

Varying Length Pendulum and Viscosity of the Liquid: A Review

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Abstract: Simple pendulum is basic tool to verify acceleration due to gravity. This system can be extended and employed to the sloshing problem. This paper reviews work in viscosity and varying length pendulum. Furthermore this paper focuses on the work in sloshing of liquid problem.

Keywords: Half-filled bob, simple pendulum, sloshing of liquid, acceleration due to gravity, viscosity.

I. INTRODUCTION

The system of simple pendulum is employed to verify the value of acceleration due to gravity in a laboratory, whereas torsional pendulum, where the restoring force is torsion, is a tool to determine the viscosity of the liquid. The cylindrical container carrying viscous liquid is an interesting physical problem for study. The sloshing of the viscous liquid inside the container poses the problem of inverted pendulum. The problem of viscous liquid inside the container and the effect on the time period of simple pendulum when it is allowed to oscillate in the viscous liquid, both are interesting physical phenomenon. These two problems addressed separately by the researchers.

II. VARYING LENGTH PENDULUM

A simple experiment demonstrate the change in the period of a pendulum when the length of the string is varied [1]. In the paper 'The Lengthening pendulum' the change in the time period of pendulum when its length is altered is confirmed [2]. The varying length pendulum is an interesting variation of fixed length pendulum. Dynamics of weightless rod with a point mass sliding along the rod axis according to periodic law was discussed in 'Dynamics of the pendulum with periodically varying length' [3].

An approximate relation between the amplitude of periodic orbit of the pendulum and excitation frequency was mentioned [4]. Change of energy and first instability region for pendulum with variable piecewise constant length were discussed by Magnus et al [5]. The stability analysis of the lower vertical position of the pendulum with damping and arbitrary periodic excitation was carried out by Seranian et al [6]. The qualitative analysis of periodic solutions and their stability for PPVL no damping and orbiting excitation amplitude was studied in [7,8]. The PPVL is much less studied than the pendulum with oscillating support which is often referred to simply as a parametrically driven pendulum or parametric pendulum; These two pendula are described by different analytical models and consequently, possess different dynamical properties. For example, the PPVL cannot be stabilized in the inverted vertical position. Nevertheless, the methods used for dynamical analysis of one pendulum are applicable for the other one. The methodological peculiarity of this work is in the assumption of quasi-linearity of the system which allows us to derive higher order approximations by the averaging method.

The pendulum with varying length and the pendulum with oscillating support was compared [9]. The focus of investigation was on the persistent attractive periodic orbit and their basins of attractions using both analytical and numerical techniques. The investigation proposes unlike the pendulum with oscillating support, the pendulum with varying length cannot be stabilized around the upward position.

III. TIME PERIOD AND VISCOSITY

The oscillation properties of simple pendulum was studied by Zhe Hoe et al [10]. The nonlinear equation of motion for the pendulum was established by considering viscous damping. The relation between oscillation period and the viscosity of the liquid was derived. The experimental data was inconsistency with the theoretical predictions. An interesting investigation was put forward by Sohif Shamim et al [11] about the study of viscous damping using webcam. The mechanical oscillator was tracked using webcam and an image processing algorithm recorded the position of geometrical center as function of time.

A dissertation [12] addressed the oscillations of a sphere in a cylindrical tube containing a viscous liquid. The effective inertia of accelerating body that is 'virtual mass' was taken into account to identify induced mass' or 'added masses. This mass was considered to determine the kinetic energy of the system $T=1/2 \mu v^2$. It suggested analytical approach and experimental approach for

solving fluid mechanics problems. J.R. Ries et al [13] studied the motion of a flat plate pendulum in viscous fluid. An analytical solution was obtained through the simultaneous solution of equations of motions of plate. An experimental investigation was performed in water and light oil.

The system of Pendulum can be made to oscillate in viscous liquid and time period variations can be attributed to the viscosity of the liquid. The dry and viscous damped pendulum was studied by L.F.C. Zonetti et al[14]. They have demonstrated these type of oscillations and have compared the same one. One can sense the relation between viscosity and the time period of pendulum. The best way for approach is torsional pendulum. Determination of viscosity using torsional pendulum was put forward[15]. The apparatus was designed to exhibit relatively high dependence of period on viscosity for a range from 10 to 150 cSt. Above and below this range, the dependence is markedly weaker.

IV. THE PROBLEM OF SLOSHING

The pendulum system was also successfully employed to address the problem of sloshing by M.I. Salem et al[16]. The lateral fluid sloshing during turning and sudden lane changing causes the low rollover threshold. They proposed a novel idea of an elliptical terminal pendulum for elliptical tank geometry. Sayyed M. Hasheminejad et al[17] developed the analytical mathematical model to study the transient liquid sloshing characteristics in half-full horizontal cylindrical container.

V. CONCLUSION

The simple pendulum with hollow bob which is partially filled with viscous liquid is equally interesting physical phenomenon. The approach of studying and relating the time period and viscosity was to consider the pendulum and viscous liquid as independent physical entities, but a new approach can be considered in which the viscous liquid can be made constituent element of the suspended bob. The emerging system has many degrees of freedom to study. In this review we do not come across to the system in which the viscous liquid is responsible for the change in the length of the problem during oscillations.

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