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Strong form of Fuzzy Closed Sets in Fuzzy Topological Space

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Abstract: In this paper we have introduced a new class of fuzzy sets called fuzzy strongly (gsp)*-closed sets, properties of this set are investigated and we introduce new fuzzy spaces namely, fuzzy $T_s(gsp)^*$ -space, fuzzy $gT_s(gsp)^*$ -space, fuzzy $g^*T_s(gsp)^*$ -space and fuzzy $g^{**}T_s(gsp)^*$ -space. Further strongly (gsp)*-continuous mappings are also introduced and investigated.

Keywords: fuzzy Strongly (gsp)*-closed sets, fuzzy Strongly (gsp)*-continuous maps, fuzzy $T_s(gsp)^*$ -space, fuzzy $gT_s(gsp)^*$ -space, fuzzy $g^*T_s(gsp)^*$ -space and fuzzy $g^{**}T_s(gsp)^*$ -space.

I. PRELIMINARIES

Throughout this paper (X, τ) and (Y, σ) represent non-empty fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A of a space (X, τ) , $cl(A)$ and $int(A)$ denote the fuzzy closure and the fuzzy interior of A respectively.

A. Definition 1.1

A Subset A of fuzzy topological space (X, τ) is called;

- 1) Fuzzy semi open set if $A \subseteq cl(int(A))$ and a fuzzy semi-closed set if $int(cl(A)) \subseteq A$.
- 2) Fuzzy semi pre-open set if $A \subseteq cl(int(cl(A)))$ and a fuzzy semi-pre closed set if $int(cl(int(A))) \subseteq A$
- 3) Fuzzy regular -open set if $int(cl(A))=A$ and a fuzzy regular -closed.

B. Definition 1.2

A Subset A of fuzzy topological space (X, τ) is called;

- 1) Fuzzy generalized closed set (briefly fuzzy g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy open in (X, τ)
- 2) Fuzzy g^* -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy g open in (X, τ)
- 3) Fuzzy g^{**} -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy g^* open in (X, τ)
- 4) Fuzzy wg - closed set if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy open in (X, τ)
- 5) Fuzzy regular generalized closed set (briefly fuzzy rg-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy regular open in (X, τ)
- 6) Fuzzy sg^{**} - closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy g^{**} open in (X, τ)
- 7) Fuzzy sg^* - closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy g^* open in (X, τ)
- 8) Fuzzy generalized semi-closed set (briefly fuzzy gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy open in (X, τ)
- 9) Fuzzy gsp - closed set if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy open in (X, τ)
- 10) Fuzzy (gsp)*- closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U fuzzy gsp is open in (X, τ)

C. Definition 1.3

A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called ;

- 1) fuzzy g – continuous if $f^{-1}(V)$ is a fuzzy g-closed set of (X, τ) for every fuzzy closed set V of (Y, σ)
- 2) fuzzy g^* – continuous if $f^{-1}(V)$ is a fuzzy g^* -closed set of (X, τ) for every fuzzy closed set V of (Y, σ)
- 3) fuzzy g^{**} – continuous if $f^{-1}(V)$ is a fuzzy g^{**} -closed set of (X, τ) for every fuzzy closed set V of (Y, σ)
- 4) fuzzy rg – continuous if $f^{-1}(V)$ is a fuzzy rg -closed set of (X, τ) for every fuzzy closed set V of (Y, σ)
- 5) fuzzy wg – continuous if $f^{-1}(V)$ is a fuzzy wg -closed set of (X, τ) for every fuzzy closed set V of (Y, σ)
- 6) fuzzy (gsp)* – continuous if $f^{-1}(V)$ is a fuzzy (gsp)* -closed set of (X, τ) for every fuzzy closed set V of (Y, σ)

D. Definition 1.4

A fuzzy topological space (X, τ) is said to be;

- 1) Fuzzy $T_{1/2}^*$ space if every fuzzy g^* -closed set in it is fuzzy closed.
- 2) Fuzzy T_d space if every fuzzy g_s -closed set in it is fuzzy g -closed.

II. FUZZY STRONGLY (GSP)* - CLOSED SETS IN FUZZY TOPOLOGICAL SPACE

We introduce the following definition

A. Definition 2.1

A subset A of a fuzzy Topological space (X, τ) is said to be a fuzzy strongly (gsp)*-closed set if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy gsp-open.

- 1) *Lemma 2.1:* Every fuzzy closed set is fuzzy strongly (gsp)*-closed.
- 2) *Proof:* Let A be a fuzzy closed. Then $cl(A) = A$. Let us prove that A is fuzzy strongly (gsp)* - closed. Let $A \subseteq U$ and U be fuzzy gsp-open. Then $cl(A) \subseteq U$. Since A is fuzzy closed $cl(int(A)) \subseteq cl(A) \subseteq U$. Then $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy gsp - open. so A is fuzzy strongly (gsp)* - closed. The converse of the above proposition need not be true in general as seen in the following example.

B. Lemma 2.2

Every fuzzy g -closed set is fuzzy strongly (gsp)*-closed.

- 1) *Proof:* Let A be fuzzy g -closed. Then $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy open in (X, τ) . To prove A is fuzzy strongly (gsp)* - closed. Then $A \subseteq U$ and U be fuzzy (gsp) open. We have $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy open in (X, τ) . Since every fuzzy open set is (fuzzy gsp) -open. We have $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy (gsp)-open in (X, τ) . But $cl(int(A)) \subseteq cl(A) \subseteq U$ whenever $A \subseteq U$ and U is (fuzzy gsp)- open in (X, τ) then $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy (gsp) - open in (X, τ) so A is fuzzy strongly (gsp)*-closed. The converse of the above proposition need not be true in general as seen in the following example.

C. Lemma 2.3

Every fuzzy g^* -closed set is fuzzy strongly (gsp)* - closed.

The converse of the above proposition need not be true in general as seen in the following example.

D. Lemma 2.4

Every fuzzy rg - closed set is fuzzy strongly (gsp)* - closed.

- 1) *Proof:* Let A be fuzzy rg -closed set. Then $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy regular-open in (X, τ) . To prove A is fuzzy strongly (gsp)* - closed. Let $A \subseteq U$ and U be fuzzy (gsp) open. Since every fuzzy regular-open set is fuzzy (gsp)-open. We have $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy (gsp)-open in (X, τ) . But $cl(int(A)) \subseteq cl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy (gsp)-open in (X, τ) . so $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy (gsp)-open in (X, τ) . So A is fuzzy strongly (gsp)*-closed.
- 2) *Remark 2.1:* fuzzy Strongly (gsp)* - closedness is independent of fuzzy semi-closedness

E. Lemma 2.5

Every fuzzy (gsp)* - closed set is fuzzy strongly (gsp)* - closed set

- 1) *Proof:* Let A be fuzzy (gsp)* -closed set. Then $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy gsp-open in (X, τ) . To prove A is fuzzy strongly (gsp)* - closed. Let $A \subseteq U$ and U be fuzzy (gsp) open. Since every fuzzy (gsp)* - open set is fuzzy (gsp)-open. We have $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy (gsp)-open in (X, τ) . But $cl(int(A)) \subseteq cl(A) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy (gsp)-open in (X, τ) $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy (gsp)-open in (X, τ) so A is fuzzy strongly (gsp)* -closed. The converse of the above proposition need not be true in general as seen in the following example.

- 2) *Theorem 2.1:* Every fuzzy g^{**} -closed set is fuzzy strongly $(gsp)^*$ -closed. The converse of the above proposition need not be true in general .
- 3) *Remark 2.2:* fuzzy Strongly $(gsp)^*$ -closed is independent of fuzzy sg^{**} -closed.
- 4) *Remark 2.3:* Strongly fuzzy $(gsp)^*$ -closed is independent of fuzzy sg^* -closed.

F. Lemma 2.6

Every fuzzy wg -closed set is fuzzy strongly $(gsp)^*$ -closed.

- 1) *Proof:* Let A be fuzzy wg -closed .Then $cl(int(A)) \subseteq U$ Whenever $A \subseteq U$ and U is fuzzy open in (X, τ) .To prove A is fuzzy strongly $(gsp)^*$ -closed .Let $A \subseteq U$ and U be fuzzy (gsp) open. Since every fuzzy wg -open set is fuzzy (gsp) - open .We have $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy (gsp) -open in (X, τ) then $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is fuzzy (gsp) -open in (X, τ) . A is fuzzy strongly $(gsp)^*$ -closed.

III. FUZZY STRONGLY (GSP)* -CONTINUOUS MAPS

We introduce the following definitions

A. Definition 3.1

A function $f:(X, \tau) \rightarrow (Y, \sigma)$ is called fuzzy Strongly $(gsp)^*$ -continuous if $f^{-1}(V)$ is a fuzzy strongly $(gsp)^*$ -closed set in (X, τ) for every fuzzy closed set V of (Y, σ) .

fuzzy continuous map is fuzzy strongly $(gsp)^*$ -continuous

Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy continuous map. Let us prove that f is fuzzy strongly $(gsp)^*$ -continuous. Let F be a fuzzy closed set in (Y, σ) . Since f is fuzzy continuous $f^{-1}(F)$ is closed in (X, τ) then $f^{-1}(F)$ is fuzzy strongly $(gsp)^*$ -closed so f is fuzzy strongly $(gsp)^*$ -continuous the converse of the above Theorem is not true

B. Theorem 3.2

Every fuzzy g -continuous map is fuzzy strongly $(gsp)^*$ -continuous

- 1) *Proof:* Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy g -continuous. Let F be a fuzzy closed set in (Y, σ) . Since f is fuzzy g -continuous $f^{-1}(F)$ is fuzzy g -closed in (X, τ) .then , $f^{-1}(F)$ is fuzzy strongly $(gsp)^*$ -closed so f is fuzzy strongly $(gsp)^*$ -continuous fuzzy closed g^{**} -closed fuzzy g -closed fuzzy wg -closed fuzzy sg^{**} -closed fuzzy g^* -closed fuzzy strongly $(gsp)^*$ -closed fuzzy $(gsp)^*$ -closed fuzzy Semi-closed sg^* -closed sg -closed .The converse of the above Theorem is not true

C. Theorem 3.3

Every g^* -continuous map is strongly $(gsp)^*$ -continuous.

- 1) *Proof:* Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a g^* -continuous. Let F be a closed set in (Y, σ) . Since f is g^* -continuous $f^{-1}(F)$ is g^* -closed in (X, τ) .By Theorem (3.6), $f^{-1}(F)$ is strongly $(gsp)^*$ -closed so f is strongly $(gsp)^*$ -continuous The converse of the above Theorem is not true

D. Theorem 3.4

Every fuzzy g^{**} -continuous map is fuzzy strongly $(gsp)^*$ -continuous.

- 1) *Proof:* Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a fuzzy g^{**} -continuous. Let F be a fuzzy closed set in (Y, σ) . Since f is fuzzy g^{**} -continuous $g^{-1}(F)$ is g^{**} -closed in (X, τ) .then $f^{-1}(F)$ is fuzzy strongly $(gsp)^*$ -closed so f is fuzzy strongly $(gsp)^*$ -continuous The converse of the above Theorem is not true.

IV. APPLICATIONS OF FUZZY STRONGLY (GSP)* - CLOSED SETS

As application of fuzzy strongly $(gsp)^*$ -closed sets, new spaces, namely fuzzy $Ts(gsp)^*$ space , $gTs(gsp)^*$ space , fuzzy $g^*T s(gsp)^*$, fuzzy $g^{**}Ts(gsp)^*$ space are introduced. We introduced the following definitions.

A. Definition 4.1

A fuzzy space (X, τ) is called a fuzzy $Ts(gsp)^*$ -space if every fuzzy strongly $(gsp)^*$ -closed set is closed.

B. Definition 4.2

A fuzzy space (X, τ) is called a fuzzy $gTs(gsp)^*$ - space if every fuzzy strongly $(gsp)^*$ - closed set is fuzzy g closed.

C. Definition 4.3

A fuzzy space (X, τ) is called a fuzzy $g^*Ts(gsp)^*$ -space if every fuzzy strongly $(gsp)^*$ - closed set is g^* -closed

D. Definition 4.4

A fuzzy space (X, τ) is called a fuzzy $g^{**}Ts(gsp)^*$ -space if every fuzzy strongly $(gsp)^*$ - closed set is fuzzy g^{**} -closed.

2) **Theorem 4.1:** Every fuzzy $Ts(gsp)^*$ -space is fuzzy $T_{1/2}^*$ -space.

3) **Proof:** Let (X, τ) be a fuzzy $Ts(gsp)^*$ - space. Let us prove that (X, τ) is a fuzzy $T_{1/2}^*$ - space. Let A be a fuzzy g^* - closed set. Since every fuzzy g^* - closed set is fuzzy strongly $(gsp)^*$ -closed, A is fuzzy strongly $(gsp)^*$ - closed. Since (X, τ) is a fuzzy $Ts(gsp)^*$ - space, A is fuzzy closed. (X, τ) is a fuzzy $T_{1/2}^*$ - space.

4) **Theorem 4.2:** Every fuzzy $Ts(gsp)^*$ -space is fuzzy $gTs(gsp)^*$ -space.

5) **Proof:** Let A be a fuzzy strongly $(gsp)^*$ -closed set. Then A is fuzzy closed. Since the fuzzy space is $Ts(gsp)^*$ - space. And every closed set is fuzzy g -closed. Hence A is fuzzy g - closed. (X, τ) is a fuzzy $gTs(gsp)^*$ - space. The converse is not true.

6) **Theorem 4.3:** Every fuzzy $g^*Ts(gsp)^*$ -space is fuzzy $gTs(gsp)^*$ -space.

7) **Proof:** Let A be a fuzzy strongly $(gsp)^*$ -closed. Then A is fuzzy g^* -closed, since the fuzzy space is a fuzzy $g^*Ts(gsp)^*$ -space since. Every fuzzy g^* -closed set is fuzzy g -closed. Hence A is fuzzy g -closed then (X, τ) is a fuzzy $gTs(gsp)^*$ -space. The converse is not true.

8) **Theorem 4.4:** Every $g^*Ts(gsp)^*$ -space is $g^{**}Ts(gsp)^*$ -space.

9) **Proof:** Let A be a strongly $(gsp)^*$ -closed set. Then A is g^* -closed, since the space is a $g^*Ts(gsp)^*$. Since Every g^{**} -closed set is g^* -closed. Hence A is g^{**} -closed. (X, τ) is a $g^{**}Ts(gsp)^*$ -space.

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