \in A$) is a semigroup of type $\Im$, then S is called as a ζ - band A of $\Im$– semigroup.

The concept that is being defined now is due to Clifford and Preston (1). Let B be a semigroup. With each i of B, associate a set G_i consisting i (i in B) which are mutually disjoint. Let G=$\cup G_i$ (i in B) and let the product in B be extended to a product in G by defining xy=ij if x is in Gi and y in Gj (i,j in B). Then G is a semigroup which is called an Inflation of B.The following result is also due to the above authors:

9) The definition of inflation as given below is due to Tamura (8).

Let B be a given semigroup. Let S be any semigroup. Then S is an inflation of B if and only if,
S contains B as a semigroup,
B contains a homomorphic image of S,

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10) An element d of a semigroup S is called a dominator element of S if dyd=d for all y in S. By D,the dominator of S, is meant the set of all dominator elements of S.

III.DOMINATOR OF AN M-SEMIGROUP

A. Definition

An element x of a semigroup S is said to be a dominator of S if and only if xyx = x for all $y \in S$ [2]. The set D of all dominators of S is called the dominator of S denoted by D. The dominator of a semigroup may be empty.

1) Examples: The following are examples of M-semigroups in which the dominator $D = \phi$.

(i)

	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	e	f
b	a	b	e	f

(ii)

	e	f	a	b	c	d
e	e	f	a	b	c	d
f	e	f	a	b	c	d
a	a	b	a	b	c	d
b	a	b	a	b	c	d
c	c	d	c	d	a	b
d	c	d	c	d	a	b

The following are two examples of M-semigroups which contains a proper dominator D.

(i)

	e	f	a	b
e	e	f	a	b
f	e	f	a	b
a	a	b	a	b
b	a	b	a	b

Here $D = \{a, b\}$.

(ii)

	e	f	a	b	c	d
e	e	f	a	b	c	d
f	e	f	a	b	c	d
a	a	b	a	b	a	b
b	a	b	a	b	a	b
c	c	d	a	b	a	b
d	c	d	a	b	a	b

Here $D = \{a, b\}$.

Examples of M-semigroups in which the dominator is itself is aright zero semigroup.

B. Lemma: A dominator $D \neq \phi$ of an M-semigroup $M \cong R \times S$ has the following properties:

(i) $D \cap R = \phi$ or $D = R = M$

(ii) $D \cap Me \cong D \cap Mf; \quad e, f \in R$

(iii) $D = \bigcup_{e \in R} (D \cap Me)$.

1) Proof: (i) If $D \cap R \neq \phi$.Let e belongs to $D \cap R$. For any x belongs to M, exe = xe.

But, exe = e since $e \in D$.
Therefore xe = e, for all x belongs to M.
Therefore for any a belongs to M, ea = xea.
That is, a = xa, for all x belongs to M.
That is, every element of M is a right zero element and hence M is a right zero semigroup [2.2] which is a rectangular band.
Therefore M = R = D.ie, $R \cap D = \phi$ or $R = M = D$.

(ii) Follows from for any ideal I of an M-semigroup M $I \cap Me \cong I \cap Mf$,; $e, f \in R$.[4]

A semigroup S contains a dominator if and only if it contains an ideal I which is a rectangular band. Then I ise dominator of S [2].

Each $I \cap Me, e \in R$ is a left ideal of I[4].

C. Lemma

Every ideal I of an M-semigroup $M \cong R \times S$ is a disjoint union of subsemigroup $I \cap Me, e \in R$. That is,

$I = \bigcup_{e \in R} (I \cap Me)$. [4]

D. Lemma

A semigroup S contains a dominator if and only if it contains an ideal I which is a rectangular band. Then I is the dominator of S [1]. Follows from 3.3

The following lemmas gives the conditions for the existence of the dominator in an M-semigroup.

E. Lemma:

In an M-semigroup $M \cong R \times S$, if the dominator D exists then $D \subset E \setminus R$ where E is the set of idempotents of M.

1) Proof: Since D is a rectangular band ideal ,

$D \subseteq E$ being a rectangular band and $D \cap R = \phi$ being an ideal.

If R = E and D exists, then $D \cap E = \phi$ and $D \subseteq E$ implies $D = \phi$.

Hence the lemma.

F. Lemma

In a left cancellative M-semigroup $M \cong R \times S$, $D = \phi$ or $D = M$.

1) Proof: If M is a left cancellative then every idempotent of M is a left identity. That is, E = R.

Therefore $D \subset E = R$.

From 3.2(i), $D = \phi$ or $D = M = R$.

G. Lemma

In an M-semigroup $M \cong R \times S$, if the dominator D of M is equal to M then M is left cancellative.

1) Proof: D = M implies $D \cap R \neq \phi$ and $D \neq \phi$

implies D = R by 4.8(i)

implies $D = M = R$.

That is xyx = x = x^2 for all x, y belongs to R = M.

That is, if xy = xz, then y = z since x belongs to R.

Hence M is left cancellative, and hence the lemma. From 3.6 and 3.7 we have:

H. Theorem

If an M-semigroup $M \cong R \times S$ has a nonempty dominator D, then D = M if and only if M is left cancellative.

I. Lemma

In an M-semigroup $M \cong R \times S$ if any one of the left identities e of R is primitive then $D \neq M$ implies $D = \phi$.

1) Proof: Let a particular $e \in R$ be primitive. For any idempotent g of Me, ge = g and eg = g.

Therefore, ge = eg = g.

That is,e = g, since e is primitive.

Hence, e is the only idempotent in Me.

Let $D \neq M$, if $D \neq \phi$, D being the kernel of M, D intersects all Me, $e \in R$ and $D \cap R = \phi$. This implies, there are idempotent elements other than e in Me, for all e belongs to R. This contradicts the property that e is primitive.

Hence the lemma.

J. Theorem

An M-semigroup $M \cong R \times S$ contains a dominator D, if and only if S contains a rectangular band ideal.

1) Proof: Let S contains a rectangular band ideal De. That is Me, $e \in R$ contains a rectangular band ideal De, $e \in R$. Consider

$$D = \bigcup_{e \in R} De.$$

Since.

For any x belongs to De, a $Me \cong Mf, De \cong Df \quad (e, f \in R)$nd a belongs to Df,

$$ax = axx = (ax)x$$
$$= (\text{element of } Me) \times \in De.$$
$$xa = xaa \in Df.$$
$$xax = xaxx = x(ax)x = x.$$

Therefore D is a rectangular band. For any xe belongs to De, and af belongs to Mf, $e, f \in R$,

$$xe \cdot af = xaf = xf \cdot af \in Df, \quad \text{since } xf \in Df.$$
$$af \cdot xe = axe \in De.$$

That is D is a rectangular band ideal and hence D is the dominator.

Conversely, let M contain a dominator D. D is a rectangular band ideal of M. That is,

$$D = \bigcup_{e \in R} (D \cap Me), \text{ for a fixed } e \in D \cap Me \subseteq Me.$$

Since D is an ideal of M, $D \cap Me$ is an ideal of Me.

Let xe, ye belong to $D \cap Me$.

Then $xe \cdot xe = xe$, since xe belongs to D.

$xe \cdot ye \cdot xe = xe$, since xe, ye belongs to D.

Therefore $D \cap Me$ is a rectangular band for all $e \in R$.

Since $S \cong Me$, S contains a rectangular band ideal.

Since the dominator of an M-semigroup is an ideal, we have the following:

IV. CONCLUSION

This paper discussed the "Properties of Domination of an M semi group".

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