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A Study on Decomposition of graphs

M.Sujatha[1], R. Sasikala[2], R. Mathivanan[3]
[1,2,3]Pg Department Of Mathematics, K.S.Rangasamy College Of Arts And Science(Autonomous)

Abstract: Decomposition of graphs is one of the prominent areas of research in graph theory and Combinatorial Design Theory. Various types of Decomposition have been suggested by different authors. This paper is a vast study on the decomposition of graphs. It mainly focuses the results related to decomposition of various graphs published so for.
***Keywords*: Decomposition of graphs , Product of graphs, Star, Path, Hamilton Cycle.**

I. INTRODUCTION

Graph theory is a major part of Discrete Mathematics. Among the many aspects of graph theory, graph decomposition is a vast area of research. Recently a number of survey articles and several papers on decomposition of graphs have been published. This paper focuses more closely on a few specific topics and result, for which significant progress has been made within few years. In this paper, we have given various problems. Theorems and Conjectures of different authors which will be useful for clear understanding of this topic.

A. Basic Definitions

A *graph* G is an ordered triple(V(G), E(G), ψ(G)) consisting of a non-empty set V(G) of vertices, a set E(G) disjoint from V(G) of edges and an incidence function ψ_G that associates with each edge of G an unordered pair of vertices of G.A simple graph in which each pair of distinct vertices is joined by an edge is called a complete graph.Let K_n be a complete graph of order n and let λ be a positive integer. We denote by λK_n the complete multi graph obtained by replacing each edge of K_n by λ parallel edges that have the same end nodes. A walk with distinct vertices is called a Path.
A closed trail whose origin and internal vertices are distinct is a Cycle. A Star S_k is the complete bipartite graph $K_{1,k}$. Partition of G into edge-disjoint subgraphs $G_1,G_2,\ldots,G_r$ such that $E(G)=E(G_1)\cup E(G_2)\cup E(G_3)\cup\ldots E(G_r)$ is called decomposition of G and we write $G=G_1\cup G_2\cup\ldots\cup G_r$. In particular, if each $G_i\cong H$, $1\leq i\leq r$, then we say that G has a H-decomposition and we denote it by H|G.

II.REVIEW OF LITERATURE

Excellent surveys of graph decompositions and factorizations are provided by Akiyama and kano[1,2]. As well as book on graph decompositions by Bosk[3] provide informations about decompositions and factrorizations. Various decompositions are available such as Cycle decomposition, Path decomposition, Star decomposition, Clique decomposition and so on.

A. Decomposition of Complete graphs

Abueida and Devan [4] gave necessary and sufficient conditions for decomposing K_n into cycles of kedges and stars k-1 edges for k=4 & k=5.Abueida and O'neil [5] extended this decomposition for the complete multigraph λK_n when k=3,4,5 and they conjectured the result for any integer $k\geq 3$ and $n\geq k$. In [6], Priyadharsini and Muthusamy showed the above conjecture to be true for n=k. More recently Abueida and Lian [7] gave necessary and sufficient conditions for decomposing K_n into cycles and stars of k-edges, for $n\geq 4k$ and k even or n odd.Shyu [9] proved that K_n can be decomposed into p copies P_{k+1} and q copies of S_{k+1} when $n\geq 4k$, $k(p+q) = nc_2$ and either k is even and $p\geq k/2$ or k is odd and $p\geq k$. In[10] Shyu also suggested some results do decompose K_n into paths & cycles. He investigated the necessary and sufficient conditions for decomposing K_n int p copies of P_5 and q copies of C_4 for $p\geq 0$ and $q\geq 0$. In 1967, A.Rosa introduce some important types of vertex labelling. Graceful labeling & rosy labelling are useful tools for decompositions of complete graphs K_{2n+1} into graphs with n edges. A.Rosa proved that if a graph G with n edges has a graceful or rosy labelling then K_{2n+1} can be cyclically decomposed into 2n+1 copies of G.Dalibor Froncek[26] gave generalization of Rosa's result. Rosa's result is given in the following theorem. "Let G be a graph with n edges. If G allows a rosy labelling, then it decomposes K_{2n+1}, if G allows an α-labelling, then it decomposes K_{2np+1} for every p>o." Dalibor Froncek [26] showed that every bipartite graph H which decomposes K_k and K_m also decomposes K_{km}.

B. Cycle decomposition of Complete Multipartite Graphs

A complete m-partite graph with part sizes $k_1,k_2,\ldots k_m$is denoted by $K_{k_1,k_2,\ldots,k_m}$.The general problem of determining necessary and sufficient conditions for the existence of a C_k-decomposition of a tripartite graph $K_{r,s,t}$ remains open when k>3 is odd. The decomposition of a complete tripartite graph into cycles of edges 5 was first investigated by Mahmoodian and Mirzakhani[11]. Cavenagh[12] proved the necessary and sufficient conditions for the existence of a decomposition of the complete tripartite graphs $K_{r,s,t}$ into C_5-decomposition. He proved that r,s&t all must be even. Cavenagh[13] has also given the necessary and sufficient conditions for the C_k decomposition of $K_3*\overline{K_{n.}}$

Muthusamy and Paulraja[14] have investigated the factorizations of product graphs into cycles of uniform length. Billington[15,16] established some results. He has proved that C_k-decomposition of $K_4*\overline{K_{n.}}$ exists if and only if $k\geqq 3$, n is even, $k|6n^2$ and $k\leq 4n$. Also they proved that C_k-decomposition of $K_5*\overline{K_{n.}}$ exists only if $k\geqq 3$, $k|10n^2$ and $k\leq 5n$. C_7-decomposition and C_p-decomposition of $K_m*\overline{K_{n.}}$ was studied by Manikandan and Paulraja[17,18]. They have shown that the necessary conditions or sufficient for the existence of C_7-decomposition and C_p-decomposition of $K_m*\overline{K_{n.}}$, $p\geqq 11$ is prime. When p is prime and k is odd, $K_m*\overline{K_{n.}}$ can be decomposed into $C_{2p}, C_{3p}\&C_{k^2}$. This result was proposed by Smith[19,20,21]. In [22] he has also given the necessary and sufficient conditions for the existence of a C_p-decomposition of λ-fold complete equipartite graphs, when p is prime.

C. Hamilton Cycle Decomposition

Hamilton cycle decomposition of various product graphs has been investigated by many authors. Hamilton cycle decomposition of cartesian product of graphs and wreath product of graphs were introduced by Alspach, B.; Aubert, J; Scheider B.; Laskar and hetyei. Bermond[23]conjectured that if both G & H are Hamilton cycle decomposable graphs, then Cartesian product and wreath products of G & H are also Hamilton cycle decomposable. The analogue of the above conjecture was also verified by Bermond[23] for tensor product of graphs. Balakrishnan & Paulraja [24,25] have given many results related to decomposition of tensor product of Hamilton cycle decomposable graphs.

D. Path Decomposition Of Graphs

More results on path decompositions can be found in [29,30], Tarsi [27] Studied the decomposition of complete multigraphs into pats of equal length and gave the necessary conditions for the decomposition of complete multigraphs into paths. Truszezynki [31] stated that If $k,m,n\in\mathbb{N}$ with m,n even and $m\geqq n$, then $K_{m.n}$ has a P_{k+1} decomposition iff $m\geq\left\lceil\frac{k+1}{2}\right\rceil$; $n\geq\left\lceil\frac{k}{2}\right\rceil$ and $m,n\equiv 0(\text{mod } k)$. Chartrand and Lesniak [36] suggested that a non trivial connected graph G has a P_3-decomposition if and only if G has even size. C.Sunil Kumar [42] showed that a complete r-partite graph is P_4-decomposable if and only if its size is a multiple of 3. He also gave an example of a 2-connected graph of size 3k which is not P_4-decomposable.

E. Star Decomposition of Graphs

C.Lin[33] gave necessary and sufficient conditions for the existence of star decomposition in complete graphs. In [34] Shyu obtained four necessary conditions for a decomposition of K_n into C_l and S_k and gave the necessary and sufficient conditions for l=k=4.

F. Multi Decomposition

Lee[35] gave the necessary and sufficient conditions for the existence of the (C_k,S_k) decomposition of a complete bipartite graph. Shyu[30] proved that K_n has a $\{P_4,S_4\}_{\{p,q\}}$ decomposition if and only if $n\geq 6$ and $3(p+q)=\binom{n}{2}$. Also proved that K_n has a $\{P_k,S_k\}_{\{p,q\}}$ decomposition with a restriction $p\geq k/2$, where k even ($p\geq k$, when k odd). Lee et. al.[35,38] have suggested a necessary and sufficient conditions on (p,q) for the existence of $\{P_k,S_k\}_{\{p,q\}}$ decomposition of $K_{n,n}$ and $K_{m,n}$. Shyu[37] has given necessary and sufficient conditions on (p,q) for the existence of $\{P_4,S_4\}_{\{p,q\}}$ decomposition of $K_{m,n}$. Jeevadoss and Muthusamy [40] have given necessary and sufficient conditions for $\{P_{k+1},C_k\}_{\{p,q\}}$ decomposition of $K_{m,n}$. Also they have established necessary and sufficient conditions for the existence of $\{P_5,C_4\}_{\{p,q\}}$ decomposition of tensor product and Cartesian product of complete graphs in [41].

Sarvate and Zhang[43] obtained necessary and sufficient conditions for the existence of $\{pP_3, qK_3\}$ decomposition of $K_n(\lambda)$, when p=q. Fairouz Beggas, and Mohammed Haddad[8] investigated necessary and sufficient conditions for the existence of decomposition of λK_n into edge disjoint stars S_k's and cycles C_k's. The following results are obtained by Fairouz Beggas. let n>4 and λ>1 be positive integers. There exists a (P_4, S_4) decomposition if and only if $\lambda n(n-1)/2 \equiv 0[4]$. n,k are positive integers such that n$\geq$4k and $n(n-1)/2 \equiv 0[k]$. Then the graph K_n is (S_k, C_k)-decomposable. Let n,k and λ be positive integers. If $\lambda n(n-1)/2 \equiv 0[k]$ and

n$\geq 4k$ orB n$\geq 2k$ and $\lambda > 1$ is even or gcd(λ, k)=1, then λK_n is (C_k, S_k) decomposable.

) Let n, k and λ>1 be positive integers. Then λK_n is (C_k, S_k) decomposable if $\lambda n(n-1)/2 \equiv 0[k]$ and

Recently S.Lakshmi and K.Kanchana[44] have investigated decomposition of line graph of K_n into P copies of P_5 and q copies of $C_4$for all possible values of p≥ 0 and q$\geq$0.

II. CONCLUSION

A number of papers have dealt with necessary and sufficient conditions for decomposing Complete graphs, Complete bipartite graphs, Complete multigraphs into stars, cycles and paths. In future we can try for the situation when resolvable decompositions are possible.A.A.Abueida and C.Lian[7] have given more results to obtain $\{C_m, S_m\}$ decomposition of $K_m$for different values of m. We can try for $\{P_m, S_m\}$ or $\{P_m, C_m\}$ decomposition for all possible values of m.

REFERENCES

[1] J.Akiyama; M.Kano. Path factors of a graph- Graphs and applications.

[2] J.Akiyamma.; M.Kano. Factors and factrorizations of graphs-a survey, Graph theory,9(1985) no.1, 1-42 (springer colo.,2011).

[3] Decomposition of graphs. J. Bosk Kluwar; norwell , MA, 1990.

[4] A.A.Abueida and M.Devan, Multidecompositions of the complete graph, Ars. Combin.72(2004)17-22

[5] A.A.Abueida and T.O'neil, Multidecomposition of λK_m into small cycles and claws,Bull.Inst. Combin. Appl. 49(2007)32-40.

[6] H.M. Priyadharsini and A.Muthusamy ,(G_m,H_m) Multifactorization of λK_m , J. combin. Math. Combin. Comput.69(2009)145-150.

[7] A.A Abueida and C.Lian,On the decompositions of complete graphs into cycles and stars on the same number of edges, Discuss. Math. Graph theory 34 (2014) 113-125

[8] Fairouz Beggas, Mohammed Haddad and Hamamache khiddouci, ecomposition of complete multigraphs into stars and cycles, Discussiones mathematicae Graph Theory 35(2015) 629-639

[9] T.Shyu, Decomposition of complete graphs into paths and stars Discrete.Math.310(2010)2164-2169.doi:10.1016/j.disc.2010.04.00

[10] T.Shyu, Decomposition of complete graphs into paths and cycles, Ars. Combin.97(2010) 257-270.

[11] E.S.Mahmoodian, M.Mirza khani. Decomposition of complete tripartite graphs into 5- cycles combinatorics and advances[Eds. C.J.Colboum and

[12] N.J Cavenagh, Further decomposition of complete tripartite graphs into 5-cycles,Discreate Math. 256(2002),55-81.

[13] N.J Cavenagh, Decompositions of complete tripartite graphs into k-cycles, Austral. J.combin., 18(1998), 193-200.

[14] A.Muthusamy ; P.Paulraja. Factorizations of product graphs into cycles of uniform length,Graphs.Combin.,11(1995),69-90.

[15] E.J.Billington,;N.J.Cavenagh,; Smith, B.R. Path and cycle decompositions of complete equipartite graphs: Four parts,Discrete Math., 309(2009), 3061-3073.

[16] E.J.Billington; N.J.Cavenagh,; B.R.Smith. Path and cycle decompositions of complete equipartite graphs: 3 and 5 parts,Discrete Math., 310(2010), 241-254

[17] R.S.Manikandan, P.Paulraja. C_5-decomposition of the tensor product of complete graphs, Austral.J.combin.,37(2007),285-294

[18] R.S.Manikandan; P.Paulraja. C_P-decomposition of some regular graphs,Discrete Math,306(2006),429-451

[19] B.R.Smith. Decomposing complete equipartite graphs into cycles of length 2p, J.Combin. Designs, 16(2008),244-252

[20] B.R.Smith.Complete equipartite 3P cycle systems, Austral.J.combin.45(2009), 125-138

[21] B.R.Smith, N.J.Cavenagh. Decomposing complete equipartite graphs into odd square length cycle: number of parts even, Discrete Math.,(2012) 1611-1622

[22] B.R.Smith, Cycle decomposition of λ-fold complete equipartite graphs ,Austral.J. combin., 47(2010), 145-156.

[23] J.C.Bermond. Hamiltonian decompositions of graphs, digraphs and hyper graphs, Ann. Discrete Math.,3(1978),21-28, Bibliography 86

[24] R.BalaKrishnan,P.Paulraja.Hamilton cycles in tensor product of graphs, Discrete Math.,186(1998)1-13

[25] R.Balakrishanan, J.C.Bermond,P. Paulraja. Yu,M.L. on Hamilton cycle decompositions of the tensor product of complete graphs, Discrete Math., 268(2003), 49-58

[26] Dalibor Froncek. Decomposition of complete graphs into small graphs. Opuscula Mathematica volume 30, No.3-2010

[27] M.Tarsi. Decomposition of a complete multigraph into simple paths, J.Combin. Theory Ser .A, 34(1983), 60-70

[28] M.Tarsi. Decomposition of Multigraph into stars, Discrete Math., 36(1981),299-304. Bibiliography 102

[29] T.W.Shyu. Decomposition of complete graphs into cycles and paths, Ars.combin., 97(2010), 257-270

[30] T.W.Shyu. Decompostion of complete graphs into paths and stars with same number of edges, Discrete Math.,(2013),DOI: 10.1016/j.disc.2012.12.020.

[31] .Truszczynski. Note on the decomposition of $\lambda K_{m,n}\left(\lambda K^*_{m,n}\right)$into paths. Discrete Math.55(1985)89-96.

[32] A.Muthusamy, P.Paulraja. path factorization of complete multipartite graphs, Discrete Math., 195(1999), 181-201.

[33] C.Lin,T.W. Shyu. A necessary and sufficient conditions for the decomposition of complete graphs J. Graph theory,23(1996), 361-364. Bibiliography 96

[34] T.W.Shyu. Decomposition of complete graphs into cycles and stars, Graph. Combin(2011). 1-13.

[35] H.C.Lee. Multidecomposition of complete bipartite graphs into cycles and stars Ars.Combin. 108(2013)355-364.

[36] G.Chartrand.L.Lesniak, Graphs and Digraphs, 2nd Edition. Wadsworth, Belmont(1986).

[37] T.W.Shyu. Decomposition of complete bipartite graphs into paths and stars with same number of edges. Discrete Math.313(2013) 865-871.

[38] H.C.Lee,Y.P.Chu, Multidecomposition of the balanced complete tripartite graphs into paths and stars. ISRN combin.(2013).doi : 10.1155/2013/398473.

[39] S.Jeevadoss,A.Muthusamy, Decomposition of Complete bipartite graphs into paths and cycles. Discrete Math. (2014)331. 98-108.

[40] S.Jeevadoss. A.Muthusamy. Decomposition of product graphs into paths and cycles of length four. Graphs and combinatorics DOI 10.100/s00373-015-1564-z.

[41] C.Sunil Kumar ,On P_4 decomposition of graphs http://www.math.nthu.edu.tw/tjm/.

[42] D.G.Sarvate, L.Zhang, Decomposition of λK_v into equal number of K's and P's, Bull.Inst.comb.Appl. 67(2013)43-48.

[43] S.Lakshmi, K.Kanchana.,Decomposition of Line graph into paths and cycles.e-ISSN:2278-5728, P-ISSN : 2319-7 65X.Volume 11, Issue 5 ver-IV(2015).31-33.