Study on Complete Fuzzy Graph and Strong Fuzzy Graph

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Abstract: In this paper we define some basic definitions and some properties of fuzzy graph. We compare the general concept of complete fuzzy graph and strong fuzzy graph.
Key words: fuzzy graph, complete fuzzy graph, strong fuzzy graph.

I. INTRODUCTION

It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relation between objects. Here objects are represented by vertices and relations are represented by edges. Azriel Rosenfeld introduced fuzzy graphs in 1975. It has been growing fast and has numerous applications in various fields. The first definition of a fuzzy graph was by Kaufmann in 1973, based on zadeh's fuzzy relations. But it was azriel Rosenfeld who considered fuzzy relation on fuzzy sets and developed the theory of fuzzy graph in 1975. The concept of signed fuzzy graph is used if the relationship between each pair of nodes is symmetric

II. BASIC PRELIMINARIES

A. Definition 2.1

Let V be a non empty set. A fuzzy graph is a pair of function. G: (σ, μ) where σ is a fuzzy subset of V and μ is a symmetric fuzzy relation on σ. i.e σ: V x [0,1] and μ: V x V→ [0,1] such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all u,v in V. Here ^ denote minimum.
We denote the underlying (crisp) graph of G: (σ, μ) by $G^*(\sigma^*, \mu^*)$ where σ^* is referred to as the (nonempty) set V of nodes and $\mu^*=E\leq$ V x V .Note that the crisp graph (V,E) is a special case of a fuzzy graph with each vertex and edge of (V,E) having degree of membership 1.

B. Definition 2.2

The fuzzy graph H:(Ʈ, v) is called a fuzzy sub graph of G: (σ, μ) induced by P if P≤V, Ʈ(u)= σ(u) for all u,v ε P. Remark σ is a fuzzy sub sets of a non empty set V and μ is symmetric fuzzy relation on σ

C. Definition 2.3

The graph G^* = (V,E) is called the underlying crisp graph of the fuzzy graph G = (σ, μ) where V = { u: σ(u) >0) = support of σ and E= {(u,v) ε V x V : μ (u,v) > 0} = support of μ

D. Definition 2.4

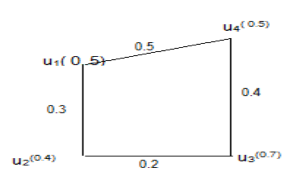
A fuzzy graph G: (σ, μ) is said to be strong fuzzy graph if $\mu(u,v)= \sigma(u) \wedge \sigma(v)$ if for all (u,v) ε E

E. Definition 2.5

A fuzzy graph G: (σ, μ) is said to be complete fuzzy graph if $\mu(u,v)= \sigma(u) \wedge \sigma(v)$ if for all u,v ε V

F. Definition 2.6

The degree of a vertex of G is denoted by $d_G(u)=\Sigma_{u\neq v}\ \mu(u,v)$. The minimum degree of G is δ(G)=min{ d(v)} and the maximum degree of G is Δ(G)=max{d(v)}
Example of fuzzy graph



Fuzzy graph (F1)

Consider G : (V,E) where V = {u_1,u_2,u_3,u_4 } and E = { u_1u_2 ,u_3u_4 , u_2,u_3 , u_4 u_1 }

G. Theorem 2.1:

In fuzzy graph $\Sigma d_G\sigma$ (u) =2 Σ μ (u,v) that is sum of degree of vertices of G is equal to twice the sum of degree of membership of all those edges which are incident on a vertex σ (u)

From figure F1 , d(σ (u_1))= μ (u_1,u_2) + μ (u_1,u_4) = 0.3+0.5 = 0.8

Like that d(σ (u_2)) = 0.5 , d(σ (u_3)) = 0.6 , d(σ (u_4)) = 0.9

We get $\Sigma d_G\sigma$ (u) = 0.8+0.5+0.6+0.9 = 2.8

Now Σ μ (u,v) = μ (u_1,u_2) + μ (u_2,u_3) + μ (u_2,u_4) + μ (u_4,u_1)

= 0.3+0.2+0.4+0.5

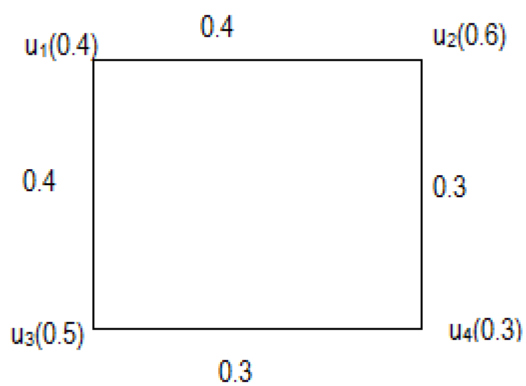
= 1.4

Then we get $\Sigma d_G\sigma$ (u) =2 Σ μ (u,v).

H. Theorem2.2

Every complete fuzzy graph is strong fuzzy graph but converse is not true

1) Proof: A fuzzy graph G: (σ, μ) is strong fuzzy graph if if μ (u,v)= σ(u) ^ σ(v) if for all (u,v) ε E and it is complete fuzzy graph if μ (u,v)= σ(u) ^ σ(v) if for all u,v ε V



Complete fuzzy graph (fig2)

E = { u_1u_2 ,u_2u_4 , u_4,u_3 , u_1 u_3 } ,V={ u_1,u_2 ,u_3,u_4 }

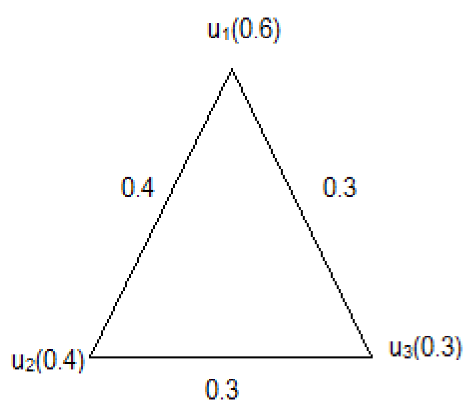
Here μ (u_1,u_2) = σ (u_1) ^ σ (u_2) = 0.4^0.6 = 0.4

μ (u_2,u_4) = σ (u_2) ^ σ (u_4) = 0.6^0.3 = 0.3

μ (u_3,u_4) = σ (u_3) ^ σ (u_4) = 0.5^0.3 = 0.3

μ (u_3,u_1) = σ (u_3) ^ σ (u_1) = 0.5^0.4 = 0.4

All edges satisfies the condition of complete graph i.e, μ (u,v) = σ(u) ^ σ(v) if for all (u,v) ε E



Strong fuzzy graph (fig 3)

E = { u_1u_2 ,u_2u_3 , u_3,u_1 } ,V={ u_1,u_2 ,u_3 }

Here $\mu(u_1,u_2) = \sigma(u_1) \wedge \sigma(u_2) = 0.6\wedge0.4 = 0.4$

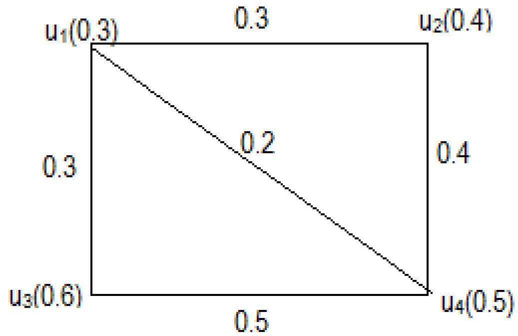
$\mu(u_2,u_3) = \sigma(u_2) \wedge \sigma(u_3) = 0.4\wedge0.3 = 0.3$

$\mu(u_3,u_1) = \sigma(u_3) \wedge \sigma(u_1) = 0.3\wedge0.6 = 0.3$

All vertices satisfies the condition of strong graph i.e, $\mu(u,v)= \sigma(u) \wedge \sigma(v)$ if for all u,v ε V

Now we compare fig 1 and fig 2, in complete fuzzy graph all elements of E satisfies the condition $\mu(u,v)= \sigma(u) \wedge \sigma(v)$ so clearly edge set E covers all the vertices therefore it automatically become strong fuzzy graph.

Conversely, Some time particular edges covers all vertices in that case given graph is satisfies only strong fuzzy graph but it does not satisfy complete fuzzy graph.



(Fig 4)

Given fuzzy graph satisfy strong fuzzy graph

i.e, $\mu(u_1,u_2) = \sigma(u_1) \wedge \sigma(u_2) = 0.3\wedge0.4 = 0.3$

$\mu(u_2,u_4) = \sigma(u_2) \wedge \sigma(u_4) = 0.3\wedge0.4 = 0.3$

$\mu(u_1,u_3) = \sigma(u_1) \wedge \sigma(u_3) = 0.3\wedge0.6= 0.3$

but it is not complete fuzzy graph (since $\mu(u_1,u_4) \neq \sigma(u_1) \wedge \sigma(u_4)$)

III. CONCLUSION

We have discussed the complete fuzzy graph and strong fuzzy graph. Finally every complete fuzzy graph is strong fuzzy graph but not conversely.

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