

Strongly $(\hat{g})^*$ Closed Sets In Topological Space

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Abstract: In this paper, we study the concept of strongly $(\hat{g})^*$ -closed sets and strongly $(\hat{g})^*$ -continuous functions and check how they deal with the topological spaces and their sub-sets. We also read out how the strongly $(\hat{g})^*$ -closed sets and maps inter-relates with other sets with a change or transformation in their properties.

Key words: $cl(A)$, $int(A)$, strongly $(\hat{g})^*$ -closed set, g -closed set, g^* -closed set, $(\hat{g})^*$ -closed set, strongly $(\hat{g})^*$ -continuous map, g -continuous map, g^* -continuous map, $(\hat{g})^*$ -continuous map

I. INTRODUCTION

Levine [4] introduced the class of g -closed sets in 1970. Veerakumar [5] introduced \hat{g} -closed sets in 1991. A. Gayathri [8] introduced the class of $(\hat{g})^*$ sets in 2014. The intention of this paper is to give the basic properties of strongly $(\hat{g})^*$ -closed set and strongly $(\hat{g})^*$ -continuous map and how they work in relation with other sets and maps.

II. PRELIMINARIES

We see the non-empty topological space (X, τ) , a subset A of X and an open set U of X . We also see the terms of closure of A i.e. $Cl(A)$ and interior of A i.e. $int(A)$.

- A. *Definition 2.1:* Let A be a subset of a topological space (X, τ) . The interior of A is defined as the union of all open sets contained in A . It is denoted by $int(A)$.
- B. *Definition 2.2:* Let A be a subset of a topological space (X, τ) . The closure of A is defined as the intersection of all closed sets containing A . It is denoted by $cl(A)$.
- C. *Definition 2.3:* A subset A of the topological space (X, τ) is called
- 1) a pre-open set [7] if $A \subseteq int(cl(A))$
 - 2) a pre-closed set [7] if $cl(int(A)) \subseteq A$
 - 3) a semi-open set [4] if $A \subseteq cl(int(A))$
 - 4) a semi-closed set [4] if $int(cl(A)) \subseteq A$
 - 5) a semi-pre open set [1] if $A \subseteq cl(int(cl(A)))$
 - 6) a semi-pre closed set [1] if $int(cl(int(A))) \subseteq A$
- D. *Definition 2.4:* A subset A of a topological space (X, τ) is called
- 1) g -closed or generalized closed set [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
 - 2) g^* -closed set [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in (X, τ) .
 - 3) \hat{g} -closed set [6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ) .
 - 4) $(\hat{g})^*$ -closed set [8] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in (X, τ) .
- E. *Definition 2.5:* A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called
- 1) g -continuous [2] if $f^{-1}(V)$ is a g -closed set of (X, τ) for every closed set V of (Y, σ) .
 - 2) g^* -continuous [5] if $f^{-1}(V)$ is a g^* -closed set of (X, τ) for every closed set V of (Y, σ) .
 - 3) $(\hat{g})^*$ -continuous [8] if $f^{-1}(V)$ is a $(\hat{g})^*$ -closed set of (X, τ) for every closed set V of (Y, σ) .

III. BASIC PROPERTIES OF STRONGLY $(\hat{G})^*$ -CLOSED SET

A. *Definition 3.1*

A subset 'A' of a topological space (X, τ) is said to be a strongly $(\hat{g})^*$ -closed set, if $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is \hat{g} -open in X .

Theorem 3.2:

Every closed set is strongly $(\hat{g})^*$ -closed.

Proof Let (X, τ) be a topological space.

And $A \subseteq (X, \tau)$ is a closed set.

i.e. $Cl(A) = A$.

To prove: A is strongly $(\hat{g})^*$ -closed.

Let $A \subseteq U$ and U be \hat{g} open.

Then $cl(A) \subseteq U$.

Also, $cl(int(A)) \subseteq cl(A)$

We get, $cl(int(A)) \subseteq cl(A) \subseteq U$

i.e. $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is \hat{g} open.

$\therefore A$ is strongly $(\hat{g})^*$ -closed.

B. Theorem 3.3

Every g -closed set is strongly $(\hat{g})^*$ -closed.

1) Proof: Let A be a g -closed set.

By the definition 2.4.1,

$Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .

To prove: A is strongly $(\hat{g})^*$ -closed.

Let $A \subseteq U$ and U is \hat{g} open.

We've, $cl(A) \subseteq U$

Also, $cl(int(A)) \subseteq cl(A)$

Then, $cl(int(A)) \subseteq cl(A) \subseteq U$

i.e. $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is \hat{g} open in (X, τ) .

$\therefore A$ is strongly $(\hat{g})^*$ -closed.

The converse of the above theorem need not be true as shown in the following example.

C. Example 3.4

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{a, c\}\}$

Closed sets are $X, \phi, \{b, c\}, \{b\}$.

Semi-open sets are $\{a, c\}, \{a, b\}, \{a\}, \phi, X$.

\hat{g} -open sets are $\{a, c\}, \{a\}, \phi, X$.

Strongly $(\hat{g})^*$ -closed sets are $\{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X$.

g -closed sets are $\{b\}, \{a, b\}, \{b, c\}, \phi, X$.

$\therefore A = \{c\}$ is a strongly $(\hat{g})^*$ -closed set but not g -closed.

Hence, every strongly $(\hat{g})^*$ -closed set need not be g -closed.

D. Theorem 3.5

Every g^* -closed set is strongly $(\hat{g})^*$ -closed.

1) Proof: Let A be a g^* -closed set.

By the definition 2.4.2,

$Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is g -open in (X, τ) .

To prove: A is strongly $(\hat{g})^*$ -closed.

Let $A \subseteq U$ and U is \hat{g} -open.

We've, $cl(A) \subseteq U$

Also, $cl(int(A)) \subseteq cl(A)$

Then, $cl(int(A)) \subseteq cl(A) \subseteq U$

i.e. $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is \hat{g} -open in (X, τ) .

$\therefore A$ is strongly $(\hat{g})^*$ -closed.

The converse of the above theorem need not be true as shown in the following example.

E. Example 3.6

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{a, c\}\}$

Closed sets are $X, \phi, \{b, c\}, \{b\}$

g -open sets are $\{a, c\}, \{c\}, \{a\}, \phi, X$.

g^* -closed sets are $\{b\}, \{a, b\}, \{b, c\}, \phi, X$.

Strongly $(\hat{g})^*$ -closed sets are $\{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X$.

$\therefore A = \{c\}$ is a strongly $(\hat{g})^*$ -closed set but not g^* -closed.

Hence, every strongly $(\hat{g})^*$ -closed set need not be g^* -closed.

F. Theorem 3.7

Every $(\hat{g})^*$ -closed set is strongly $(\hat{g})^*$ -closed.

1) *Proof:* Let A be a $(\hat{g})^*$ -closed set.

By the definition 2.4.4,

$Cl(A) \subseteq U$, whenever $A \subseteq U$ and U is \hat{g} -open in (X, τ) .

To prove: A is strongly $(\hat{g})^*$ -closed.

Let $A \subseteq U$ and U is \hat{g} -open.

We've, $cl(A) \subseteq U$

Also, $cl(int(A)) \subseteq cl(A)$

Then, $cl(int(A)) \subseteq cl(A) \subseteq U$

i.e. $cl(int(A)) \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ) .

$\therefore A$ is strongly $(\hat{g})^*$ -closed.

The converse of the above theorem need not be true as shown in the following example.

G. Example 3.8:

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{a, c\}\}$

Closed sets are $X, \phi, \{b, c\}, \{b\}$

g -open sets are $\{a, c\}, \{c\}, \{a\}, \phi, X$.

$(\hat{g})^*$ -closed sets are $\{b\}, \{a, b\}, \{b, c\}, \phi, X$.

Strongly $(\hat{g})^*$ -closed sets are $\{b\}, \{c\}, \{a, b\}, \{b, c\}, \phi, X$.

$\therefore A = \{c\}$ is a strongly $(\hat{g})^*$ -closed set but not $(\hat{g})^*$ -closed.

Hence, every strongly $(\hat{g})^*$ -closed set need not be $(\hat{g})^*$ -closed.

IV. BASIC PROPERTIES OF STRONGLY $(\hat{G})^*$ -CLOSED CONTINUOUS MAPS

A. Definition 4.1

A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a strongly $(\hat{g})^*$ -continuous map if $f^{-1}(V)$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) for every closed set V of (Y, σ) .

B. Theorem 4.2

Every continuous map is strongly $(\hat{g})^*$ -continuous.

1) *Proof:* Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous map.

To prove: f is strongly $(\hat{g})^*$ -continuous.

Let V be a closed set in (Y, σ) .

Since, f is continuous; there exists a closed set $f^{-1}(V)$ in (X, τ) .

By theorem 3.2,

"Every closed set is a strongly $(\hat{g})^*$ -closed."

Hence, $f^{-1}(V)$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) .

$\therefore f$ is strongly $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

C. Example 4.3:

Let $X = Y = \{a, b, c\}$

And $\tau = \{\emptyset, X, \{a, b\}\}$

Closed sets in (X, τ) are $X, \emptyset, \{c\}$.

And $\sigma = \{\emptyset, Y, \{b, c\}\}$

Closed sets in (Y, σ) are $Y, \emptyset, \{a\}$.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map.

Semi-open sets are $\{a, b\}, \emptyset, X$.

\hat{g} -open sets are $\{a, b\}, \{a\}, \{b\}, \emptyset, X$.

Strongly $(\hat{g})^*$ -closed sets are $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \emptyset, X$.

$\therefore f^{-1}\{a\} = \{a\}$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) but not a closed set in (X, τ) .

Thus, the converse of the above theorem is not true.

Hence, every strongly $(\hat{g})^*$ -continuous map need not be continuous.

D. Theorem 4.4

Every g -continuous map is strongly $(\hat{g})^*$ -continuous.

1) *Proof:* Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous map.

By the definition 2.5.1,

$f^{-1}(V)$ is a g -closed set of (X, τ) for every closed set V of (Y, σ) .

To prove: f is strongly $(\hat{g})^*$ -continuous.

Let V be a closed set in (Y, σ) .

Since, f is g -continuous; there exists a g -closed set $f^{-1}(V)$ in (X, τ) .

By theorem 3.3,

“Every g -closed set is strongly $(\hat{g})^*$ -closed.”

Hence, $f^{-1}(V)$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) .

$\therefore f$ is strongly $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

E. Example 4.5

Let $X = Y = \{a, b, c\}$

And $\tau = \{\emptyset, X, \{a, b\}\}$

Closed sets in (X, τ) are $X, \emptyset, \{c\}$.

And $\sigma = \{\emptyset, Y, \{b, c\}\}$

Closed sets in (Y, σ) are $Y, \emptyset, \{a\}$.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map.

g -closed sets are $\{c\}, \{b, c\}, \{a, c\}, \emptyset, X$.

Strongly $(\hat{g})^*$ -closed sets in (X, τ) are $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \emptyset, X$.

$\therefore f^{-1}\{b\} = \{b\}$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) but not a g -closed set in (X, τ) .

Thus, the converse of the above theorem is not true.

Hence, every strongly $(\hat{g})^*$ -continuous map need not be g -continuous.

F. Theorem 4.6

Every g^* -continuous map is strongly $(\hat{g})^*$ -continuous.

1) *Proof:* Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous map.

By the definition 2.5.2,

$f^{-1}(V)$ is a g^* -closed set of (X, τ) for every closed set V of (Y, σ) .

To prove: f is strongly $(\hat{g})^*$ -continuous.

Let V be a closed set in (Y, σ) .

Since, f is g^* -continuous; there exists a g^* -closed set $f^{-1}(V)$ in (X, τ) .

By theorem 3.5,

“Every g^* -closed set is strongly $(\hat{g})^*$ -closed.”

Hence, $f^{-1}(V)$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) .

$\therefore f$ is strongly $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

G. Example 4.7

Let $X = Y = \{a, b, c\}$

And $\tau = \{\phi, X, \{a, b\}\}$

Closed sets in (X, τ) are $X, \phi, \{c\}$.

And $\sigma = \{\phi, Y, \{b, c\}\}$

Closed sets in (Y, σ) are $Y, \phi, \{a\}$.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map.

g -open sets in (X, τ) are $\{a, b\}, \{a\}, \{b\}, \phi, X$.

g^* -closed sets are $\{c\}, \{b, c\}, \{a, c\}, \phi, X$.

Strongly $(\hat{g})^*$ -closed sets in (X, τ) are $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \phi, X$.

$\therefore f^{-1}\{b\} = \{b\}$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) but not a g^* -closed set in (X, τ) .

Thus, the converse of the above theorem is not true.

Hence, every strongly $(\hat{g})^*$ -continuous map need not be g^* -continuous.

H. Theorem 4.8

Every $(\hat{g})^*$ -continuous map is strongly $(\hat{g})^*$ -continuous.

1) *Proof:* Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a continuous map.

By the definition 2.5.3,

$f^{-1}(V)$ is a $(\hat{g})^*$ -closed set of (X, τ) for every closed set V of (Y, σ) .

To prove: f is strongly $(\hat{g})^*$ -continuous.

Let V be a closed set in (Y, σ) .

Since, f is $(\hat{g})^*$ -continuous; there exists a $(\hat{g})^*$ -closed set $f^{-1}(V)$ in (X, τ) .

By theorem 3.7,

“Every $(\hat{g})^*$ -closed set is strongly $(\hat{g})^*$ -closed.”

Hence, $f^{-1}(V)$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) .

$\therefore f$ is strongly $(\hat{g})^*$ -continuous.

The converse of the above theorem need not be true as shown in the following example.

I. Example 4.9

Let $X = Y = \{a, b, c\}$

And $\tau = \{\phi, X, \{a, b\}\}$

Closed sets in (X, τ) are $X, \phi, \{c\}$.

And $\sigma = \{\phi, Y, \{b, c\}\}$

Closed sets in (Y, σ) are $Y, \phi, \{a\}$.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map.

\hat{g} -closed sets are $\{c\}, \{b, c\}, \{a, c\}, \phi, X$.

\hat{g} -open sets are $\{a, b\}, \{a\}, \{b\}, \phi, X$.

$(\hat{g})^*$ -closed sets are $\{c\}, \{b, c\}, \{a, c\}, \phi, X$.

Strongly $(\hat{g})^*$ -closed sets in (X, τ) are $\{a\}, \{b\}, \{c\}, \{b, c\}, \{a, c\}, \phi, X$.

$\therefore f^{-1}\{b\} = \{b\}$ is a strongly $(\hat{g})^*$ -closed set in (X, τ) but not a $(\hat{g})^*$ -closed set in (X, τ) .

Thus, the converse of the above theorem is not true.

Hence, every strongly $(\hat{g})^*$ -continuous map need not be $(\hat{g})^*$ -continuous.

III. CONCLUSION

Hence, I would like to conclude my paper by giving the properties of strongly $(\hat{g})^*$ -closed set and strongly $(\hat{g})^*$ - continuous function. And also with further results and solutions we can bring in the comparison of strongly $(\hat{g})^*$ -closed set and function with other sets and functions as well in a given topological space.

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