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# Boundary Conditions for the Asymptotic Fields

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**Abstract:** One considers within the hydrodynamic approximation a three layer model is which both the surface layer and the bulk metal can sustain longitudinal fields, within the LWL one present explicit analytical results for  $d_{\perp}(\omega)$ ,  $d_{\parallel}(\omega)$  and for the ellipsometry parameters, which contain previous results of ABELES and LOPEZRIOS as special cases and may be useful for the interpretation of experimental data on metal films absorbed on metallic substrates. One discuss surface plasmons in terms of the response functions  $d_{\perp}(\omega)$ ,  $d_{\parallel}(\omega)$ , Especially the treatment of “multipole” surface plasmons yields some understanding of the frequency dependence of  $d_{\perp}(\omega)$ .

**Keywords:** Metal film, “Multipole” surface plasmons, ellipsometry parameters

## I. INTRODUCTION

### A. Boundary Conditions for the Asymptotic Fields

One assume that far from the surface the exact electromagnetic fields reduce to transverse fields and compare the exact solution  $E(r) = E(z) \exp[ik_x x - \omega t]$  of Maxwell’s equations in the whole space with a reference field defined by

$$E^0(z;a) = E^<(z)\theta(a-z) + E^>(z)\theta(z-a) \quad (4.1)$$

Where the transverse fields  $E^<(z)$  and  $E^>(z)$  are the extrapolations of the asymptotic limits of  $E(z)$  on the vacuum side and on the metal side, respectively, towards a plane  $z = a$  in the surface region. The reference field (4.1) together with the corresponding  $B^0$ -field and the displacement field.

$$D^0(z;a) = \epsilon_a E^<(z)\theta(a-z) + \epsilon_t E^>(z)\theta(z-a) \quad (4.2)$$

is assumed to solve Maxwell’s equations with the local dielectric constants  $\epsilon_a$  in the halfspace  $z < a$  and  $\epsilon_t$  in the metallic halfspace  $z > a$ . Here and in the following we assume the local approximation  $\epsilon_t(k, \omega) = \epsilon_t(0, \omega) = \epsilon_t$  to be sufficient for the bulk response of the metal to transverse waves. Furthermore we consider the slightly more general case that to the left of the surface we have a dielectric described by  $\epsilon_a$ , rather than vacuum.

Since the reference fields are determined by the asymptotic values of the exact field, the reference fields will in general not satisfy the standard matching conditions at the plane  $z=a$ . That means, the reference field is not the solution of the classical Fresnel problem with dielectric constants  $\epsilon_a$  and  $\epsilon_t$  in  $z < a$  and  $z > a$ , respectively. On the contrary, the reference field contains by definition the full information about the reflection and transmission properties of the nonlocal surface problem. Following APELL<sup>2</sup>, and more closely, recent work by KMPA and GERHARDTS we now derive the exact matching conditions for the reference field.

To be specific, we consider first the case of p polarization and write the field in the dielectric in the form

$$E_x^<(z) = -\frac{c p_a}{\omega} E_0 (e^{iz p_a} - r_p e^{-iz p_a}) \quad (4.3a)$$

$$E_z^<(z) = -\frac{c k_x}{\omega} E_0 (e^{iz p_a} - r_p e^{-iz p_a}) \quad (4.3b)$$

With  $E_0$  the amplitude of the incident field,  $r_p$  the reflection amplitude and  $k_x^2 + p_a^2 = \epsilon_a \omega^2 / c^2$ . The asymptotic transverse field inside the metal is written as

$$E_x^>(z) = E_x^t e^{iz p_t} \quad (4.4a)$$

$$E_z^>(z) = -\frac{k_x}{p_t} E_x^t e^{iz p_t} \quad (4.4b)$$

With  $k_x^2 + p_t^2 = \epsilon_t \omega^2 / c^2$ . By construction of the reference field there exist  $\geq$  values  $\xi_1 < a$  and  $\xi_2 < a$  (slightly) outside the surface region, so that the exact field agree practically with the reference fields, e.g.  $D(z) \approx D^0(z;a)$ , for  $z < \xi_1$  and  $z < \xi_2$ . Since both the exact fields and the reference fields satisfy in the halfspace  $z > a$  and  $z < a$  Maxwell’s equations, although with different constitution equations, we can use  $V \cdot D = ik_x D_x + D_z^1 = 0$  for both  $D(r)$  and  $D^0(r;a)$  to evaluate

$$\int_a^{\xi_2} dz (D_z^1(z) - D_z^{01}(z; a)) = [D_z(a) - D_x^>(a)]$$

$$= ik_x \int_a^{\xi_2} dz [D_x(z) - D_x^0(z; a)] \quad (4.5)$$

where  $D_z(\xi_2) = D_z^0(\xi_2; a) = D_z^>(\xi_2)$  has been taken into account. Adding the corresponding integral over the interval  $\xi_1 < z < a$ , we obtain, since  $D_z(z)$  is continuous at  $z=a$ ,

$$D_z^>(a) - D_z^<(z) = -ik_x \int_{\xi_1}^{\xi_2} dz D_x(z) - D_x^0(z; a)] \quad (4.6)$$

This matching condition for the asymptotic fields replaces the standard boundary condition " $D_z(z)$  continuous".

A second matching condition for the asymptotic fields, corresponding to the standard boundary condition " $E_x(z)$  continuous", is obtained from Faraday's Law  $\nabla \times E = -c^{-1} \partial B / \partial t$  i.e.  $ik_x E_z - E_x^1 = i\omega B_y / c$  which yields for instance

$$\int_a^{\xi_2} dz E_x^1(z) - E_x(\xi_2) - E_x(a) + ik_x \int_a^{\xi_2} dz E_z(z) + i \frac{\omega}{c} [\xi_2 B_y(\xi_2) - a B_y(a^+)] + \frac{\omega^2}{c^2} \int_a^{\xi_2} dz (z D_x(z)) \quad (4.7)$$

Here we have integrated by parts, using  $B_y^1 = i\omega D_x / c$ , the x component of Ampere's law  $\nabla \times H = -c^{-1} \partial B / \partial t$ . We now subtract from (4.7) the corresponding expression for the reference fields and add the results to that obtained in the same way by integrating over the interval  $\xi_1 < z < a$ . This yields the matching condition.

Where the explicit boundary terms at  $z = \xi_1$  and  $z = \xi_2$  have cancelled according to the definition of the reference fields. The boundary terms  $-a[B_y^>(a) - B_y^<(a)]$  have been included in the integral of (4.8) using  $ik_x B_y = i\omega D_y / c$  and (4.6).

For a compact notation we define the following moments of "surface solutions" LWL. i.e. of differences between exact and reference fields :

## REFERENCES

- [1] K.Kempa, R.R. Gerhardt : Solid State Commun 53, 579 (1985)
- [2] C. Schwatz, W.L. Schaich : Phys. Rev B30, 1059 (1984)
- [3] A. Bagchi, R.G. Barrera, A.K. Rajagopal:Phys. Rev B20, 4824 (1979)
- [4] J.D.E. McIntyre, D.E. Aspnes : Surf. Sci, 24, 417 (1971)
- [5] J.E. Slupe : Phys. Rev. B22, 1589 (1980)
- [6] A.J. Bennett : Phys. Rev. B1.203 (1970)
- [7] L. Genzel, T.P. Martin, U. Kreibitz : Z. Phys. B21, 339 (1975)
- [8] J.C. Maxwell-Garnett : Phil. Trans. Roy. Soc. 203, 385 (1904) : 205, 237 (1906)
- [9] A. Leibesly, B.N.J. Persson : J. Phys C : Solid State Phys 16, 5375 (1983)
- [10] R. Rupin : Phys. Rev B11, 2871 (1975)
- [11] A.D. Boardmann, B.V. Prajape Y.O. Nakanura : Phys. State. Sol (b) 75, 347 (1976)
- [12] J.E. Inglesfield, E.L. Wikborg : J. Phys F5, 1706 (1975)





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