

Anti- Fuzzy Soft Sets in Subtraction Algebras

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Abstract: In this project, we derived Support Anti Intuitionistic Fuzzy A-Ideal subtraction BCI-Algebras theorems and Some Operations on Support Anti Intuitionistic Fuzzy A-ideal theorems and examples are derived.

I. INTRODUCTION

Fuzzy set theory was introduced by L-Zadeh since 1965. Immediately it became a useful method to study in the problems of imprecision and uncertainty. Since, a lot of new theories treating imprecision and uncertainty have been introduced. For instance intuitionistic fuzzy sets were introduced in 1986 by K. Atanassov which is a generalization of the notion of a fuzzy set. When fuzzy set give the degree of membership of an element in a given set, intuitionistic fuzzy set give a degree of membership and a degree of non-membership of an element in a given set. Then, the concept of fuzzy relations and intuitionistic fuzzy relations introduced. They together with their logic operators and are applied in many different fields.

In this paper, we combine a intuitionistic fuzzy set with a fuzzy set. This raise a new concept called support- intuitionistic fuzzy (SIF) set. In which there are three membership function of an element in a given set.

To develop the theory of BCI-algebras, the ideal theory plays an important role. Liu and meng introduced the notation of q-ideals and a-ideals in BCI-algebras. Liu and Zhang discussed the fuzzification of h-ideals, gave relations between fuzzy ideals, fuzzy h-ideals and fuzzy p-ideals. They also considered characterizations of fuzzy h-ideals. Using the notion of fuzzy h-ideals, they provided characterization of associative BCI-algebras. After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this fundamental concept.

The important concept that addresses uncertain information is the soft set theory originated by Molodtsov. This concept is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory. Molodtsov has successfully applied the soft set theory in many different fields such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, and probability.

II. PRELIMINARIES

1) *Definition 2.1:* A **fuzzy set** A on the universe U is an object of the form

$$A = \{x, \mu_A(x) : x \in U\}$$

Where $\mu_A(x) (X \in [0,1])$ is called the degree of membership of x in A.

2) *Definition 2.2:* An Intuitionistic fuzzy (IF) set A on the universe U is an object of the form $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in U\}$ where $\mu_A(x) (\in [0,1])$ is called the “degree of membership of x in A “, $\lambda_A(x) (X \in [0,1])$ is called the “ degree of non-membership of x in A “ and where μ_A and λ_A satisfy the following condition $\mu_A(x) + \lambda_A(x) \leq 1, (\forall x \in U)$.

3) *Definition 2.3:* A support-intuitionist fuzzy (SIF) set A on the universe U is an object of the form $A = \{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) : \forall x \in U\}$ where $\mu_A(x) (\in [0,1])$ is called the “degree of membership of x in A”, $\lambda_A(x) (\in [0,1])$ is called the “ degree of non-membership of x in A”, and $\gamma_A(x)$ is called the “degree of support membership of x in A”, and where μ_A, λ_A and γ_A satisfy the following condition. $\mu_A(x) + \lambda_A(x) \leq 1, 0 \leq \gamma_A(x) \leq 1, (\forall x \in U)$.

The family of all support intuitionistic fuzzy set in U is denoted by SIFS (U)

4) *Definition 2.4:* Algebra $(X; *, 0)$ of type (2,0) is called a **BCI-algebra** if it satisfies the following conditions:

a) $\forall x, y, z \in X, ((x*y) *(x*z)) *(z*y) = 0,$

b) $\forall x, y \in X, (x*(x*y)) *y = 0,$

c) $\forall x \in X, x*x = 0,$

d) $\forall x, y \in X, x*y = 0, y*x = 0 \implies x = y$

5) *Definition 2.5:* A nonempty subset A of a BCI-algebra X is called an **ideal** of X if it satisfies: (I) $0 \in A$, (II) $\forall x, y \in X, \forall y \in A, x-y \in A \implies x \in A$.

6) *Definition 2.6:* A nonempty subset A of a BCI-algebra X is called **a-ideal** of X if it satisfies: (I) $0 \in A$, (II) $x, y \in X, \forall z \in A$
 $((x-z) - (0-y) \in A \Rightarrow y - x \in A)$

7) *Definition 2.7:* An intuitionist fuzzy set A in a nonempty set X is an object having the form $A = \{ x, \mu_A(x), \lambda_A(x) \}: \forall x \in X$,
 Where the function $\mu_A: X \rightarrow [0,1]$ and $\lambda_A: X \rightarrow [0,1]$ denoted the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$.

8) *Definition 2.8:* An intuitionist fuzzy set $A = \langle X, \mu_A, \lambda_A \rangle$ in a BCI- algebra X is called an **intuitionist fuzzy sub-algebra** of X if it satisfies: $(\forall x, y \in X) (\mu_A(x-y) \geq \min\{\mu_A(x), \mu_A(y)\}), \forall x, y \in X (\lambda_A(x-y) \leq \max\{\lambda_A(x), \lambda_A(y)\})$.

9) *Definition 2.9:* An intuitionist fuzzy set $A = \langle X, \mu_A, \lambda_A \rangle$ in X is called an **intuitionist fuzzy ideal** of X, if it satisfies the following axioms:

a) $\mu_A(0) \geq \mu_A(x)$ and $\lambda_A(0) \leq \lambda_A(x)$,

b) $\mu_A(x) \geq \min\{\mu_A(x-y), \mu_A(y)\}$

c) $\lambda_A(x) \leq \max\{\lambda_A(x-y), \lambda_A(y)\}, \forall x, y \in X$.

10) *Definition 2.10:* An intuitionist fuzzy set $A = \langle X, \mu_A, \gamma_A \rangle$ in X is called an **intuitionist fuzzy ideal** of X, if it satisfies the following axioms

(I) $\lambda_A(x) \leq \max\{\lambda_A(x-y), \lambda_A(y)\}, \forall x, y \in X$.

And $(\forall x, y, z \in X) (\mu_A(y-x) \geq \min\{\mu_A((x-z) - (0-y)), \mu_A(z)\})$,

$(\forall x, y, z \in X) (\lambda_A(y-x) \leq \max\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\})$.

11) *Definition 2.11:* An intuitionist fuzzy set $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$ in X is called an **Support Intuitionistic Fuzzy set** X, if it satisfies the following axioms For all A, B $\in X$

a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x), \lambda_A(x) \geq \lambda_B(x)$ and $\gamma_A(x) \leq \gamma_B(x), \forall x \in X$

b) $A \supseteq B$ iff $\mu_A(x) \geq \mu_B(x), \lambda_A(x) \leq \lambda_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$

c) $A = B$ iff $A \subseteq B$ and $B \subseteq A$

12) *Definition 2.12:* Let A and B be any two Support Intuitionistic Fuzzy set X then $A \cup B$ is also Support Intuitionistic Fuzzy set X, if it satisfies the following axioms

$A \cup B = \{(x, \mu_{A \cup B}(x), \lambda_{A \cup B}(x), \gamma_{A \cup B}(x)) / x \in X\}$

Where $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$,

$\lambda_{A \cup B}(x) = \min\{\lambda_A(x), \lambda_B(x)\}$,

$\gamma_{A \cup B}(x) = \max\{\gamma_A(x), \gamma_B(x)\}, \forall x \in X$.

13) *Definition 2.13:* Let A and B be any two Support Intuitionistic Fuzzy set X then $A \cap B$ is also Support Intuitionistic Fuzzy set X, if it satisfies the following axioms

$A \cap B = \{(x, \mu_{A \cap B}(x), \lambda_{A \cap B}(x), \gamma_{A \cap B}(x)) / x \in X\}$

Where $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$,

$\lambda_{A \cap B}(x) = \max\{\lambda_A(x), \lambda_B(x)\}$,

$\gamma_{A \cap B}(x) = \min\{\gamma_A(x), \gamma_B(x)\}, \forall x \in X$.

14) *Definition 2.14:* Let A be any Support Intuitionistic Fuzzy set X then $\sim A$ is also Support intuitionistic Fuzzy set X, if it satisfies the following axioms

The complement of A

$\sim A = \{(x, \mu_A(x), \lambda_A(x), 1 - \gamma_A(x)) / \forall x \in X\}$.

15) *Definition 2.15:* Let U and V be two universes and let $A = \{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) / \forall x \in U\}$ and $B = \{(y, \mu_B(y), \lambda_B(y), \gamma_B(y)) / \forall y \in U\}$ be two Support Intuitionistic Fuzzy sets on U and V respectively. We define the Cartesian product of these two Support Intuitionistic Fuzzy sets

$A \times B = \{((x,y), \mu_{A \times B}(x,y), \lambda_{A \times B}(x,y), \gamma_{A \times B}(x,y)) / \forall x \in U, \forall y \in U\}$

Where $\mu_{A \times B}(x,y) = \mu_A(x) \mu_B(y)$

$\lambda_{A \times B}(x,y) = \lambda_A(x) \lambda_B(y)$

$\gamma_{A \times B}(x,y) = \gamma_A(x) \gamma_B(y)$

16) *Definition 2.16:* Let U and V be two universes and let $A = \{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) / \forall x \in U\}$ and $B = \{(y, \mu_B(y), \lambda_B(y), \gamma_B(y)) / \forall y \in U\}$ be two Support Intuitionistic Fuzzy sets on U and V respectively. We define the Cartesian product of these two Support Intuitionistic Fuzzy sets

$A \otimes B = \{((x,y), \mu_{A \otimes B}(x,y), \lambda_{A \otimes B}(x,y), \gamma_{A \otimes B}(x,y)) / \forall x \in U, \forall y \in V\}$ Where $\mu_{A \otimes B}(x) = \min\{\mu_A(x), \mu_B(x)\}$,

$\lambda_{A \otimes B}(x) = \max\{\lambda_A(x), \lambda_B(x)\}$,

$$\gamma_{A \otimes B}(x) = \min\{\gamma_A(x), \gamma_B(x)\}, \forall x \in U, y \in V.$$

17) Definition 2.17: Let U and V be two universes and let A = $\{(x, \mu_A(x), \lambda_A(x), \gamma_A(x)) / \forall x \in U\}$ and B = $\{(y, \mu_B(y), \lambda_B(y), \gamma_B(y)) / \forall y \in U\}$ be two Support Intuitionistic Fuzzy sets on U and V respectively. We define the Cartesian product of these two Support Intuitionistic Fuzzy sets

$$A \boxtimes B = \{(x,y), \mu_{A \boxtimes B}(x,y), \lambda_{A \boxtimes B}(x,y), \gamma_{A \boxtimes B}(x,y)) / \forall x \in U, \forall y \in V\}$$

$$\text{Where } \mu_{A \boxtimes B}(x) = \max\{\mu_A(x), \mu_B(x)\},$$

$$\lambda_{A \boxtimes B}(x) = \min\{\lambda_A(x), \lambda_B(x)\},$$

$$\gamma_{A \boxtimes B}(x) = \max\{\gamma_A(x), \gamma_B(x)\}, \forall x \in U, y \in V.$$

III. SUPPORT ANTI INTUITIONISTIC FUZZY A-IDEAL IN SUBTRACTION BCI-ALGEBRAS THEOREMS

A. Theorem 3.1

If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then A \cup B is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

Proof: An AIFS A = $\langle X, \mu_A, \lambda_A, \gamma_A \rangle$ in X is called an Anti intuitionistic fuzzy a-ideal of X if $\mu_A(0) \leq \mu_A(x)$, $\mu_B(0) \leq \mu_B(x)$ and

$$(i) \max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(x)\}$$

$$\mu_{A \cup B}(0) \leq \mu_{A \cup B}(x), \forall x \in X.$$

$$\lambda_A(0) \geq \lambda_A(x), \lambda_B(0) \geq \lambda_B(x) \text{ and}$$

$$\min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(x)\}$$

$$\lambda_{A \cup B}(0) \geq \lambda_{A \cup B}(x), \forall x \in X.$$

$$\gamma_A(0) \leq \gamma_A(x), \gamma_B(0) \leq \gamma_B(x) \text{ and}$$

$$\text{Max}\{\gamma_A(0), \gamma_B(0)\} \leq \max\{\gamma_A(x), \gamma_B(x)\}$$

$$\gamma_{A \cup B}(0) \leq \gamma_{A \cup B}(x), \forall x \in X.$$

$$(ii) \mu_A(y-x) \leq \max\{\mu_A((x-z) - (0-y)), \mu_A(z)\} \text{ and}$$

$$\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$$

$$\max\{\mu_A(y-x), \mu_B(y-x)\} \leq \max\{\max\{\mu_A((x-z) - (0-y)), \mu_A(z)\}, \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}\}$$

$$\mu_{A \cup B}(y-x) \leq \max\{\max\{\mu_A((x-z) - (0-y)), \mu_B((x-z) - (0-y))\}, \max\{\mu_A(z), \mu_B(z)\}\}$$

$$\mu_{A \cup B}(y-x) \leq \max\{\mu_{A \cup B}((x-z) - (0-y)), \mu_{A \cup B}(z)\}$$

$$\text{If } \lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\} \text{ and}$$

$$\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}$$

$$\min\{\lambda_A(y-x), \lambda_B(y-x)\} \geq \min\{\min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\}, \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}\}$$

$$\lambda_{A \cup B}(y-x) \geq \min\{\min\{\lambda_A((x-z) - (0-y)), \lambda_B((x-z) - (0-y))\}, \min\{\lambda_A(z), \lambda_B(z)\}\}$$

$$\lambda_{A \cup B}(y-x) \geq \min\{\lambda_{A \cup B}((x-z) - (0-y)), \lambda_{A \cup B}(z)\}.$$

$$\text{If } \gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\} \text{ and}$$

$$\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}$$

$$\text{Max}\{\gamma_A(y-x), \gamma_B(y-x)\} \leq \max\{\max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}, \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}\}$$

$$\gamma_{A \cup B}(y-x) \leq \max\{\max\{\gamma_A((x-z) - (0 - y)), \gamma_B((x-z) - (0 - y))\}, \text{Max}\{\gamma_A(z), \gamma_B(z)\}\}$$

$$\gamma_{A \cup B}(y-x) \leq \max\{\gamma_{A \cup B}((x-z) - (0 - y)), \gamma_{A \cup B}(z)\}.$$

Hence AUB is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

1) *Example 3.1:* Consider a BCI-algebra $X = \{0, a, b\}$ with the following Cayley table:

| | | | |
|---|---|---|---|
| - | 0 | a | B |
| 0 | 0 | b | A |
| A | A | 0 | B |
| B | B | a | 0 |

| | | | |
|-------------|-----|-----|-----|
| X | 0 | a | B |
| μ_A | 0.2 | 0.7 | 0.2 |
| λ_A | 0.5 | 0.2 | 0.5 |
| γ_A | 0.4 | 0.6 | 0.4 |

| | | | |
|-------------|-----|-----|-----|
| X | 0 | a | B |
| μ_B | 0.3 | 0.8 | 0.3 |
| λ_B | 0.7 | 0.4 | 0.7 |
| γ_B | 0.6 | 0.7 | 0.6 |

I) Let $x=a$, $\max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(x)\}$

$$\max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(a), \mu_B(a)\}$$

$$\max\{(0.2), (0.3)\} \leq \max\{(0.7), (0.8)\}$$

$$0.3 \leq 0.8$$

Let $x=b$, $\min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(x)\}$

$$\min\{0.5, 0.7\} \geq \min\{\lambda_A(b), \lambda_B(b)\}$$

$$0.5 \geq \min\{0.5, 0.7\}$$

$$0.5 \geq 0.5$$

And let $x=a$, $\max\{\gamma_A(0), \gamma_B(0)\} \leq \max\{\gamma_A(x), \gamma_B(x)\}$

$$\text{Max}\{(0.4), (0.6)\} \leq \max\{\gamma_A(a), \gamma_B(a)\}$$

$$0.6 \leq \max\{0.6, 0.7\}$$

0.6 ≤ 0.7 is satisfied.

II) If $x=b, y=a, z=0$

$$\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0-y)), \mu_A(z)\} \text{ and}$$

$$\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$$

$$\max\{\mu_A(y-x), \mu_B(y-x)\} \leq \max\{\max\{\mu_A((x-z) - (0-y)), \mu_A(z)\}, \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}\}$$

$$\max\{\mu_A(a-b), \mu_B(a-b)\} \leq \max\{\max\{\mu_A((b-0) - (0-a)), \mu_A(0)\}, \max\{\mu_B((b-0) - (0-a)), \mu_B(0)\}\}$$

$$\max\{\mu_A(b), \mu_B(b)\} \leq \max\{\max\{\mu_A(b-b), \mu_A(0)\}, \max\{\mu_B(b-b), \mu_B(0)\}\}$$

$$\max\{0.2, 0.3\} \leq \max\{\max\{0.2, 0.2\}, \max\{0.3, 0.3\}\}$$

$$\max\{0.2, 0.3\} \leq \max\{0.2, 0.3\}$$

0.3 ≤ 0.3 is satisfied.

If $x=b, y=a, z=0$

$$\lambda_A(y-x) \geq \min\{\lambda_A((x-z) * (0-y)), \lambda_A(z)\} \text{ and}$$

$$\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}$$

$$\min\{\lambda_A(y-x), \lambda_B(y-x)\} \geq \min\{\min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\}, \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}\}$$

$$\min\{\lambda_A(a-b), \lambda_B(a-b)\} \geq \min\{\min\{\lambda_A((b-0) - (0-a)), \lambda_A(0)\}, \min\{\lambda_B((b-0) - (0-a)), \lambda_B(0)\}\}$$

$$\min\{\lambda_A(b), \lambda_B(b)\} \geq \min\{\min\{\lambda_A(b-b), \lambda_A(0)\}, \min\{\lambda_B(b-b), \lambda_B(0)\}\}$$

$$\min\{0.5, 0.7\} \geq \min\{\min\{\lambda_A(0), \lambda_A(0)\}, \min\{\lambda_B(0), \lambda_B(0)\}\}$$

$$\min\{0.5, 0.7\} \geq \min\{\min\{0.5, 0.5\}, \min\{0.7, 0.7\}\}$$

$$0.7 \geq \min\{0.5, 0.7\}$$

0.7 ≥ 0.7 is satisfied.

If $x=b, y=a, z=0$

$$\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\} \text{ and}$$

$$\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}$$

$$\max\{\gamma_A(y-x), \gamma_B(y-x)\} \leq \max\{\max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}, \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}\}$$

$$\max\{\gamma_A(a-b), \gamma_B(a-b)\} \leq \max\{\max\{\gamma_A((b-0) - (0-a)), \gamma_A(0)\}, \max\{\gamma_B((b-0) - (0-a)), \gamma_B(0)\}\}$$

$$\max\{\gamma_A(b), \gamma_B(b)\} \leq \max\{\max\{\gamma_A(b-b), \gamma_A(0)\}, \max\{\gamma_B(b-b), \gamma_B(0)\}\}$$

$$\max\{0.4, 0.6\} \leq \max\{\max\{\gamma_A(0), \gamma_A(0)\}, \max\{\gamma_B(0), \gamma_B(0)\}\}$$

$$\max\{0.4, 0.6\} \leq \max\{\max\{0.5, 0.5\}, \max\{0.6, 0.6\}\}$$

$$0.6 \leq \max\{0.5, 0.6\}$$

0.6 ≤ 0.6 is satisfied.

This completes the proof.

B. Theorem 3.2

If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then $A \cap B$ is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

Proof: An AIFS $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$ in X is called an Anti intuitionistic fuzzy a-ideal of X if $\mu_A(0) \leq \mu_A(x)$, $\mu_B(0) \leq \mu_B(x)$ and

I)
$$\min\{\mu_A(0), \mu_B(0)\} \leq \min\{\mu_A(x), \mu_B(x)\}$$

$$\mu_{A \cap B}(0) \leq \mu_{A \cap B}(x), \forall x \in X.$$

$$\lambda_A(0) \geq \lambda_A(x), \lambda_B(0) \geq \lambda_B(x) \text{ and}$$

$$\max\{\lambda_A(0), \lambda_B(0)\} \geq \max\{\lambda_A(x), \lambda_B(x)\}$$

$$\lambda_{A \cap B}(0) \geq \lambda_{A \cap B}(x), \forall x \in X.$$

$$\gamma_A(0) \leq \gamma_A(x), \gamma_B(0) \leq \gamma_B(x) \text{ and}$$

$$\min\{\gamma_A(0), \gamma_B(0)\} \leq \min\{\gamma_A(x), \gamma_B(x)\}$$

$$\gamma_{A \cap B}(0) \leq \gamma_{A \cap B}(x), \forall x \in X.$$

II) If $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0-y)), \mu_A(z)\}$ and

$$\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$$

$$\min\{\mu_A(y-x), \mu_B(y-x)\} \leq \min\{\max\{\mu_A((x-z) - (0-y)), \mu_A(z)\},$$

$$\max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}\}$$

$$\mu_{A \cap B}(y-x) \leq \max\{\min\{\mu_A((x-z) - (0-y)), \mu_B((x-z) - (0-y))\}, \min\{\mu_A(z), \mu_B(z)\}\}$$

$$\mu_{A \cap B}(y-x) \leq \max\{\mu_{A \cap B}((x-z) - (0-y)), \mu_{A \cap B}(z)\}.$$

If $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\}$ and

$$\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}$$

$$\max\{\lambda_A(y-x), \lambda_B(y-x)\} \geq \max\{\min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\},$$

$$\min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}\}$$

$$\lambda_{A \cap B}(y-x) \geq \min\{\max\{\lambda_A((x-z) - (0-y)), \{\lambda_B((x-z) - (0-y))\}, \max\{\lambda_A(z), \lambda_B(z)\}\}$$

$$\lambda_{A \cap B}(y-x) \geq \max\{\lambda_{A \cap B}((x-z) - (0-y)), \lambda_{A \cap B}(z)\}.$$

If $\gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}$ and

$$\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}$$

$$\min\{\gamma_A(y-x), \gamma_B(y-x)\} \leq \min\{\max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}, \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}\}$$

$$\gamma_{A \cap B}(y-x) \leq \max\{\min\{\gamma_A((x-z) - (0-y)), \gamma_B((x-z) - (0-y))\}, \min\{\gamma_A(z), \gamma_B(z)\}\}$$

$$\gamma_{A \cap B}(y-x) \leq \max\{\gamma_{A \cap B}((x-z) - (0-y)), \gamma_{A \cap B}(z)\}.$$

Hence $A \cap B$ is also support intuitionistic fuzzy a-ideal in BCI-algebra X .

1) *Example 3.2:* Consider a BCI-algebra $X = \{0, a, b\}$ with the following Cayley table:

| | | | |
|-----|---|---|---|
| $-$ | 0 | A | B |
| 0 | 0 | B | A |
| A | A | 0 | B |
| B | B | A | 0 |

| | | | |
|-------------|-----|-----|-----|
| X | 0 | a | b |
| μ_A | 0.2 | 0.7 | 0.2 |
| λ_A | 0.5 | 0.2 | 0.5 |
| γ_A | 0.4 | 0.6 | 0.4 |

| | | | |
|-------------|-----|-----|-----|
| X | 0 | a | B |
| μ_A | 0.3 | 0.8 | 0.3 |
| λ_A | 0.7 | 0.4 | 0.7 |
| γ_A | 0.6 | 0.7 | 0.6 |

I) Let $x=a$, $\min\{\mu_A(0), \mu_B(0)\} \leq \min\{\mu_A(x), \mu_B(x)\}$

$$\min\{(0.2), (0.3)\} \leq \min\{(0.7), (0.8)\}$$

$$0.2 \leq 0.7$$

Let $x=b$, $\max\{\lambda_A(0), \lambda_B(0)\} \geq \max\{\lambda_A(x), \lambda_B(x)\}$

$$\max\{0.5, 0.7\} \geq \max\{\lambda_A(b), \lambda_B(b)\}$$

$$0.7 \geq \max\{0.5, 0.7\}$$

$$0.7 \geq 0.7$$

And let $x=a$, $\min\{\gamma_A(0), \gamma_B(0)\} \leq \min\{\gamma_A(x), \gamma_B(x)\}$

$$\min\{(0.4), (0.6)\} \leq \min\{\gamma_A(a), \gamma_B(a)\}$$

$$0.4 \leq \min\{0.6, 0.7\}$$

$$0.4 \leq 0.6 \text{ is satisfied.}$$

II) If $x=b$, $y=a$, $z=0$

If $\mu_A(y-x) \leq \min\{\mu_A((x-z) - (0-y)), \mu_A(z)\}$ and

$$\mu_B(y-x) \leq \min\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$$

$$\min\{\mu_A(y-x), \mu_B(y-x)\} \leq \min\{\max\{\mu_A((x-z) - (0-y)), \mu_A(z)\},$$

$$\max\{\mu_B((x-z) - (0 - y)), \mu_B(z)\}$$

$$\min\{\mu_A(a-b), \mu_B(a-b)\} \leq \max\{\min\{\mu_A((b-0) - (0 - a)), \mu_A(0)\},$$

$$\min\{\mu_B((b-0) - (0 - a)), \mu_B(0)\}\}$$

$$\min\{\mu_A(b), \mu_B(b)\} \leq \max\{\min\{\mu_A(b-b), \mu_A(0)\}, \min\{\mu_B(b-b), \mu_B(0)\}\}$$

$$\min\{0.2, 0.3\} \leq \max\{\min\{0.2, 0.2\}, \min\{0.3, 0.3\}\}$$

$$\min\{0.2, 0.3\} \leq \max\{0.2, 0.3\}$$

0.2 ≤ 0.3 is satisfied.

If x=b, y=a, z=0

$$\text{If } \lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\} \text{ and}$$

$$\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}$$

$$\max\{\lambda_A(y-x), \lambda_B(y-x)\} \geq \max\{\min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\}, \min\{\lambda_B((x-z) - (0 - y)), \lambda_B(z)\}\}$$

$$\max\{\lambda_A(a-b), \lambda_B(a-b)\} \geq \min\{\max\{\lambda_A((b-0) - (0 - a)), \lambda_A(0)\}, \max\{\lambda_B((b-0) - (0 - a)), \lambda_B(0)\}\}$$

$$\max\{\lambda_A(b), \lambda_B(b)\} \geq \min\{\max\{\lambda_A(b-b), \lambda_A(0)\}, \max\{\lambda_B(b-b), \lambda_B(0)\}\}$$

$$\max\{0.5, 0.7\} \geq \min\{\max\{\lambda_A(0), \lambda_A(0)\}, \max\{\lambda_B(0), \lambda_B(0)\}\}$$

$$\max\{0.5, 0.7\} \geq \min\{\max\{0.5, 0.5\}, \max\{0.7, 0.7\}\}$$

$$0.7 \geq \min\{0.5, 0.7\}$$

0.7 ≥ 0.5 is satisfied.

If x=b, y=a, z=0

$$\text{If } \gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\} \text{ and}$$

$$\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}$$

$$\min\{\gamma_A(y-x), \gamma_B(y-x)\} \leq \min\{\max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\}, \max\{\gamma_B((x-z) - (0 - y)), \gamma_B(z)\}\}$$

$$\min\{\gamma_A(a-b), \gamma_B(a-b)\} \leq \max\{\min\{\gamma_A((b-0) - (0 - a)), \gamma_A(0)\}, \min\{\gamma_B((b-0) - (0 - a)), \gamma_B(0)\}\}$$

$$\min\{\gamma_A(b), \gamma_B(b)\} \leq \max\{\min\{\gamma_A(b-b), \gamma_A(0)\}, \min\{\gamma_B(b-b), \gamma_B(0)\}\}$$

$$\min\{0.4, 0.6\} \leq \max\{\min\{\gamma_A(0), \gamma_A(0)\}, \min\{\gamma_B(0), \gamma_B(0)\}\}$$

$$\min\{0.4, 0.6\} \leq \max\{\min\{0.5, 0.5\}, \min\{0.6, 0.6\}\}$$

$$0.4 \leq \max\{0.5, 0.6\}$$

0.4 ≤ 0.6 is satisfied.

This completes the proof.

C. Theorem 3.3

If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then ~A is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

Proof:

An AIFS $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$ in X is called an Anti intuitionistic fuzzy a-ideal of X if

I) $\mu_A(0) \leq \mu_A(x), \lambda_A(0) \geq \lambda_A(x), \gamma_A(0) \leq \gamma_A(x).$

II) $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0 - y)), \mu_A(z)\}, (\forall x, y, z \in X)$

$\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\} (\forall x, y, z \in X)$ and

$1-\gamma_A(y-x) \leq 1 - \max\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\} (\forall x, y, z \in X)$

$1-\gamma_A(y-x) \leq \min\{\gamma_A((x-z) - (0 - y)), \gamma_A(z)\} (\forall x, y, z \in X).$

1) Example: 3.3

Consider a BCI-algebra $X = \{0, a, b\}$ with the following Cayley table:

| | | | |
|---|---|---|---|
| - | 0 | a | B |
| 0 | 0 | b | A |
| A | A | 0 | B |
| B | B | a | 0 |

| | | | |
|-------------|-----|-----|-----|
| X | 0 | A | B |
| μ_A | 0.3 | 0.8 | 0.3 |
| λ_A | 0.6 | 0.3 | 0.6 |
| γ_A | 0.6 | 0.7 | 0.6 |

Let $x=b, y=a, z=0.$

$\mu_A(0) \leq \mu_A(x) \Rightarrow \mu_A(0.3) \leq \mu_A(0.3),$

$\lambda_A(0) \geq \lambda_A(x) \Rightarrow \lambda_A(0.6) \geq \lambda_A(0.6)$ and

$\gamma_A(0) \leq \gamma_A(x) \Rightarrow \gamma_A(0.6) \leq \gamma_A(0.6)$

Then, $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0 - y)), \mu_A(z)\}$

$\mu_A(a-b) \leq \max\{\mu_A((b-0) - (0 - a)), \mu_A(0)\}$

$\mu_A(b) \leq \max\{\mu_A((b-b), \mu_A(0)\}$

$\mu_A(b) \leq \max\{\mu_A(0), \mu_A(0)\}$

$0.3 \leq \max\{0.3, 0.3\}$

$0.3 \leq 0.3$

$\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0 - y)), \lambda_A(z)\} (\forall x, y, z \in X)$

$\lambda_A(a-b) \geq \min\{\lambda_A((b-0) - (0 - a)), \lambda_A(0)\}$

$\lambda_A(b) \geq \min\{\lambda_A(b-b), \lambda_A(0)\}$

$0.6 \geq \min\{\lambda_A(0), \lambda_A(0)\}$

$$0.6 \geq \min \{0.6, 0.6\}$$

$$0.6 \geq 0.6.$$

And $1 - \gamma_A(y-x) \leq 1 - \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\}$ ($\forall x, y, z \in X$)

$$1 - \gamma_A(a-b) \leq 1 - \max\{\gamma_A((b-0) - (0-a)), \gamma_A(0)\}$$

$$1 - \gamma_A(b) \leq 1 - \max\{\gamma_A(b-b), \gamma_A(0)\}$$

$$1 - \gamma_A(b) \leq 1 - \max\{\gamma_A(0), \gamma_A(0)\}$$

$$1-0.6 \leq \min \{0.6, 0.6\}$$

$$0.5 \leq 1 - 0.6$$

$0.5 \leq 0.5$. This completes the proof.

D. Theorem 3.4

If A and B is a support intuitionistic fuzzy a-ideal in BCI-algebra X, then $A \times B$ is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

Proof:

An AIFS $A = \langle X, \mu_A, \lambda_A, \gamma_A \rangle$ in X is called an Anti intuitionistic fuzzy a-ideal of X if

$$\mu_A(0) \leq \mu_A(x), \mu_B(0) \leq \mu_B(x) \text{ and}$$

I) $\max\{\mu_A(0), \mu_B(0)\} \leq \max\{\mu_A(x), \mu_B(x)\}$

$$\mu_{A \times B}(0) \leq \mu_{A \times B}(x), \forall x \in X.$$

$$\lambda_A(0) \geq \lambda_A(x), \lambda_B(0) \geq \lambda_B(x) \text{ and}$$

$$\min\{\lambda_A(0), \lambda_B(0)\} \geq \min\{\lambda_A(x), \lambda_B(x)\}$$

$$\lambda_{A \times B}(0) \geq \lambda_{A \times B}(x), \forall x \in X.$$

$$\gamma_A(0) \leq \gamma_A(x), \gamma_B(0) \leq \gamma_B(x) \text{ and}$$

$$\max\{\gamma_A(0), \gamma_B(0)\} \leq \max\{\gamma_A(x), \gamma_B(x)\}$$

$$\gamma_{A \times B}(0) \leq \gamma_{A \times B}(x), \forall x \in X.$$

II) If $\mu_A(y-x) \leq \max\{\mu_A((x-z) - (0-y)), \mu_A(z)\}$ and

$$\mu_B(y-x) \leq \max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}$$

$$\{\mu_A(y-x), \mu_B(y-x)\} \leq \{\max\{\mu_A((x-z) - (0-y)), \mu_A(z)\},$$

$$\max\{\mu_B((x-z) - (0-y)), \mu_B(z)\}\}$$

$$\mu_{A \times B}(y-x) \leq \max\{\{\mu_A((x-z) - (0-y)), \mu_B((x-z) - (0-y))\}, \{\mu_A(z), \mu_B(z)\}\}$$

$$\mu_{A \cup B}(y-x) \leq \max\{\mu_{A \times B}((x-z) - (0-y)), \mu_{A \times B}(z)\}.$$

If $\lambda_A(y-x) \geq \min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\}$ and

$$\lambda_B(y-x) \geq \min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}$$

$$\{\lambda_A(y-x), \lambda_B(y-x)\} \geq \{\min\{\lambda_A((x-z) - (0-y)), \lambda_A(z)\},$$

$$\min\{\lambda_B((x-z) - (0-y)), \lambda_B(z)\}\}$$

$$\lambda_{AXB}(y-x) \geq \min\{\{\lambda_A((x-z) - (0-y)), \{\lambda_B((x-z) - (0-y))\}, \{\lambda_A(z), \lambda_B(z)\}\}$$

$$\lambda_{AXB}(y-x) \geq \min\{\lambda_{AXB}((x-z) - (0-y)), \lambda_{AXB}(z)\}.$$

$$\text{If } \gamma_A(y-x) \leq \max\{\gamma_A((x-z) - (0-y)), \gamma_A(z)\} \text{ and}$$

$$\gamma_B(y-x) \leq \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}$$

$$\{\gamma_A(y-x), \gamma_B(y-x)\} \leq \{\max\{\gamma_A((x-z) - (0-y)),$$

$$\gamma_A(z)\}, \max\{\gamma_B((x-z) - (0-y)), \gamma_B(z)\}\}$$

$$\gamma_{AXB}(y-x) \leq \max\{\{\gamma_A((x-z) - (0-y)), \gamma_B((x-z) - (0-y))\}, \{\gamma_A(z), \gamma_B(z)\}\}$$

$$\gamma_{AXB}(y-x) \leq \max\{\gamma_{AXB}((x-z) - (0-y)), \gamma_{AXB}(z)\}.$$

Hence AX B is also support intuitionistic fuzzy a-ideal in BCI-algebra X.

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