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Anti- Homomorphism of Fuzzy Soft Subhemirings of a Hemiring

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Abstract: In this paper, we made an attempt to study the algebraic nature of an anti-homomorphism of fuzzy soft subhemirings of a hemiring. 2000 AMS Subject classification: 05C38, 15A15, 05A15, 15A18.

Keywords: Fuzzy soft set, fuzzy soft subhemiring, anti-fuzzy soft subhemiring, and pseudo Fuzzy soft coset.

I. INTRODUCTION

There are many concepts of universal algebras generalizing an associative ring (R; +; .). Some of them in particular, nearrings and several kinds of semirings have been proven very useful. Semirings (called also halfrings) are algebras (R; +; .) share the same properties as a ring except that (R ; +) is assumed to be a semigroup rather than a commutative group. Semirings appear in a natural manner in some applications to the theory of automata and formal languages. An algebra (R ; +, .) is said to be a semiring if (R ; +) and (R ; .) are semigroups satisfying a. (b + c) = a. b + a. c and (b + c) .a = b. a + c. a for all a, b and c in R. A semiring R is said to be additively commutative if a + b = b + a for all a, b and c in R. A semiring R may have an identity 1, defined by 1. a = a = a. 1 and a zero 0, defined by 0 + a = a = a + 0 and a. 0 = 0 = 0. a for all a in R. A semiring R is said to be a hemiring if it is an additively commutative with zero. After the introduction of fuzzy sets by L.A. Zadeh [17], several researchers explored on the generalization of the concept of fuzzy sets. M. Borah, T. J. Neog and D. K. Sut,[5] were developed some operations of fuzzy soft sets, On operations of soft sets was developed by

A.Sezgin and A. O. Atagun,[13] and KumudBorgohain and ChittaranjanGohain,[7] was developed some New operations on Fuzzy Soft Sets, In this paper, we introduce some Theorems in Fuzzy soft subhemirings of a hemiring.

II. PRELIMINARIES

1) *Definition:* A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)(\varepsilon\epsilon E)$ from this family may be considered as the set of ε -elements of the soft sets (F, E) or as the set of ε - approximate elements of the soft set.

2) *Definition:* Let (U,E) be a soft universe and A ⊆ E. Let $\mathcal{F}$(U) be the set of all fuzzy subsets in U. A pair ($\widetilde{F}$,A) is called a fuzzy soft set over U, where$\widetilde{F}$, is a mapping given by

$\tilde{F}$: A → $\mathcal{F}$(U).

3) *Definition:* Let R be a hemiring. A Fuzzy soft subset (F,A) of R is said to be an Fuzzy soft subhemiring (FSHR) of R if it satisfies the following conditions:

(i) $\mu_{(F,A)}(x + y) \geq \min\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\}$,

(ii) $\mu_{(F,A)}(xy) \geq \min\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\}$, for all x and y in R.

4) *Definition:* Let (R, +,.) be a hemiring. An Fuzzysoftsubhemiring (F,A) of R is said to be an Fuzzysoft normal subhemiring (FSNSHR) of R if it satisfies the following conditions:

(i) $\mu_{(F,A)}(xy) = \mu_{(F,A)}(yx)$, (ii) $\nu_{(F,A)}(xy) = \nu_{(F,A)}(yx)$, for all x and y in R.

5) *Definition:* If (R, +, **.**) and (R', +, **.**) are any two hemirings, then the function f: R → R'is called a **homomorphism** if f(x+y) = f(x)+f(y) and f(xy)=f(x)f(y), for all x and y in R.

6) *Definition:* If (R, +, **.**) and (R', +, **.**) are any two hemirings, then the function f: R → R'is called an **anti-homomorphism** if f(x+y) = f(y)+f(x) and f(xy)=f(y)f(x), for all x and y in R.

7) *Definition:* Let (R, +,.) and (R', +, .) be any two hemirings. Then the function f: R → R'be a hemiringhomomorphism.If f is one-to-one and onto, then f is called a **hemiring isomorphism**.

8) *Definition:* Let (R, +,.) and (R', +, .) be any two hemirings. Then the function f: R → R'be a hemiring anti-homomorphism. If f is one-to-one and onto, then f is called a **hemiring anti-isomorphism**.

9) *Definition:* Let R and R^{I}be any two hemirings. Let f: R → R^{I}be any function and let A be an Fuzzy soft subhemiring in R, V be an Fuzzy soft subhemiring in f(R) = R^{I}, defined by

$\mu_{V(y)} = \sup_{x\in f^{-1}(y)} (\mu_{(F,A)}(x)$ for all x in R and y in R^{I}. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

10) *Definition:* Let (F,A) be an Fuzzy soft subhemiring of a hemiring (R, +, ·) and a in R. Then the pseudo Fuzzy soft coset $(a(F,A))^p$ is defined by $((a\mu_{(F,A)})^p)(x) = p(a)\mu_{(F,A)}(x)$, for every x in R and for some p in P.

III. FUZZY SOFT SUBHEMIRINGS OF A HEMIRING

1) *Theorem: If* (F, A) is an Fuzzy soft subhemiring of a hemiring (R, +,.), then (F, □A) is an Fuzzy soft subhemiring of R.

a) *Proof:* Let (F, A) be an fuzzy soft subhemiring of a hemiring R. Consider (F,A) = {⟨x, $\mu_{(F,A)}$ (x), ⟩}, for all x in R, we take (F, □A) = (F,B) ={⟨x, $\mu_{(F,B)}$ (x), ⟩}, where $\mu_{(F,B)}$ (x) = $\mu_{(F,A)}$ (x),. Clearly, $\mu_{(F,B)}$ (x+y) ≥ min {$\mu_{(F,B)}$ (x), $\mu_{(F,B)}$ (y)}, for all x and y in R and $\mu_{(F,B)}$ (xy) ≥ min {$\mu_{(F,B)}$ (x), $\mu_{(F,B)}$ (y)}, for all x and y in R. Since A is an fuzzy soft subhemiring of R, we have $\mu_{(F,A)}$ (x+y) ≥ min {$\mu_{(F,A)}$ (x), $\mu_{(F,A)}$ (y)}, for all x and y in R, And $\mu_{(F,A)}$ (xy) ≥ min { $\mu_{(F,A)}$ (x), $\mu_{(F,A)}$ (y) }, for all x and y in R, for all x and y in R. Hence (F,B) = (F, □A) is an fuzzy soft subhemiring of a hemiring R.

2) *Theorem:* If (F,A) is an fuzzy soft subhemiring of a hemiring (R, +, .), then (F, ◊A) is an fuzzy soft subhemiring of R.

a) *Proof:* Let (F,A) be an fuzzy soft subhemiring of a hemiring R. That is

(F,A) = {⟨x, $\mu_{(F,A)}$ (x)⟩}, for all x in R. Let (F,◊A) = (F,B)= ⟨x, $\mu_{(F,B)}$ (x)⟩}, for all x and y in R. Since (F,A) is an fuzzy soft subhemiring of R, which implies that 1–$\mu_{(F,B)}$ (xy) ≤ max {(1–$\mu_{(F,B)}$ (x)), (1–$\mu_{(F,B)}$ (y))},which implies that $\mu_{(F,B)}$ (xy) ≥ 1– max {(1–$\mu_{(F,B)}$ (x)), (1–$\mu_{(F,B)}$ (y))}= min {$\mu_{(F,B)}$ (x), $\mu_{(F,B)}$ (y)}. Therefore, $\mu_{(F,B)}$ (xy) ≥ min {$\mu_{(F,B)}$ (x), $\mu_{(F,B)}$ (y) }, for all x and y in R. Hence (F,B) = (F,◊A) is an fuzzysoftsubhemiring of a hemiring R.

3) *Theorem:* Let (R, +, .) be a hemiring and (F,A) be a non-empty subset of R. Then (F,A) is a subhemiring of R if and only if (F,B) = ⟨$\chi_{(F,A)}$, $\bar{\chi}_{(F,A)}$⟩is a fuzzy soft subhemiring of R, where $\chi_{(F,A)}$ is the characteristic function.

a) *Proof:* Let (R, +, .) be a hemiring and (F,A) be a non-empty subset of R. First let (F,A) be a subhemiring of R. Take x and y in R.

Case (i): If x and y in (F,A), then x+y, xy in (F,A), since (F,A) is a subhemiring of R, $\chi_{(F,A)}(x)$ =$\chi_{(F,A)}(y)$ =$\chi_{(F,A)}(x+y)$= $\chi_{(F,A)}(xy)$ =1 and $\chi_{(F,A)}(x)$ =$\chi_{(F,A)}(y)$= $\chi_{(F,A)}(x+y)$ =$\chi_{(F,A)}(xy)$ = 0. So, $\chi_{(F,A)}(x+ y)$ ≥ min {x), $\mu_{(F,A)}\chi(y)$ }, for all x and y in R, $\chi_{(F,A)}(xy)$ ≥ min{ $\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }, for all x and y in R. So, $\chi_{(F,A)}(x+ y)$ ≤ max{ $\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }, for all x and y in R, $\chi_{(F,A)}(xy)$ ≤ max{ $\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }, for all x and y in R.

Case (ii): If x in (F,A), y not in (F,A) (or x not in (F,A), y in (F,A)), then x+y, xy may or may not be in (F,A), $\chi_{(F,A)}(x)$ =1, $\chi_{(F,A)}(y)$ = 0 (or) $\chi_{(F,A)}(x)$ =0, $\chi_{(F,A)}(y)$ =1), $\chi_{(F,A)}(x+y)$ =$\chi_{(F,A)}(xy)$ = 1 (or 0) and $\chi_{(F,A)}$ (x) = 0, $\chi_{(F,A)}(x)$ (y) = 1(or) $\chi_{(F,A)}$ (x)=1, $\chi_{(F,A)}(y)$ =0)= $\chi_{(F,A)}$ (x+y) =$\chi_{(F,A)}$ (xy) = 0 (or 1). Clea rly $\chi_{(F,A)}$ (x+ y) ≥ min{ $\chi_{(F,A)}$ (x), $\chi_{(F,A)}$ (y) }, for all x and y in R $\chi_{(F,A)}$ (xy) ≥ min{ $\chi_{(F,A)}$ (x), $\chi_{(F,A)}$ (y)}, for all x and y in R,and$\chi_{(F,A)}$ (x + y) ≤ ma x{ $\chi_{(F,A)}(x)$, $\chi_{(F,A)}$ (y) }, for all x and y in R $\chi_{(F,A)}$ (xy) ≤ ma x{ $\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }, for all x and y in R.

Case (iii): If x and y not in (F,A) , then x+y, xy may or may not be in (F,A) , $\chi_{(F,A)}$ (x)= $\chi_{(F,A)}$ (y) = 0, $\chi_{(F,A)}(x+y)$= $\chi_{(F,A)}$(xy) =1 or 0 and $\chi_{(F,A)}(x)$ = $\chi_{(F,A)}(y)$ =1, $\chi_{(F,A)}(x+y)$= $\chi_{(F,A)}(xy)$= 0 or 1. Clearly $\chi_{(F,A)}(x+ y)$ ≥ min{ $\chi_{(F,A)}$ (x), $\chi_{(F,A)}(y)$ }, for all x and y in R $\chi_{(F,A)}(xy)$ ≥ min{ $\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }, for all x and y in R, and $\chi_{(F,A)}(x + y)$ ≤ ma x {$\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }, for all x and y in R $\chi_{(F,A)}$ (xy) ≤ ma x {$\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }, for all x a nd y in R. So in all the three cases, we have B is a fuzzy soft subhemiring of (F,A) hemiring R. Conversely, let x and y in (F,A) , since (F,A) is (F,A) non empty subset of R, so, $\chi_{(F,A)}$ (x) =$\chi_{(F,A)}$ (y) = 1, $\chi_{(F,A)}(x)$ = $\chi_{(F,A)}(y)$ = 0.Since B = ⟨$\chi_{(F,A)}$, $\bar{\chi}_{(F,A)}$⟩is a fuzzy soft subhemiring of R, we have $\chi_{(F,A)}(x + y)$ ≥ min {$\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }= min {1, 1} = 1, $\chi_{(F,A)}(xy)$ ≥ min{ $\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }= min {1,1 } = 1. Therefore $\chi_{(F,A)}(x + y)$ =$\chi_{(F,A)}(xy)$ =1.and, $\chi_{(F,A)}(x + y)$ ≤ ma x {$\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ }= max{0, 0 } = 0, $\chi_{(F,A)}(xy)$ ≤ max {$\chi_{(F,A)}(x)$, $\chi_{(F,A)}(y)$ } = ma x { 0, 0 } = 0.Therefore $\chi_{(F,A)}(x + y)$ = $\chi_{(F,A)}(xy)$ = 0. Hence x + y and xy in (F,A) , so (F,A) is a subhemiring of R.

In the following Theorem ◦ is the composition operation of functions:

4) *Theorem:* Let (F,A) be an fuzzy soft subhemiring of a hemiring H and f is an isomorphism from a hemiring R onto H. Then (F,A)◦f is a fuzzy soft subhemiring of R.

a) *Proof:* Let x and y in R and (F,A) be an fuzzy soft subhemiring of a hemiring H. Then we have, ($\mu_{(F,A)}$◦f)(x+y) = $\mu_{(F,A)}$ (f(x+y))= $\mu_{(F,A)}$ (f(x)+ f(y)) , as f is an isomorphism ≥ min { $\mu_{(F,A)}$ (f(x)), $\mu_{(F,A)}$ (f(y))}, = min {($\mu_{(F,A)}$◦f)(x), ($\mu_{(F,A)}$◦f)(y)},which implies that ($\mu_{(F,A)}$◦f)(x+y) ≥ min {($\mu_{(F,A)}$◦f)(x), ($\mu_{(F,A)}$◦f)(y)}.And, ($\mu_{(F,A)}$◦f)(xy) = $\mu_{(F,A)}$ (f(xy))= $\mu_{(F,A)}$ (f(x)f(y)), as f is an isomorphism ≥ min {$\mu_{(F,A)}$ (f(x)), $\mu_{(F,A)}$ (f(y))} = min {($\mu_{(F,A)}$◦f)(x), ($\mu_{(F,A)}$◦f)(y)}, which implies that ($\mu_{(F,A)}$◦f)(xy) ≥ min{($\mu_{(F,A)}$◦f)(x), ($\mu_{(F,A)}$◦f)(y)}.Therefore (F,A)◦f is a Fuzzy soft subhemiring of a hemiring R.

5) *Theorem:* Let (F,A) be an fuzzy soft subhemiring of a hemiring h and f is an anti-isomorphism from a hemiring r onto h. then (F,A)$\circ$f is a fuzzy soft subhemiring of R.

a) *Proof:* Let x and y in R and (F,A) be an fuzzysoftsubhemiring of a hemiring H. Then we have,

$(\mu_{(F,A)} \circ f)(x+y) = \mu_{(F,A)}(f(x+y)) = \mu_{(F,A)}(f(y)+f(x))$, as f is an anti-isomorphism $\geq \min\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\} = \min\{(\mu_{(F,A)}\circ f)(x), (\mu_{(F,A)}\circ f)(y)\}$,which implies that $(\mu_{(F,A)}\circ f)(x+y) \geq \min\{(\mu_{(F,A)}\circ f)(x), (\mu_{(F,A)}\circ f)(y)\}$.And, $(\mu_{(F,A)}\circ f)(xy) = \mu_{(F,A)}(f(xy)) = \mu_{(F,A)}(f(y)f(x))$, as f is an anti-isomorphism $\geq \min\{\mu_{(F,A)}(f(x)), \mu_{(F,A)}(f(y))\}$, $= \min\{(\mu_{(F,A)}\circ f)(x), (\mu_{(F,A)}\circ f)(y)\}$, which implies that $(\mu_{(F,A)}\circ f)(xy) \geq \min\{(\mu_{(F,A)}\circ f)(x), (\mu_{(F,A)}\circ f)(y)\}$. Therefore (F,A)$\circ$f is an fuzzysoftsubhemiring of the hemiring R.

6) *Theorem:* Let (F,A) be an fuzzy soft subhemiring of a hemiring (R, +, .), then the pseudo fuzzy soft coset $(a(F,A))^p$ is an fuzzy soft subhemiring of a hemiring R, for every a in R.

a) *Proof:* Let (F,A) be an fuzzy soft subhemiring of a hemiring R. For every x and y in R, we have, $((a\mu_{(F,A)})^p)(x+y) = p(a)\mu_{(F,A)}(x+y) \geq p(a)\min\{(\mu_{(F,A)}(x), \mu_{(F,A)}(y)\} = \min\{p(a)\mu_{(F,A)}(x), p(a)\mu_{(F,A)}(y)\} = \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$.Therefore, $((a\mu_{(F,A)})^p)(x+y) \geq \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Now, $((a\mu_{(F,A)})^p)(xy) = p(a)\mu_{(F,A)}(xy) \geq p(a)\min\{\mu_{(F,A)}(x), \mu_{(F,A)}(y)\} = \min\{p(a)\mu_{(F,A)}(x), p(a)\mu_{(F,A)}(y) = \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$.Therefore, $((a\mu_{(F,A)})^p)(xy) \geq \min\{((a\mu_{(F,A)})^p)(x), ((a\mu_{(F,A)})^p)(y)\}$. Hence $(a(F,A))^p$ is an fuzzy soft subhemiring of a hemiring R.

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