International Journal for Research in Applied Science & Engineering Technology (IJRASET)
ISSN: 2321-9653; IC Value: 45.98; SJ Impact Factor: 6.887
Volume 6 Issue III, March 2018- Available at www.ijraset.com

# Construction of The Diophantine Triple involving Pentatope Number

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***Abstract: We search for three distinct polynomials with integer coefficients such that the product of any two numbers increased by a non-zero integer (or polynomials with integer coefficients) is a perfect square.***
***Keywords: Diophantine triples, Pentatope number, Polynomials & Perfect square. 2010 Mathematics Subject Classification: 11D25.***

## I. INTRODUCTION

In Mathematics, a Diophantine equation is a polynomial equation, usually in two or more unknowns, such that only the integer solutions are sought or studied (an integer solution is a solution such that all the unknowns take integer values).The word Diophantine refers to the Hellenistic mathematician of the 3rd century, Diophantus of Alexandria, who made a study of such equations and was one of the first mathematician to introduce symbolism into algebra. The mathematical study of Diophantine problems that Diophantus initiated is now called Diophantine analysis.While individual equations present a kind of puzzle and have been considered throughout history, the formulation of general theories of Diophantine equations (beyond the theory of quadratic forms) was an achievement of the twentieth century.

In [1-6], theory of numbers were discussed. In [7-9], diophantine triples with the property $D(n)$ for any arbitrary integer $n$ and also for any linear polynomials were discussed. Recently, in [10&11] pentatope numbers were analysed for its special dio-triples and evaluated using z-transform. This paper aims at constructing Dio-Triples where the product of any two members of the triple with the addition of a non-zero integer or a polynomial with integer coefficients satisfies the required property. Also, we present three sections where in each of which we find the Diophantine triples from Pentatope number of different ranks with their corresponding properties.

### A. *Notation*

$\mathrm{PT}_n =$ Pentatope number of rank $n$.

### B. *Basic Definition*

A set of positive integers $\{a_1, a_2, ..., a_m\}$ is said to have the property $D(n)$ if $a_i a_j + n$ is a perfect square for all $1 \le i < j \le m$; such a set is called a Diophantine m-tuple of size $m$, where $n$ may be non-zero integer or polynomial with integer coefficients.

## II. METHOD OF ANALYSIS

### A. *Section A*

Let $a = 24\,PT_n$ and $b = 24\,PT_{n-1}$ be Pentatope numbers of rank $n$ and $n-1$ respectively such that $ab + (4n^2 + 8n + 4)$ is a perfect square say $X^2$.

Let $c$ be any non-zero integer such that

$$ac + (4n^2 + 8n + 4) = Y^2 \tag{1}$$

$$bc + (4n^2 + 8n + 4) = Z^2 \tag{2}$$

Setting $Y = a + \alpha$ and $Z = b + \alpha$ and subtracting (1) from (2), we get

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$$c(b-a) = Z^2 - Y^2 = (Z+Y)(Z-Y)$$
$$= (a+b+2X)(b-a)$$

Thus, we get $c = a+b+2X$

Similarly by choosing $Y = a - X$ and $Z = b - X$, we obtain $c = a+b-2X$

Here we have $X = n^4 + 4n^3 + 3n^2 - 2n - 2$ and thus two values of $c$ are given by $c = 4n^4 + 16n^3 + 16n^2 - 4$ and $c = 4n^2 + 8n + 4$.

Thus, we observe that $\{24PT_n, 24PT_{n\text{-}1}, 6PT_{2n} + (4n^3 + 5n^2 - 3n - 4)\}$ and $\{24PT_n, 24PT_{n\text{-}1}, 6PT_{2n} - (4n^4 + 12n^3 + 7n^2 - 5n - 4)\}$ are Diophantine triples with the property $D(4n^2 + 8n + 4)$.

Some numerical examples are given below in the following table.

Table 1

| $n$ | Diophantine Triples | $D(4n^2+8n+4)$ |
|---|---|---|
| 1 | (24, 0, 32) & (24, 0, 16) | 16 |
| 2 | (120, 24, 252) & (120, 24, 36) | 36 |
| 3 | (360, 120, 896) & (360, 120, 64) | 64 |

In general, it is noted that the triples $(24\,PT_n, 24\,PT_{n\text{-}1}, 6\,PT_{2n} - (4n^3 + 5n^2 - 3n + (2k-4)))$ & $(24\,PT_n, 24\,PT_{n\text{-}1}, 6\,PT_{2n} - (4n^4 + 12n^3 + 7n^2 - 5n + (2k-4)))$ are Dio 3-tuples with the property $D(2k\,n^4 + 8k\,n^3 + (6k+4)n^2 - (4k-8)n + (k-2)^2)$, where $k = 0, 1, 2, \ldots$

*B. Section B*

Let $a = 24PT_n$ and $b = 24PT_{n-2}$ be Pentatope numbers of rank $n$ and $n-2$ respectively such that $ab + (16n^2 + 16n + 4)$ is a perfect square say $X^2$.

Let $c$ be any non-zero integer such that

$$ac + (16n^2 + 16n + 4) = Y^2 \tag{3}$$
$$bc + (16n^2 + 16n + 4) = Z^2 \tag{4}$$

Applying the procedure as mentioned in section A, we have

$c = 4n^4 + 8n^3 + 4n^2 - 4$ and $c = 16n^2 + 16n + 4$.

Thus, we observe that $\{24PT_n, 24PT_{n\text{-}2}, 6PT_{2n} - (4n^3 + 7n^2 + 3n + 4)\}$ and $\{24PT_n, 24PT_{n\text{-}2}, 6PT_{2n} - (4n^4 + 12n^3 - 5n^2 - 13n - 4)\}$ are Diophantine triples with the property $D(16n^2 + 16n + 4)$.

Some numerical examples are given below in the following table.

Table 2

| $n$ | Diophantine Triples | $D(16n^2+16n+4)$ |
|---|---|---|
| 1 | (24, 0, 12) & (24, 0, 36) | 36 |
| 2 | (120, 0, 140) & (120, 0, 100) | 100 |
| 3 | (360, 24, 572) & (360, 24, 196) | 196 |

In general, it is noted that the triples $\left(24\,PT_n,\ 24\,PT_{n\text{-}2},\ 6\,PT_{2n}-(4n^3+7n^2+3n-(2k-4))\right)$ & $\left(24\,PT_n,\ 24\,PT_{n\text{-}2},\ 6\,PT_{2n}-(4n^4+12n^3-5n^2-13n+(2k-4))\right)$ are Dio 3-tuples with the property $D\left(2k\,n^4+4k\,n^3-(6k-16)n^2-(8k-16)n+(k-2)^2\right)$, where $k=0,1,2,\ldots$

*C. Section C*

Let $a=24\,PT_n$ and $b=24\,PT_{n-3}$ be Pentatope numbers of rank $n$ and $n-3$ respectively such that $ab+36n^2$ is a perfect square say $X^2$.

Let $c$ be any non-zero integer such that

$$ac+36n^2=Y^2 \tag{5}$$

$$bc+36n^2=Z^2 \tag{6}$$

Proceeding in the same way as in section A, we have

$c=4n^4+8n^2$ and $c=36n^2$.

Thus, we observe that $\left\{24\,PT_n, 24\,PT_{n\text{-}3}, 6\,PT_{2n}-(12n^3+3n^2+3n)\right\}$ and $\left\{24\,PT_n, 24\,PT_{n\text{-}3}, 6\,PT_{2n}-(4n^4+12n^3-25n^2+3n)\right\}$ are Diophantine triples with the property $D(36n^2)$.

Some numerical examples are given below in the following table.

Table 3

| $n$ | Diophantine Triples | $D(36n^2)$ |
|---|---|---|
| 1 | (24, 0, 12) & (24, 0, 36) | 36 |
| 2 | (120, 0, 96) & (120, 0, 144) | 144 |
| 3 | (360, 0, 396) & (360, 0, 324) | 324 |

In general, it is noted that the triples $\left(24\,PT_n,\ 24\,PT_{n\text{-}3},\ 6\,PT_{2n}-(12n^3+3n^2+3n-2k)\right)$ & $\left(24\,PT_n,\ 24\,PT_{n\text{-}3},\ 6\,PT_{2n}-(4n^4+12n^3-25n^2+3n+2k)\right)$ are Dio 3-tuples with the property $D\left(2k\,n^4-(14k-36)n^2+k^2\right)$, where $k=0,1,2,\ldots$

## III. CONCLUSION

In this paper we have presented a few examples of constructing a Diophantine triples for Pentatope number of different rank with suitable properties. To conclude one may search for Diophantine triples for other numbers with their corresponding suitable properties.

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