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International Journal For Research in  
Applied Science and Engineering Technology



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# **INTERNATIONAL JOURNAL FOR RESEARCH**

IN APPLIED SCIENCE & ENGINEERING TECHNOLOGY

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**Volume:** TPAM-2018-03 **Issue:** conference **Month of publication:** March 2018

**DOI:**

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# Contra (gsp)\*-Continuous Function in Topological Spaces

Kalpana.M<sup>1</sup>,

<sup>1</sup>Assistant Professor, St.Joseph's College Of Arts And Science College For Women, Hosur.

**Abstract:** In this paper we have introduced a new function of contra (gsp)\*-continuous in topological spaces which is properly placed in between the class of closed sets and gsp-closed sets.

**Key Words:** contra g-continuous, contra gs-continuous,  $\alpha$ g-continuous, contra rg-continuous.

## I. INTRODUCTION

Levine [10] introduced the class of  $g$ -closed sets in 1970. Maki.et.al [12] defined  $\alpha g$ -closed sets in 1994. Arya and Tour [3] defined  $gs$ -closed sets in 1990. Dontchev [8], Gnanambal [9] Palaniappan and Rao[17] introduced gsp-closed set,  $rg$ -closed sets respectively. Veerakumar [18] introduced  $g^*$ -closed sets in 1991. J.Dontchev [8] introduced gsp-closed sets in 1995. The purpose of this paper is to introduce the concepts contra (gsp)\*-continuous function.

## II. PRELIMINARIES

A. **Definition 2.1:** A subset  $A$  of topological space  $(X, \tau)$  is called

- 1) a pre-open set [14] if  $A \subseteq \text{int}(cl(A))$  and a pre-closed set if  $cl(\text{int}(A)) \subseteq A$
- 2) a semi-open set [11] if  $A \subseteq cl(\text{int}(A))$  and a semi-closed set if  $\text{int}(cl(A)) \subseteq A$
- 3) a semi-preopen set [1] if  $A \subseteq cl(\text{int}(cl(A)))$  and a semi-preclosed set [1] if
- 4) an  $\alpha$ -open set [15] if  $A \subseteq \text{int}(cl(\text{int}(A)))$  and an  $\alpha$ -closed set [15] if  $cl(\text{int}(cl(A))) \subseteq A$
- 5) a *regular-open* set [14] if  $\text{int}cl(A)=A$  and an *regular-closed* set [14] if  $A = \text{int}cl(A)$

B. **Definition 2.2:** A subset  $A$  of topological space  $(X, \tau)$  is called

- 1) a generalized closed set (briefly  $g$ -closed) [10] if  $cl(A) \subseteq U$  whenever  $A \subseteq U$
- 2) generalized semi-closed set (briefly  $gs$ -closed) [3] if  $scl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
- 3) an  $\alpha$ -generalized closed
- 4) set (briefly  $\alpha g$ -closed) [12] if  $\alpha cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$
- 5) a regular generalized closed set (briefly  $rg$ -closed) [17] if  $spcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is regular open in  $(X, \tau)$

C. **Definition 2.3:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

- 1) Contra  $g$ -continuous [4] if  $f^{-1}(v)$  is a  $g$ -closed set of  $(x, \tau)$  for every open set  $v$  of  $(y, \sigma)$
- 2) Contra  $\alpha g$ -continuous [9] if  $f^{-1}(v)$  is a  $\alpha g$ -closed set of  $(x, \tau)$  for every open set  $v$  of  $(y, \sigma)$
- 3) Contra  $gs$ -continuous [7] if  $f^{-1}(v)$  is a  $gs$ -closed set of  $(x, \tau)$  for every open set  $v$  of  $(y, \sigma)$
- 4) Contra  $rg$ -continuous [17] if  $f^{-1}(v)$  is a  $rg$ -closed set of  $(x, \tau)$  for every open set  $v$  of  $(y, \sigma)$

D. **Definition 2.4 :** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  from a topological space  $X$  into a topological space  $Y$  is said to be contra – continuous if  $f^{-1}(V)$  is closed in  $X$  for each open set  $V$  of  $Y$ .

E. *Definition 2.5:* A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $\text{contra}(\text{gsp})^*$ -continuous if  $f^{-1}(V)$  is  $(\text{gsp})^*$ -closed in  $(X, \tau)$  for each open set  $V$  of  $(Y, \sigma)$ .

1) *Theorem:* Let a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a map, where  $(X, \tau)$ ,  $(Y, \sigma)$  are spaces then  $f$  is  $\text{contra}(\text{gsp})^*$ -continuous iff the inverse image of every closed subset of  $(Y, \sigma)$  is  $\text{gsp}$ -open in  $(X, \tau)$

2) *Proof*

Let  $F$  be a closed subset in  $(Y, \sigma)$ .

Then  $Y-F$  is open in  $(Y, \sigma)$

Since  $f$  is  $\text{contra}(\text{gsp})^*$ -continuous,  $f^{-1}(Y-F)$  is  $(\text{gsp})^*$ -closed.

But  $f^{-1}(Y-F) = X - f^{-1}(F)$

Thus  $f^{-1}(F)$  is  $\text{gsp}$  open in  $(X, \tau)$

Conversely,

Let  $G$  be an open subset in  $(Y, \sigma)$

Then  $Y-G$  is closed in  $(Y, \sigma)$

Since the inverse image of every closed subset in  $(Y, \sigma)$  is  $\text{gsp}$ -open in  $(X, \tau)$

$f^{-1}(Y-G)$  is  $\text{gsp}$  open in  $(X, \tau)$

But  $f^{-1}(Y-G) = X - f^{-1}(G)$

Thus  $f^{-1}(G)$  is  $(\text{gsp})^*$ -closed.

Therefore  $f$  is  $\text{contra}(\text{gsp})^*$ -continuous.

F. *Theorem:* Every  $\text{contra}$ -continuous function is  $\text{contra}(\text{gsp})^*$ -continuous.

1) *Proof:* Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be  $\text{contra}$ -continuous.

Let  $V$  be any open set in  $Y$ .

Then the inverse image  $f^{-1}(V)$  is closed in  $X$ .

Since every closed set is  $(\text{gsp})^*$ -closed.

$f^{-1}(V)$  is  $(\text{gsp})^*$ -closed in  $X$ .

Therefore  $f$  is  $\text{contra}(\text{gsp})^*$ -continuous.

G. *Theorem:* Every  $\text{contra}(\text{gsp})^*$ -continuous map is  $\text{contra g}$ -continuous.

But the converse is not true.

1) *Proof*

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be  $\text{contra}(\text{gsp})^*$ -continuous.

Let  $V$  be any open set in  $Y$ .

Then the inverse image  $f^{-1}(V)$  is  $(\text{gsp})^*$ -closed in  $X$ .

Since every  $\text{gsp}^*$ -closed set is  $\text{g}$ -closed.  $f^{-1}(V)$  is  $\text{g}$ -closed in  $X$ . hence  $f$  is  $\text{contra g}$ -continuous.

2) *Example*

Let  $X=Y= \{a, b, c\}$

$\tau= \{\emptyset, X, \{a\}, \{a,b\}\}$

$\sigma= \{\emptyset, Y, \{b\}\}$  Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity map. Let us prove that  $f$  is  $\text{contra g}$ -continuous.

But not  $\text{contra}(\text{gsp})^*$ -continuous.

We have proved that the  $\text{g}$ -closed sets are

$\emptyset, X, \{c\}, \{a,c\}, \{b,c\}$

And the  $\text{gsp}^*$ -closed sets are

$\emptyset, X, \{c\}, \{b,c\}$

$f^{-1}(\{a,c\}) = \{a,c\}$  is  $\text{g}$ -closed in  $(X, \tau)$

Thus the inverse of every closed set of  $(Y, \sigma)$  is  $\text{g}$ -closed in  $(X, \tau)$  but not

$(\text{gsp})^*$ -closed in  $(X, \tau)$ .

Hence  $f$  is  $\text{contra g}$ -continuous but not  $\text{contra}(\text{gsp})^*$ -continuous.

H. *Theorem:* Every  $\text{contra}(\text{gsp})^*$ -continuous map is  $\text{contra } \alpha \text{g}$ -continuous.

1) *Proof*

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be  $\text{contra}(\text{gsp})^*$ -continuous.

To prove that,  $f$  is contra  $\alpha g$  -continuous.

$V$  be any open set in  $Y$ . Then the inverse image  $f^{-1}(V)$  is  $(gsp)^*$ -closed in  $X$ . Since every  $gsp^*$ -closed set is  $\alpha g$  -closed.  $f^{-1}(V)$  is  $\alpha g$  -closed in  $X$ .

Therefore  $f$  is contra  $\alpha g$  -continuous.

## 2) Example

Let  $X=Y= \{a, b, c\}$

$\tau= \{\varphi, X, \{a\}, \{a,b\}\}$

$\sigma = \{\emptyset, Y, \{b\}\}$

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be the identity map.

Let us prove that  $f$  is contra  $\alpha g$  -continuous

But not contra  $(gsp)^*$ -continuous We have proved that the  $\alpha g$  -closed sets are  $\varphi, X, \{b\}, \{c\}, \{a,c\}, \{b,c\}$

$f^{-1}(\{a,c\}) = \{a,c\}$  is  $\alpha g$  -closed in  $(X, \tau)$  but not  $(gsp)^*$ -closed in  $(X, \tau)$ . Hence  $f$  is contra  $\alpha g$ -continuous but not contra  $(gsp)^*$ -continuous.

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