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Contra (i,j) (gsp)* - continuous Function in Bitopological Space

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Abstract: In this paper we have introduced a new function of contra (i,j) (gsp)*-continuous in bitopological spaces which is properly placed in between the class of closed sets and gsp-closed sets.

Key Words: contra (i,j) g-continuous, contra (i,j) gs-continuous, α continuous, contra (i,j) gsp-continuous.

I. INTRODUCTION

A triple (X, τ_i, τ_j) where X is a non-empty set and τ_i and τ_j are topologies on X is called a bitopological space." Kelly [20] introduced study of such spaces. In 1985, Fukutake [19] introduced the concepts of g-closed sets [10] in bitopological spaces. In the year 1994, Maki et.al [12] defined αg -closed sets in topological space. S.P. Arya and N. Tour [3] defined gs -closed sets in 1990. Dontchev [8], Gnanambal [9] and Palaniappan and Rao [17] introduced gsp-closed sets. J.Dontchev [8] introduced gsp-closed sets in 1995. Levine [10] Devi et.al is. In this paper the new function contra (i,j) (gsp)*-continuous function is introduced. The concepts contra (i,j) g-continuous, contra (i,j) gs-continuous, contra (i,j) αg -continuous, contra (i,j) gsp-continuous are defined few of their properties are studied.

II. PRELIMINARIES

A. Definition 2.1

"A subset A of topological space (X, τ_i, τ_j) is called

- 1) a pre-open set [14] if $A \subseteq \text{int}(cl(A))$ and a pre-closed set if $cl(\text{int}(A)) \subseteq A$
- 2) a semi-open set [11] if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if
- 3) a semi-pre open set [1] if $A \subseteq cl(\text{int}(cl(A)))$ and a semi-pre closed set [1] if
- 4) an α -open set [15] if $A \subseteq \text{int}(cl(\text{int}(A)))$ and an α -closed set [15] if
- 5) *regular-open*_{set [14]} if $\text{int}(cl(A)) = A$ and an *regular-closed*_{set [14]} if $A = \text{int}(cl(A))$ "

B. Definition 2.2

"A subset A of topological space (X, τ_i, τ_j) is called

- 1) a generalized closed set (briefly (i,j) g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in
- 2) generalized semi-closed set (briefly (i,j) gs-closed) [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .
- 3) an α -generalized closed set (briefly (i,j) αg -closed) [12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .
- 4) a generalized semi pre-closed set (briefly (i,j) gsp-closed) [8] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in τ_i .

C. Definition 2.3

"A function $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is called

- 1) Contra (i,j) g-continuous [4] if $f^{-1}(V)$ is a g-closed set of (X, τ_i, τ_j) for every closed set V of (Y, σ_i, σ_j) .
- 2) Contra (i,j) αg -continuous [9] if $f^{-1}(V)$ is a αg -closed set of (X, τ_i, τ_j) for every closed set V of (Y, σ_i, σ_j) .
- 3) Contra (i,j) gs-continuous [7] if $f^{-1}(V)$ is a gs-closed set of (X, τ_i, τ_j) for every closed set V of (Y, σ_i, σ_j) .
- 4) contra (i,j) gsp-continuous [8] if $f^{-1}(V)$ is a gsp-closed set of (X, τ_i, τ_j) for every closed set V of (Y, σ_i, σ_j) .

D. Definition 3

A function $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ is called contra (i,j) (gsp)*-continuous if $f^{-1}(v)$ is (i,j) (gsp)*-closed in (X, τ_i, τ_j) for each open set v of (Y, σ_i, σ_j) .

E. Theorem 3.1

Every contra continuous function is contra (i,j) (gsp)*-continuous

Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a contra continuous map

Let v be any open set in (Y, σ_i, σ_j) .

Then the inverse image of $f^{-1}(v)$ is (i,j) (gsp)*- closed in (X, τ_i, τ_j) .

Since every closed set is (i,j) (gsp)*- closed.

$f^{-1}(v)$ is (i,j) (gsp)*- closed in (X, τ_i, τ_j) .

Therefore f is contra (i,j) (gsp)*-continuous.

Converse is not true.

F. Theorem 3.2

Every contra (i,j) (gsp)*-continuous map is contra (i,j) g-continuous. But the converse is not true.

1) Proof: Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a contra (i,j) (gsp)* continuous map.

Let v be any open set in (Y, σ_i, σ_j) .

Then the inverse image of $f^{-1}(v)$ is (i,j) (gsp)*- closed in (X, τ_i, τ_j) .

Since every (i,j) (gsp)*- closed set is (i,j) g-closed.

$f^{-1}(v)$ is (i,j) g-closed in (X, τ_i, τ_j) .

Therefore f is contra (i,j) g-continuous.

G. Example 3.3

Let $X = \{a, b, c\} = Y$, $\tau_i = \{\emptyset, X, \{c\}, \{a, c\}\}$, $\tau_j = \{\emptyset, X, \{b\}, \{a, b\}\}$, $\sigma_i = \emptyset, Y, \{a, c\}$,

$\sigma_j = \emptyset, Y, \{b, c\}$.

Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be the identity map.

Let us prove that f is contra (i,j) g- continuous. But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) g-closed sets are all the subsets of X .

And the (i,j) (gsp)*- closed sets are $\emptyset, X, \{c\}, \{a, b\}, \{a, c\}$.

$f^{-1}\{a\} = \{a\}$ is (i,j) g-closed in (X, τ_i, τ_j) .

But it is not (i,j) (gsp)*- closed in (X, τ_i, τ_j) .

Hence f is contra (i,j) g- continuous but not (i,j) (gsp)*-continuous.

G. Theorem 3.4

Every contra (i,j) (gsp)*-continuous map is contra (i,j) gs-continuous

1) Proof: Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a contra (i,j) (gsp)* continuous map.

Let v be any open set in (Y, σ_i, σ_j) .

Then the inverse image of $f^{-1}(v)$ is (i,j) (gsp)*- closed in (X, τ_i, τ_j) .

Since every (i,j) (gsp)*- closed set is (i,j) gs-closed.

$f^{-1}(v)$ is (i,j) gs-closed in (X, τ_i, τ_j) .

Therefore f is contra (i,j) gs-continuous.

H. Example 3.5

Let $X = \{a, b, c\} = Y$, $\tau_i = \{\emptyset, X, \{c\}, \{a, c\}\}$, $\tau_j = \{\emptyset, X, \{b\}, \{a, b\}\}$, $\sigma_i = \emptyset, Y, \{a, c\}$,

$\sigma_j = \emptyset, Y, \{b, c\}$.

Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be the identity map.

Let us prove that f is contra (i,j) gs- continuous. But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) gs-closed sets are all the subsets of X .

And the (i,j) (gsp)*- closed sets are $\emptyset, X, \{c\}, \{a, b\}, \{a, c\}$.

$f^{-1}\{a\} = \{a\}$ is (i,j) gs-closed in (X, τ_i, τ_j) .

But it is not (i,j) (gsp)*-closed in (X, τ_i, τ_j) .

Hence f is contra (i,j) gs-continuous but not (i,j) (gsp)*-continuous.

I. Theorem 3.6

Every contra (i,j) (gsp)*-continuous map is contra (i,j) α g-continuous.

1) Proof: Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be a contra (i,j) (gsp)* continuous map.

Let v be any open set in (Y, σ_i, σ_j) .

Then the inverse image of $f^{-1}(v)$ is (i,j) (gsp)*-closed in (X, τ_i, τ_j) .

Since every (i,j) (gsp)*-closed set is (i,j) α g-closed.

$f^{-1}(v)$ is (i,j) α g-closed in (X, τ_i, τ_j) .

Therefore f is contra (i,j) α g-continuous.

J. Example 3.7

Let $X = \{a, b, c\} = Y$, $\tau_i = \{\varphi, X, \{c\}, \{a, c\}\}$, $\tau_j = \{\varphi, X, \{b\}, \{a, b\}\}$, $\sigma_i = \varphi, Y, \{a, c\}$,

$\sigma_j = \varphi, Y, \{a, c\}$.

Let $f: (X, \tau_i, \tau_j) \rightarrow (Y, \sigma_i, \sigma_j)$ be the identity map.

Let us prove that f is contra (i,j) α g-continuous. But not contra (i,j) (gsp)*-continuous.

We have proved that the (i,j) α g-closed sets are all the subsets of X .

And the (i,j) (gsp)*-closed sets are $\varphi, X, \{c\}, \{a, b\}, \{a, c\}$.

$f^{-1}\{b\} = \{b\}$ is (i,j) α g-closed in (X, τ_i, τ_j) .

But it is not (i,j) (gsp)*-closed in (X, τ_i, τ_j) .

Hence f is contra (i,j) α g-continuous but not (i,j) (gsp)*-continuous.

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