Contra (gsp)*-Continuous Function in Topological Spaces

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Abstract: In this paper we have introduced a new function of contra $(gsp)^*$-continuous in topological spaces which is properly placed in between the class of closed sets and gsp-closed sets.
Key Words: contra g-continuous, contra gs-continuous,αg-continuous,contra rg-continuous.

I. INTRODUCTION

Levine [10] introduced the class of g -closed sets in 1970. Maki.et.al [12] defined αg -closed sets in 1994. Arya and Tour [3] defined gs -closed sets in 1990. Dontchev [8], Gnanambal [9] Palaniappan and Rao[17] introduced gsp-closed set, rg -closed sets respectively.Veerakumar [18] introduced g^* -closed sets in 1991.J.Dontchev [8] introduced gsp-closed sets in 1995..The purpose of this paper is to introduce the concepts contra $(gsp)^*$-continuous function.

II. PRELIMINARIES

A. *Definition 2.1:* A subset A of topological space (X,τ) is called

1) a pre-open set[14] if $A \subseteq \text{int}(cl(A))$ and a pre-closed set if cl(int(A))$\subseteq A$
2) a semi-open set [11] if $A \subseteq cl(\text{int}(A))$ and a semi-closed set if $\text{int}(cl(A)) \subseteq A$
3) a semi-preopen set[1] if $A \subseteq cl(\text{int}(cl(A))$ and a semi-preclosed set[1] if
4) an α-open set [15] if $A \subseteq \text{int}(cl(\text{int}(A)))$ and an α-closed set [15] if $cl(\text{int}(cl(A))) \subseteq A$
5) a $regular-open$ set[14] if intcl(A)=A and an $regular-closed$ set[14] if A= intcl(A)

B. *Definition 2.2:* A subset A of topological space (X,τ) is called

1) a generalized closed set (briefly g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$
2) generalized semi-closed set(briefly) gs-closed [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).
3) an α- generalized closed
4) set (briefly αg-closed) [12] if $\alpha\, cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ)
5) a regular generalized closed set (briefly rg-closed) [17] if sp$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X,τ)

C. *Definition 2.3:*A function f: (X,τ) $\rightarrow$ (Y,σ) is called

1) Contra g-continuous [4] if f^{-1}(v) is a g-closed set of (x,τ) for every open set v of (y,σ)
2) Contra αg-continuous[9] if f^{-1}(v) is a αg-closed set of (x,τ) forevery open set v of (y,σ)
3) Contra gs-continuous [7] if f^{-1}(v) is a gs-closed set of (x,τ) for every open set v of (y,σ)
4) Contra rg-continuous [17] if f^{-1}(v) is a rg-closed set of (x,τ) for every open set v of (y,σ)

D. *Definition 2.4 :* A function *f*: (X,τ) → (Y,σ) from a topological space X into a topological space Y is said to be contra –continuous if f^{-1}(V) is closed in X for each open set V of Y.

E. Definition 2.5: A function *f*: $(X,\tau) \to (Y,\sigma)$ is called contra(gsp)*-continuous if $f^{-1}(V)$ is (gsp)*-closed in (X,τ) for each open set V of (Y,σ).

1) Theorem: Let a function *f*: $(X,\tau) \to (Y,\sigma)$ be a map, where (X,τ), (Y,σ) are spaces then f is contra(gsp)*-continuous iff the inverse image of every closed subset of (Y,σ) is gsp-open in $(X,\tau$

2) Proof

Let F be a closed subset in (Y,σ).
Then Y-F is open in (Y,σ)
Since f is contra(gsp)*-continuous, f^{-1}(Y-F) is (gsp)*-closed.
But f^{-1}(Y-F)= X-f^{-1}(F)
Thus f^{-1}(F) is gsp open in (X,τ)
Conversely,
Let G be an open subset in (Y,σ)
Then Y-G is closed in (Y,σ)
Since the inverse image of every closed subset in (Y,σ) is gsp-open in(X,τ)
f^{-1}(Y-G) is gsp open in (X,τ)
But f^{-1}(Y-G)= X-f^{-1}(G)
Thus f^{-1}(G) is (gsp)*-closed.
Therefore *f* is contra(gsp)*-continuous.

F. Theorem: Every contra-continuous function is contra(gsp)*-continuous.

1) Proof: Let *f*: $(X,\tau) \to (Y,\sigma)$ be contra-continuous.
Let V be any open set in Y.
Then the inverse image f^{-1}(V) is closed in X.
Since every closed set is (gsp)*-closed.
f^{-1}(V) is (gsp)*-closed in X.
Therefore f is contra(gsp)*-continuous.

G. Theorem: Every contra(gsp)*-continuous map is contra g-continuous.
But the converse is not true.

1) Proof

Let *f*: $(X,\tau) \to (Y,\sigma)$ be contra(gsp)*-continuous.
Let V be any open set in Y.
Then the inverse image f^{-1}(V) is (gsp)*-closed in X.
Since every gsp*-closed set is g-closed. f^{-1}(V) is g-closed in X. herefore f is contra g-continuous.

2) Example

Let X=Y= {a, b, c}
τ= {φ, X, {a},{a,b}}
σ ={ ϕ,Y,{b}} Let *f*: $(X,\tau) \to (Y,\sigma)$ be the identity map. Let us prove that f is contra g-continuous.
But not contra(gsp)*-continuous.
We have proved that the g-closed sets are
φ, X, {c},{a,c}, {b,c}
And the gsp*-closed sets are
φ, X, {c}, {b,c}
f^{-1}({a,c})={a,c} is g-closed in (X,τ)
Thus the inverse of every closed set of (Y,σ) is g-closed in (X,τ) but not (gsp)*-closed in (X,τ).
Hence f is contra g-continuous but not contra(gsp)*-continuous.

H. Theorem: Every contra(gsp)*-continuous map is contra αg-continuous.

1) Proof

Let *f*: $(X,\tau) \to (Y,\sigma)$ be contra(gsp)*-continuous.

To prove that, *f* is contra αg -continuous.
V be any open set in Y. Then the inverse image f^{-1}(V) is (gsp)*-closed in X. Since every gsp*-closed set is αg -closed.
f^{-1}(V) is αg -closed in X.
Therefore *f* is contra αg -continuous.

2) Example

Let X=Y= {a, b, c}
τ= {φ, X, {a},{a,b}}
σ ={ ϕ,Y,{b}}
Let *f*: (X,τ) →(Y,σ) be the identity map.
Let us prove that f is contra αg –continuous
But not contra(gsp)*-continuous We have proved that the αg -closed sets are φ, X, {b}, {c},{a,c}, {b,c}
φ, X,{c}, {b,c}
f^{-1}({a,c})={a,c} is αg -closed in (X,τ) but not (gsp)*-closed in (X,τ). Hence *f* is contra αg-continuous but not contra(gsp)*-continuous.

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