# Literature Review on Numerical Solution of Fractional Differential Equations

Sasikala R[1], Sujatha M[2], Subramani S[3]
[1,2,3]Assistant Professors, Department Of Mathematics K. S. Rangasamy College Of Arts & Science College, Tirunchengode

***Abstract: Finding a solution of fractional differential equation is a rising area of present trend research because such equations happen in different applied fields. This paper is focuses on review of various numerical methods which are used to find the solution of different types of fractional differential equations so far.***
***Keywords: Sequential fractional differential equation, Lane-Emden type of differential equations, Reproducing kernel method, caputo operator Jacobi polynomials, collocation methods.***

## I. INTRODUCTION

The fractional differential equations of various types plays important roles and tools not only in mathematics but also in physics, control systems, dynamical systems and engineering to create the mathematical modeling of many physical phenomena, the motion of a large thin plate in a Newtonian fluid, the process of cooling a semi-infinite body by radiation, the phenomena in electromagnetic acoustic visco-elasticity, electrochemistry and material science and so on.

These equations are more adequate for modeling physical and chemical process than integer-order differential equations. So far there have been several fundamental works on the fractional derivative and fractional differential equations. The Lane-Emden differential equations are important for mathematical modeling. The following five papers proposed how the authors obtained the numerical solution for such type of fractional differential equations.Before analyzed the five papers, we have go thrown Reference papers also. Deriving the iteration formula, proving its convergence to exact solution and studying the approximate solutions by using (VIM) are given in [1].

Derived the iteration formula for, ADM & DJIM for finding nonlinear sequential fractional differential equations for Caputo operator [2]. This result proposed a numerical solution for partial fractional differential equations [3]. Lane Emden type fractional differential equations solved by reproducing kernel method [4] This paper author proposed numerical solution of fractional differential equations by using fractional Taylor series.[5]

*A.* They found the solution of Fractional Order Integro-Differential Equations Using Variational Iteration Meth
*B.* They found Revised Variational Iteration Method for Solving System of Ordinary Differential Equations
*C.* They found, Variational Iteration Method for Solving Discrete KDV Equatio
*D.* They found of the Variational Iteration Method form Differential Equations to q-Fractional Difference Equations
*E.* They obtained solving a System of Nonlinear Fractional Differential Equations Using Adomian Decomposition,
*F.* They proved convergence of decomposition methods
*G.* They reviewed the decomposition method in applied mathematics,
*H.* They introduced Homotopy perturbation method for solving boundary problems.
*I.* A new homotopy perturbation method obtained for solving systems of partial differential equations. Computational Mathematical
*J.* Numerical inversion of Laplace transform: a survey and comparison of methods.
*K.* solution find inversion of Laplace transform.
*L.* They have given theory of reproducing kernels.
*M.* Nonlinear Numerical Analysis in the Reproducing Kernel Space.
*N.* Solving a nonlinear system of second order boundary value problems.
*O.* Method for solving nonlinear initial value problems by combining homotopy perturbation and reproducing kernel Hilbert space methods.
*P.* Nonlinear Numerical Analysis in the Reproducing Kernel Space

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## II. MAIN RESULTS

A. *Variational Iteration Method for Solving System of Fractional Order Ordinary Differential Equations,* $Nabaa\ N.Hasan's$, $Fahel\ S.Fadhel$.

These authors were proposed, approximate solution for system of linear and nonlinear fractional differential equations using variation itration method.

He considered the system of fractional differential equations

$D_*^{\alpha_i} X_i(t) = f_i(t, X_1, X_2, X_3, \ldots\ldots\ldots\ldots X_m)$,i=1,2……m,

m $\epsilon$ N, 0 $\leq$ $\alpha_i \leq 1$.------ (1.1)

Where $f_i$ continuous function is $D_*^{\alpha_i}$ is the Caputo derivative of $X_i$ of order$\alpha_i$, subject to initial condition

$X_1$(0) =$c_1$, $X_2$(0) =$c_2$, $X_3$(0) =$c_3$, ………., $X_m$(0) =$m$.

The variational iteration formula was derived from theorem.

1) *Theorem: 1:* Consider the system of fractional differential equations (1.1), then the relation variational iteration formula is

$$X_{i,n+1}(t) = X_{i,n}(t) - I^{\alpha_i}\{ D_*^{\alpha_i} X_{1,n}(t) - f_i\left(t, X_{1,n}(t)\right), X_{2,n}(t) - f_i\left(t, X_{m,n}(t)\right)\}$$

------- (1.2)

For $n^{th}$ approximation. For finding exact solution, they used to following the theorem.

2) *Theorem: 2 :* Let $X_i \in (c^2[0,T], \|\cdot\|_\infty)$ be the exact solution of the fractional differential equations (1.1) and let $X_{i,n} \in (c^2[0,T])$ be the obtained solution of the equation (1.2). If $E_{i,n}(t) = X_{i,n}(t) - X_i(t)$(t) and $f_i$ in equation (1.1) satisfies the Lipchitz condition with constants $L_i$, $such\ that\ L_i < \Gamma_{(\alpha_i)}$, then the sequence of approximations solution $X_{i,n}$, n=0,1,2, ……. , converges to exact solution $X_i$ this author, Considered the system of fractional differential equation and used variation iteration formula (1.2) for find the $X_{i,n}$ and finally, the sequence of approximations solution $X_{i,n}$ converges to$X_i$. Therefore, $X_i$ is the exact solution of equation (1.1). This method can apply for both linear & non-linear fractional differential equations.

B. *Numerical methods for sequential fractional differential equations for Caputo operator.* $G.B.Loghmani^1$ And $S.Javanmardi^2$.

This paper was contained find the solution of nonlinear sequential fractional differential equations for Captuo operator by used ADM and DJIM. Also, he has given efficiency of these methods. He used Riemann-Liouville fractional integral operator and fractional derivative in the Captuo sense. He considered sequential fractional differential equations of the general

$$\left.\begin{aligned} D_*^{\sigma}\, y(x) &= f(x,y) \\ D_* &\equiv D_*^{\alpha_n}, D_*^{\alpha_{n-1}}, \ldots\ldots\ldots D_*^{\alpha_1} \\ \sigma &= \sum_{j=1}^{n} a_j, \\ m-1 < \alpha_j &\leq m\ , (j = 1,2, \ldots\ldots n) \end{aligned}\right\} --- (2.1)$$

After the used the operator, he got

$$y(x) = \sum_{k=0}^{m-1} y^k(o)^{+} \frac{x^k}{k!} + D^{-\sigma}[f(x,y)]\text{------------------ (2.2)}$$

In ADM, f(x, y) decomposed into a series. i.e., adomain polynomial. In that process, the iteration are determined in recursive way

$$y_0(x) = \sum_{k=0}^{m-1} \frac{c_k}{k!} x^k ,$$

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$$y_{n+1}(x) = D^{-\sigma}[A_n]\ , n = 0,1,2, \dots.$$

DJIM equation (2.2) can be written as $y = f + N(y)$, $N: B \rightarrow B$ , $B$ is a Banach space. The solution $y = \sum_0^\infty y_n$ , N can be decomposed as

$$N(\sum_{n=0}^{\infty} y_n) = N\ (y_0) + \sum_{n=1}^{\infty} \left\{ \{ N \left\{ \sum_{j=0}^{n} y_j \right\} - N \left\{ \sum_{j=0}^{n-1} y_j \right\} \right\} .$$

We have iteration formula,

$$y_0(x) = c_0,$$

$$y_1(x) = D^{-\sigma}[f(x_0, y_0)] + \sum_{k=1}^{m-1} \frac{c_k}{x_k!} x^k,$$

$$y_{n+1}(x) = D^{-\sigma}[f(x, y_0 + y_1 + \cdots + y_n) - f(x, y_0 + y_1 + \cdots + y_{n-1}]\ ,$$

$$y = y(0) + \sum_{n=1}^{\infty} y_n.$$

Finally, they have given examples for find the efficiency of them two methods.

*C. Numerical solutions of fractional partial differential equations by numerical Laplace inversion technique.*
$[Mohammad\ Javidi^1\ and\ Bashir\ Ahmad^2]$

These authors were brought the homotopy perturbation method to solve the Laplace transform So, they were transformed, the partial fractional differential equations in terms of Laplace transform. After, to retrieve the time domain solution, they have used Stehfest's numerical algorithm for calculating inverse Laplace transform. They have taken homotopy perturbation method from few references for a general type of the non-linear differential equation with boundary condition. They have applied homotopy perturbation method, for linear fractional partial differential equation

$$\frac{\partial^\alpha u}{\partial t^\alpha} + A(x)\frac{\partial u}{\partial x} + B(x)\frac{\partial^2 u}{\partial x^2} + C(x)u = h(x,t), (x,t) \epsilon\ [0,1] \times [0,T]$$

With initial conditions

$$\left. \begin{array}{l} \frac{\partial^k u}{\partial t^k}(x,0) = f_k(x), \qquad k = 0,1, \dots \dots . m-1, \\ \qquad and\ the\ boundary\ conditions \\ u(0,t) = g_0(t), \qquad u(1,t) = g_1(t), \quad t \geq 0. \end{array} \right\} - - - - - (3.1)$$

Where $f_k$ , $k = 0, 1, 2, \dots \dots m-1, h, g_0, g_1,$ A and B are known and T > 0 is real number they have taken Laplace transform for equation (3.1), after construct homotopy,

$$\emptyset(x,s) = \frac{-p}{s^\alpha}\left[A(x)\frac{\partial}{\partial x} + B(x)\frac{\partial^2}{\partial x^2} + C(x)\right]\emptyset(x,s) + \frac{1}{s^m}\left[s^{m-1} f_0(x) + s^{m-2} f_1(x) + s^{m-3} f_2(x) + \dots + f_{m-1}(x) + \frac{1}{s^\alpha}\bar{h}(x,s)\right]$$

The solution is $\bar{\emptyset}(x,s) = \sum_{j=0}^{\infty} p^j\ \emptyset_j\ (x,s) - - - - - - (3.2)$

In that, they found the approximate solution of (3.1)

$$H_n(x,s) = \sum_{j=0}^{n} \emptyset_j(x,s) - - - - - - - - - - - - - (3.3).$$

The inverse Laplace transform

$$u(x,t) \cong u_n(x,t) = L^{-1}\ (H_n(x,s))$$

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They used Stehfest's algorithm to solve the inverse Laplace transform,

Finally, they have found

$u_n(x,t) = \frac{l_n(2)}{t} \sum_{j=1}^{2p} d_j\, H_n\left(x, j\frac{l_n(2)}{t}\right),$

Where P is a positive integer, and

$d_j = (-1)^{j+p} \sum_{i=\frac{j+1}{2}}^{\min(j,p)} \frac{i^p(2i)!}{(p-1)!\,i!\,(i-1)!\,(j-1)!\,(2i-j)!}$

*D. Numerical solutions of fractional differential equations of Lane- Emden type by an accurate technique.*

[*Ali akgul*[1] *Mustafa Inc*[2], *Esrakaratas*[3] *and Baleanu*[4,5]]

This paper, the authors discussed about, effectiveness of reproducing Kernal method (RKM) to solve fractional differential equations of Lane Emden type. The Lane- Emden type equation is,

$$\left.\begin{array}{l} D^\alpha y(t) + \frac{k}{t^{\alpha-\beta}} D^\beta y(t) + f(t,y) = g(t) \\ \quad \text{with the initial conditions}, \\ y(0) = A \qquad , \qquad y'(0) = B \end{array}\right\} - - - - - - - - (4.1)$$

Where $0 < t \le 1$, $k \ge 0$, $1 < \alpha \le 2$, $0 < \beta \le 1$. A, B are constants f (t, y) is a continuous real valued function and $g(t) \epsilon\, C[0,1]$.

*1) Definition:* The left and right Riemann – Liouville fractional derivatives of order $\alpha$ of h (t) are given as

$$_aD_t^\alpha\, h(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-v)^{n-\alpha-1}\, h(v)dv$$

And

$$_tD_b^\alpha\, h(t) = \frac{(-1)^n}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_t^b (v-t)^{n-\alpha-1}\, h(v)dv$$

Also they discussed about left & right Caputo fractional derivative.

Reproducing Kernel functions

Let $F \neq \varphi$. A function R: F×F→ C is called a reproducing Kernel function Hilbert space H iff

R (.v) $\epsilon$ H for all v $\epsilon$ F.

$\langle Q, R(\cdot,\gamma)\rangle = Q(v)$ For all $v\,\epsilon\, F$ and all Q $\epsilon\, H$.

*2) These authors ordered this paper like :* The reproducing Kernel function

$$v_y(t) = \begin{cases} \frac{1}{4} y^2t^2 + \frac{1}{12} y^2t^3\, 0 - \frac{1}{24} yt^4 + \frac{1}{120} t^5 , & 0 \le t \le y \le 1 \\ \frac{1}{4} y^2t^2 + \frac{1}{12} y^2t^3\, 0 - \frac{1}{24} yt^4 + \frac{1}{120} t^5 , & 0 \le y < t \le 1 \end{cases}$$

Of the reproducing Kernel Hilbert space. $^\circ\bar{w}_2^3\,[0,1]$.

Like A$\zeta = D^\alpha \zeta(t) + \frac{k}{t^{\alpha-\beta}} D^\beta \zeta(t)$, $\zeta\ \epsilon\ ^\circ\bar{\omega}_2^3\,[0,1]$. Where A is linear operator.

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A is bounded linear operator

They obtained the exact solution $\zeta$ In the way, $\zeta_n(t)$ converges to $\zeta(t)$

Also they have given numerical experiments for the above type equations using RKM and Collocation method

E. *Numerical solutions of fractional differential equations by using fractional Taylor basis.*

*Vidhya Saraswathy Krishnasamy, Somayeh Mashayekhi and Mohsen Razzaghi. l*

They have taken, properties of Riemann Liouville fractional integral operator's Caputo derivative and fractional Taylor basis.

$$(i) I^{\alpha} t^{\alpha} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\alpha+1)} t^{\gamma+\alpha}, \alpha \geq 0;\ \gamma > -1$$

$$(ii) I^{\alpha} I^{\beta} y(t) = I^{\beta} I^{\alpha} y(t) = I^{\alpha+\beta} y(t), \quad \alpha, \beta > 0$$

$$(iii) D^{\alpha} y(t) = I^{n-\alpha}\left(\frac{d^n}{dt^n} y(t)\right), n-1 < \alpha \leq n, n \in \mathbb{N}$$

$$(iv) I^{\alpha}\left(D^{\alpha} y(t)\right) = y(t) - \sum_{k=0}^{n-1} y^k(0) \frac{t^k}{k!}$$

$$(v) T_{m\gamma}(t) = [1, t^{\gamma}, t^{2\gamma}, \dots\dots, t^{m\gamma}]^T$$

Also they discussed about function approximation and Error bound for the best approximation. They derived the fractional Taylor operational matrix of the fractional integration

$$I^{\alpha}\left(T_{m\gamma}(t)\right) = t^{\alpha} G_{\alpha} * T_{m\gamma}(t)$$

They used the property,

$$I^{\alpha}\left(T_{m\gamma}(t)\, T_{m\gamma}^T(t)\right) = t^{\alpha} S_{\alpha} * (T_{m\gamma}(t) T_{m\gamma}^T(t))$$

$$\text{i.e., } I^{\alpha}\left(T_{m\gamma}(t)\, T_{m\gamma}^T(t)\right) = \begin{bmatrix} \frac{1}{\Gamma(\alpha+1)} t^{\alpha} & \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\alpha+1)} t^{\gamma+\alpha} & \dots\dots & \frac{\Gamma(m\gamma+1)}{\Gamma(m\gamma+\alpha+1)} t^{m\gamma+\alpha} \\ \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+\alpha+1)} t^{\gamma+\alpha} & \frac{\Gamma(2\gamma+1)}{\Gamma(2\gamma+\alpha+1)} t^{2\gamma+\alpha} & \dots\dots & \frac{\Gamma(m+1)\gamma+1)}{\Gamma((m+1)\gamma+\alpha+1)} t^{(m+1)\gamma+\alpha} \\ \vdots & \vdots & & \vdots \\ \frac{\Gamma(m\gamma+1)}{\Gamma(m\gamma+\alpha+1)} t^{m\gamma+\alpha} & \frac{\Gamma(m+1)(\gamma+1)}{\Gamma((m+1)\gamma+\alpha+1)} t^{(m+1)\gamma+\alpha} & \dots\dots & \frac{\Gamma(2m\gamma+1)}{\Gamma(2m\gamma+\alpha+1)} t^{2m\gamma+\alpha} \end{bmatrix}$$

From that making system of linear equation. These system of equations are solved by many methods, these authors were used Newton's iterative methods. So, the fractional differential equations was changed to fractional integration after obtained the operational matrix, that matrix were solved by numerical iterative methods. Also they have given illustrations.

## II. CONCLUSION

I start my research in the field of fractional differential equations some type of fractional differential equations have not analytic solution, for that the author have introduced the approximate solution for such type of equations. For finding approximate solution, they were used the various numerical methods. In this work, we study such type of equation were solved by numerical methods. I am going try, other numerical methods is applicable or not for instead of reproducing Kernel Methods.

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