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K-Even and K-Odd Sequential Harmonious Labeling of Graphs

M. Meenakshi[1], R. Selva Lakshmi[2]
(Assistant Professor & M.phil scholar)

Abstract: In this article we investigate some results on k - even sequential harmonious labeling of graphs and k – odd sequential harmonious labeling of graphs. Triangular snakes and twigs are k – even sequential harmonious labeling of graphs and k – odd sequential harmonious labeling of graphs.

I. INTRODUCTION

The introduction of harmonious graphs was introduced by Graham and Sloane. The establishment of odd sequential graphs was done by Singh and Varkey. Gayathri and Hemalatha gave the brief history about even sequential harmonious labeling of graphs. In the upcoming study, even sequential harmonious labeling of trees, cycle related graphs have been established. Similarly, odd sequential harmonious labeling of trees and cycle related graphs are also introduced. The assignment of integers from 1 to n for nodes, arcs and both subject to particular conditions is known as graph labeling. The mapping domain and co-domain consists of the set of nodes or arcs then the labeling is called vertex or edge labeling. If the vertices of the graph are assigned values under particular condition then that it is known as graph labeling. The introduction of graph labeling was first established in the year 1960's. About 1200 papers the graph labeling techniques have been investigated. In coding theory, X-ray crystallography, radar, astronomy, circuit design, communication network designing etc… labeling methods are applied and it becomes a effective ones.

II. DEFINITION

A. Definition 2.1

A connected graph which admits harmonious labeling is called **harmonious graphs**.

B. Definition 2.2

A twig $TW(p_n), (n \geq 3)$ is a graph obtained from a path by attaching exactly two pendant edges to the internal vertex of the path.

C. Definition 2.3

A labeling is even sequential harmonious labeling if there exists an injection f from the vertex set v to $\{0,1,2,\dots,2q\}$ such that the induced mapping f^+ from the edge set E to $\{2,4,6,\dots,2q\}$ defined by

$f^+(uv) = \begin{cases} f(u)+f(v), if\ f(u)+f(v) is\ even \\ f(u)+f(v)+1, if\ f(u)+f(v) is\ odd \end{cases}$ are distinct.

D. Definition 2.4

A labeling is K- Even sequential harmonious labeling if there exists an injection f from the vertex set v to $\{k-1, k+1, \dots, k+2q-1\}$ such that the induced mapping f^* from the edge set E to $\{2k, 2k+2, 2k+4, \dots, 2k+2q-2\}$ defined by

$f^*(uv) = \begin{cases} f(u)+f(v), if\ f(u)+f(v) is\ even \\ f(u)+f(v)+1, if\ f(u)+f(v) is\ odd \end{cases}$ are distinct.

E. Definition 2.5

A labeling is K- odd sequential harmonious labeling if there exists an injection f from the vertex set v to $\{k-1, k+1, \dots, k+2q-1\}$ such that the induced mapping f^* from the edge set E to $\{2k-1, 2k+1, 2k+3, \dots, 2k+2q-3\}$ defined by

$f^*(uv) = \begin{cases} f(u)+f(v)+1, if\ f(u)+f(v) is\ even \\ f(u)+f(v), if\ f(u)+f(v) is\ odd \end{cases}$ are distinct.

F. Definition 2.6

A labeling is odd sequential harmonious labeling if there exists an injection f from the vertex set v to $\{0,1,2,\dots,2q\}$ such that the induced mapping f^+ from the edge set E to $\{1,3,5,\dots,2q\}$ defined by

$f^+(uv) = \begin{cases} f(u) + f(v) + 1, if\ f(u) + f(v) is\ even \\ f(u) + f(v)\ , if\ f(u) + f(v) is\ odd \end{cases}$ are distinct.

III.RESULTS

A. Theorem 3.1

The triangular snake T_n $(n \geq 2)$ is a k – even sequential harmonious graph for any k.

1) Proof

Let$\{v_1, v_2, \dots, v_{n+1}, u_1, u_2, \dots, u_n\}$be the vertices and $\{e_1, e_2, \dots, e_n, e'_1, e'_2, \dots, e'_{2n}\}$be the edges of T_n which is denoted as in the figure 3.1

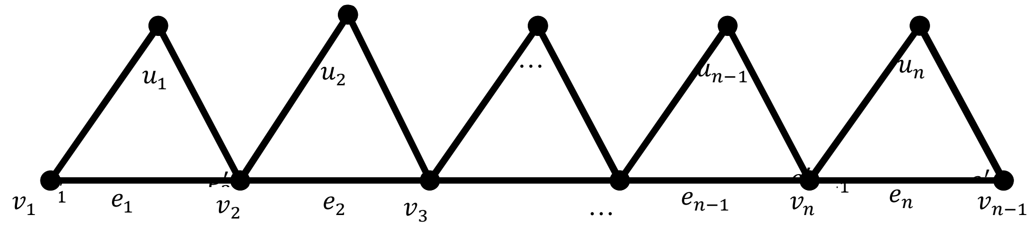

Fig 3.1: T_n with ordinary labeling

First we label the vertices as follows:

Define $f: v \rightarrow \{k-1, k+1, \dots, k+2q-1\}$ by

For $1 \leq i \leq n+1$,

$$f(v_i) = k + i - 2$$

For $1 \leq i \leq n$,

$$f(u_i) = k + 2n + 3i - 2$$

Then the induced edge labels are as follows

For $1 \leq i \leq n$,

$$f(e_i) = 2k + 2(i-1)$$

For $1 \leq i \leq 2n$,

$$f^+(e'_i) = 2k + 2n + 2(i-1)$$

Therefore,

$f^+(E) = \{2k, 2k+2, \dots, 2k+2q-2\}$.so, f is a k- even sequential harmonious labeling and

hence, the graph T_n is a k – even sequential harmonious labeling for any k.

A. Example 3.1

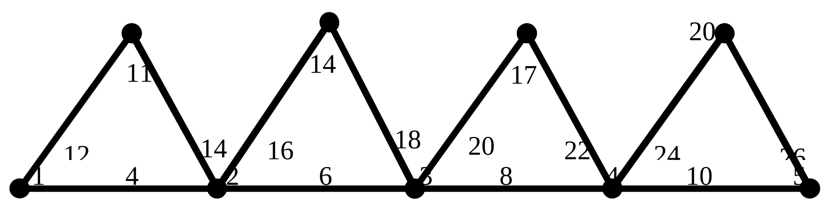

Fig 3.2: 2-ESHL of T_4

B. Theorem 3.2

The triangular snake T_n $(n \geq 2)$ is a k – odd sequential harmonious graph for any k.

1) Proof

Let$\{v_1, v_2, \dots, v_{n+1}, u_1, u_2, \dots, u_n\}$be the vertices and $\{e_1, e_2, \dots, e_n, e'_1, e'_2, \dots, e'_{2n}\}$be the edges of T_n which is denoted as in the figure 3.3

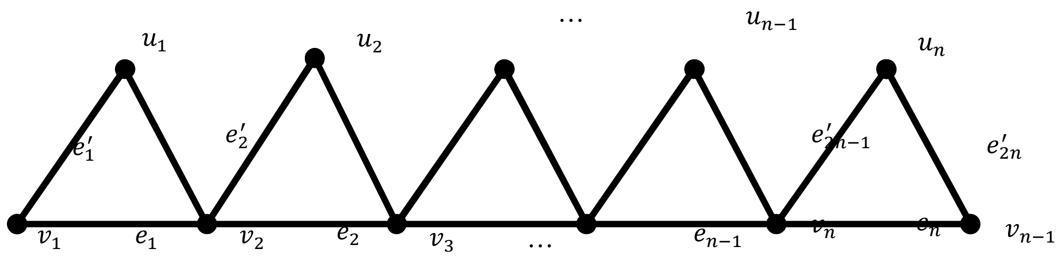

Fig 3.3: T_n with ordinary labeling

First we label the vertices as follows:

Define $f: v \rightarrow \{k-1, k+1, \dots, k+2q-1\}$ by

For $1 \leq i \leq n+1$,

$$f(v_i) = k + i - 2$$

For $1 \leq i \leq n$,

$$f(u_i) = k + 2n + 3i - 3$$

Then the induced edge labels are as follows

For $1 \leq i \leq n$,

$$f^+(e_i) = 2k + 2i - 3$$

For $1 \leq i \leq 2n$,

$$f^+(e'_i) = 2k + 2n + 2i - 3$$

Therefore,

$f^+(E) = \{2k-1, 2k+1, \dots, 2k+2q-3\}$.so, f is a k- odd sequential harmonious labeling and hence, the graph T_n is a k – odd sequential harmonious labeling for any k

C. *Example 3.2*

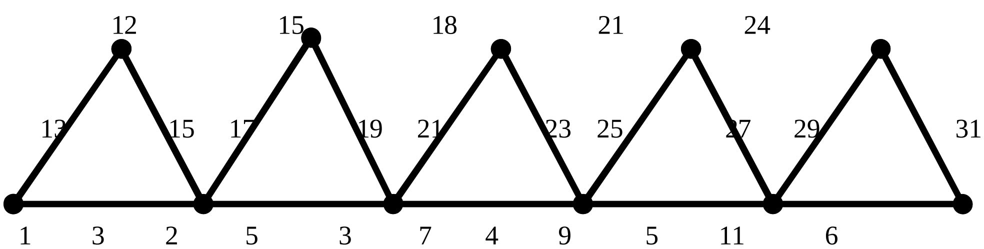

Fig 3.4: 2-OSHL of T_5

D. *Theorem 3.3*

The graph twig $TW(p_n)$, $(n \geq 3)$ is a k- even sequential harmonious graph for any k.

1) Proof

Let the vertices of $TW(p_n)$ be $\{u_i, v_j, w_j : 1 \leq i \leq n \;\&\; 1 \leq j \leq n-2\}$ and the edges of $TW(p_n)$ be $\{(u_i, u_{i+1}), (v_i, u_{i+1}), (w_i, u_{i+1}) : 1 \leq i \leq n-2\}$ which are denoted as in Fig 3.5

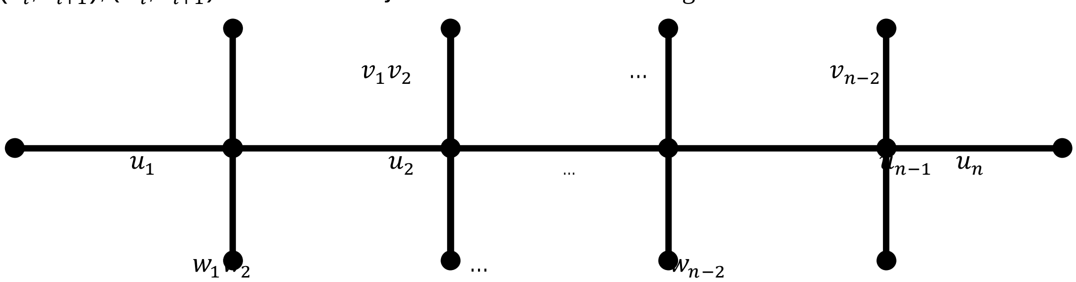

Fig 3.5:$TW(p_n)$with ordinary labeling

We first label the vertices as follows:
Define $f: v(TW(p_n)) \rightarrow \{k-1, k+1, \dots, k+2q-1\}$ by

$f(u_i) = k + i - 2, 1 \leq i \leq n$

$f(v_j) = 2n - 2 + k + 3(j-1), 1 \leq j \leq n-2$

$f(w_j) = 2n + 2 + 3(j-1), 1 \leq j \leq n-2$

Then the induced edge labels are:
$f^+(u_i u_{i+1}) = 2i + 2k - 2, 1 \leq i \leq n-1$
$f^+(v_i u_{i+1}) = 2n + 4(i-1) + 2k - 2, 1 \leq i \leq n-2$
$f^+(w_i u_{i+1}) = 2(n+1) + 4(i-1) + 2k - 2, 1 \leq i \leq n-2$
Clearly, the edge labels are even and distinct, $f^+(E) = \{2k, 2k+2, 2k+4, \dots, 2k+2q-2\}$.
Hence, the graph twig $TW(p_n), (n \geq 3)$ is a k- even sequential harmonious graph for any k.

E. Example 3.3

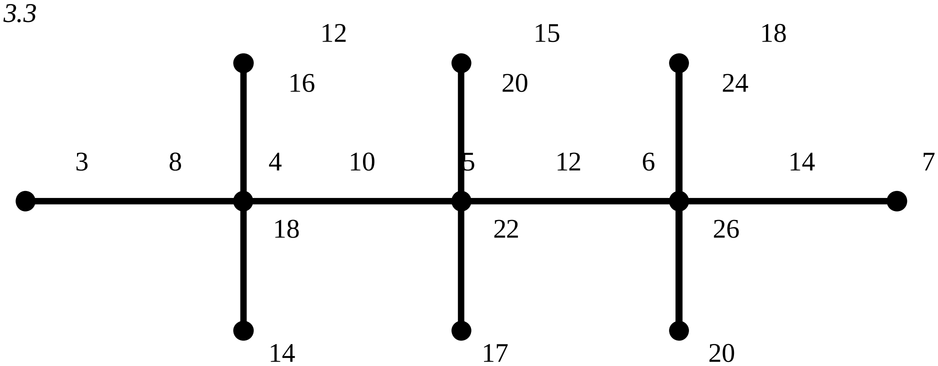

Fig 3.6:$4 - ESHL\ of\ TW(p_5)$

F. Theorem 3.4

The graph twig $TW(n), (n \geq 4)$ is a k- odd sequential harmonious graph for any k.

1) Proof

Let the vertices of $TW(n)$ be $\{v_i, 1 \leq i \leq n, u_i, w_i, 1 \leq j \leq n-2\}$ and the edges of $TW(n)$ be $\{a_i, 1 \leq i \leq n-1, b_i, c_i, 1 \leq i \leq n-2\}$ which are denoted as in Fig 3.7

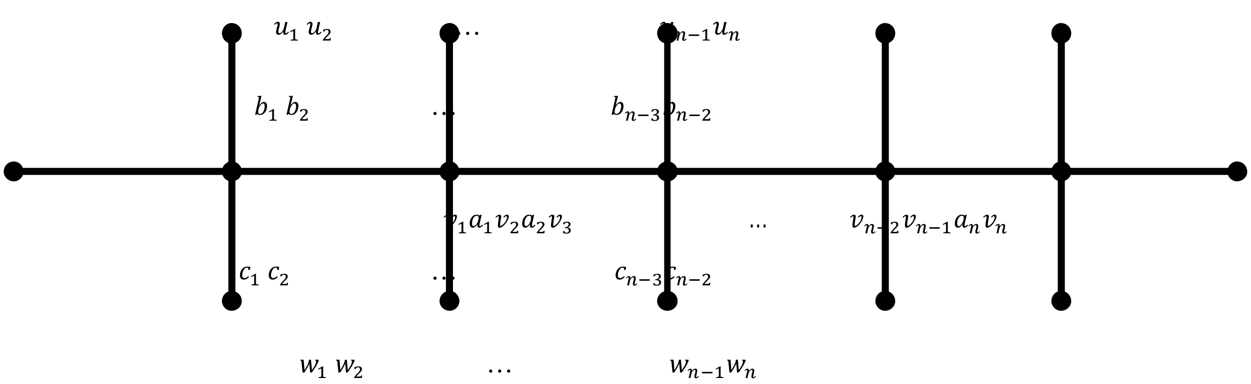

Fig 3.7:$TW(n)$with ordinary labeling

We first label the vertices as follows:
Define $f: v \rightarrow \{k-1, k+1, \dots, k+2q-1\}$ by
For $1 \leq i \leq n$,

$$f(v_i) = \begin{cases} k + 3i - 4 & i\ odd \\ k + 3i - 6 & i\ even \end{cases}$$

For $1 \leq i \leq n-2$,

$$f(u_i) = \begin{cases} k + 3i - 2 & i\ odd \\ k + 3i - 4 & i\ even \end{cases}$$

$$f(w_i) = \begin{cases} k + 3i & i\ odd \\ k + 3i - 2 & i\ even \end{cases}$$

Then the induced edge labels are:

For $1 \leq i \leq n - 1$,

$$f^+(a_i) = 2k + 6i - 7$$
$$f^+(b_i) = 2k + 6i - 5$$
$$f^+(c_i) = 2k + 6i - 3$$

Therefore,

$f^+(E) = \{2k - 1, 2k + 1, \dots, 2k + 2q - 3\}$. So, f is a k- odd sequential harmonious labeling and hence, the twig $TW(n)$, $(n \geq 4)$ is a k- odd sequential harmonious graph for any k.

G. Example 3.4

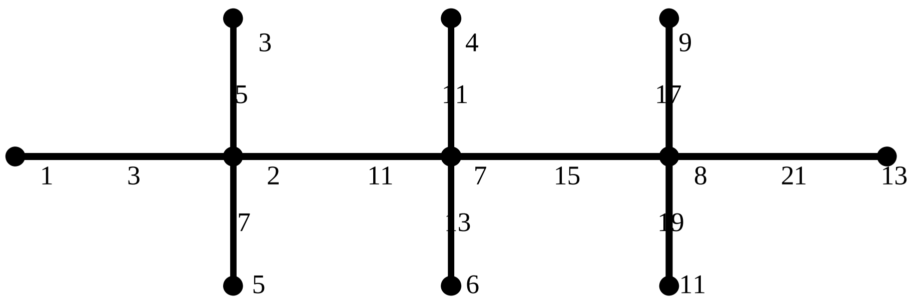

Fig 3.8: $2 - ESHL\ of\ TW(5)$

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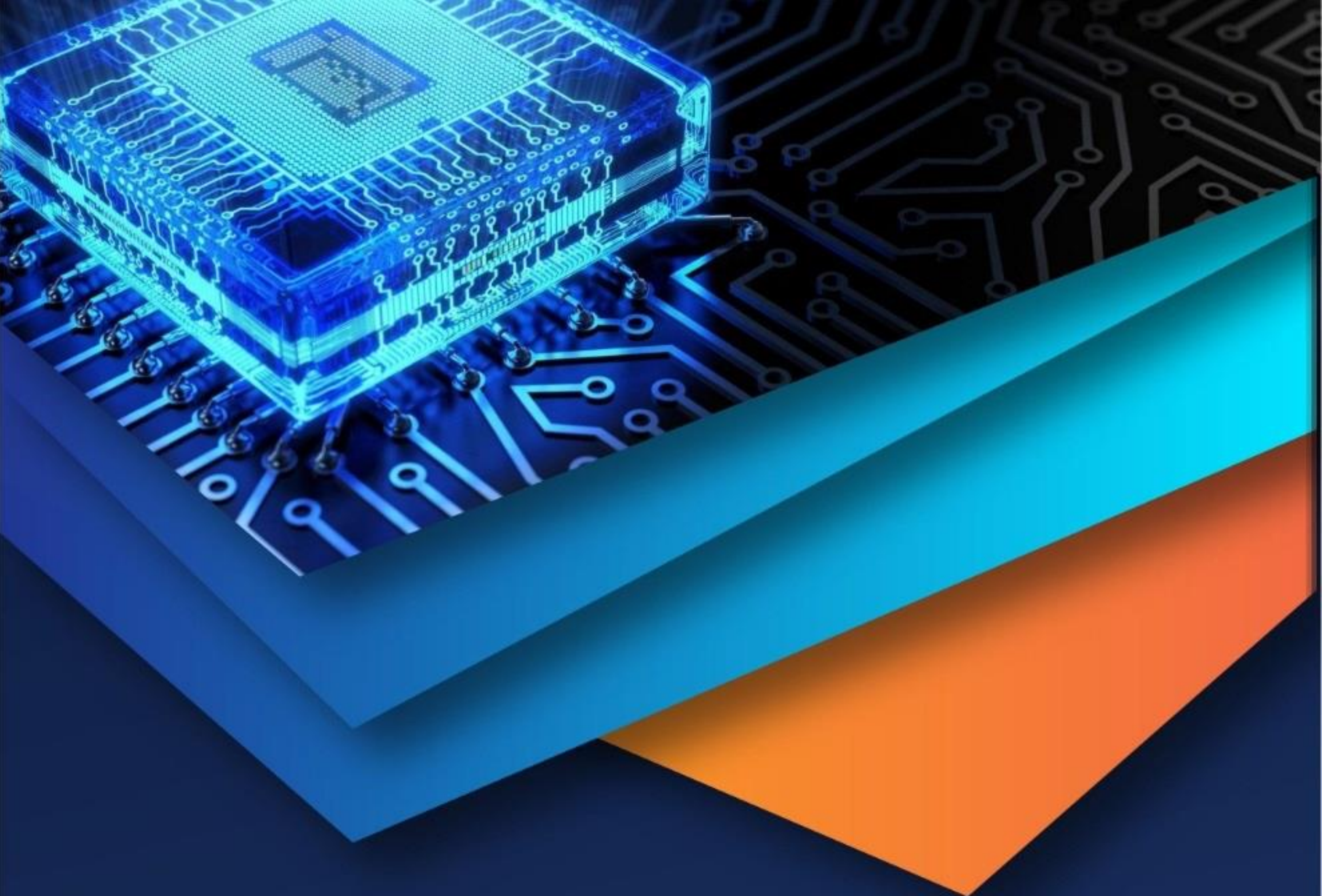

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