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LICT Subdivision Double Domination in Graphs

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Abstract: ***Let $S(G)$ be the subdivision graph of G. The lict graph of $n[S(G)]$ of $S(G)$ is a graph whose vertex set is the union of the set of edges and set of cutvertices of $S(G)$ in which two vertices adjacent if and only if the corresponding members are adjacent or incident. A subset D^d of $V[n(S(G))]$ is double dominating set of $n[S(G)]$ if for every vertex $v \in V[n(S(G))]$, $|N[v] \cap D^d| \geq 2$, that is v is in D^d and has at least one neighbour in D^d or v is in $V[n(S(G))] - D^d$ and has at least two neighbours in D^d. The lict subdivision double dominating number $\gamma_{ddns}(G)$ is a minimum cardinality of the lict subdivision double dominating set of G and is denoted by $\gamma_{ddns}(G)$. In this paper, we establish some sharp bounds for $\gamma_{ddns}(G)$. Also some upper and lower bounds on $\gamma_{ddns}(G)$ in terms of the vertices, edges and other different parameters of Gand not in terms of the element of $n[S(G)]$. Further, its relation with other different dominating parameters is also obtained.***

Subject classification number: $AMS - 05C69, 05C70$.
Keyword: Lict subdivision graph/ Dominating set/Double domination

I. INTRODUCTION

In this paper, all the graphs considered here are simple, finite, non-trivial, undirected and connected. The vertex set and edge set of graph G are denoted by $V(G) = p$ and $E(G) = q$ respectively. Terms not defined here are used in the sense of Harary [1]. The neighbourhood of a vertex $v \in V$ is defined by $N(v) = \{u \in V / uv \in E\}$. The close neighbourhood of a vertex$v$ is $N[v] = N(v) \cup \{v\}$. The order $|V(G)|$ of G is denoted by p. The degree of v is $d(v) = |N(v)|$. The maximum degree of a graph G is denoted by $\Delta(G)$ and the minimum degree is denoted by $\delta(G)$. A vertex cover in a graph G is a set of vertices that covers all the edges of G. The vertex covering number $\alpha_0(G)$ is the minimum cardinality of a vertex cover in G. A set of vertices in a graph G is called independent set if no two vertices in the set are adjacent. The vertex independence number $\beta_0(G)$ is the maximum cardinality of an independent set of vertices. For a vertex v of a graph G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The maximum eccentricity is its diameter, $diam(G)$. A set D of vertices in a graph G is called a dominating set of G if every vertex in $V - D$ is adjacent to some vertex in D. The domination number of G, denoted by $\gamma(G)$ is the minimum cardinality of a dominating set. The domination in graphs with many variations is now well studied in graph theory. A thorough study of domination appears in [2]. Let $S(G)$ be the subdivision graph of G. The lict graph of $n[S(G)]$ of $S(G)$ is a graph whose vertex set is the union of the set of edges and set of cutvertices of $S(G)$ in which two vertices adjacent if and only if the corresponding members are adjacent or incident. A subset D^d of $V[n(S(G))]$ is double dominating set of $n[S(G)]$ if for every vertex $v \in V[n(S(G))]$, $|N[v] \cap D^d| \geq 2$, that is v is in D^d and has at least one neighbour in D^d or v is in $V[n(S(G))] - D^d$ and has at least two neighbours in D^d. The lict subdivision double dominating number $\gamma_{ddnb}(G)$ is a minimum cardinality of the lict subdivision double dominating set of G and is denoted by $\gamma_{ddns}(G)$. The graph valued function related to domination parameters have been studied in [4,5,6,7,9,10,11,12,13,14,15]. Further in [8], the subdivision of G with graphvalued function and related with domination parameters has been established. In this paper, we establish some sharp bounds for $\gamma_{ddns}(G)$. Also some upper and lower bounds on $\gamma_{ddns}(G)$ in terms of the vertices, edges and other different parameters of G and not in terms of the element of $n[S(G)]$. Further, its relation with other different dominating parameters is also obtained.

II. RESULTS

We need the following theorems to prove our results.

1) *Theorem A[1]:* For any path P_n, the vertex covering number is $\alpha_0(P_n) = \begin{cases} \frac{n}{2} & \text{if n is even} \\ \frac{n-1}{2} & \text{if n is odd} \end{cases}$

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2) *Theorem B[1]:* For any path P_n, the edge covering number is $\alpha_1(P_n) = \begin{cases} \frac{n}{2} & \text{if n is even} \\ \frac{n+1}{2} & \text{if n is odd} \end{cases}$
3) *Theorem C[2]:* A graph G is eulerian if and only if G is of even degree.
4) *Theorem D[3]:* For any connected graph G, with $p \geq 4$, $G \neq K_p$, $\gamma_{ss} = \alpha_0(G)$.

III. THE LICT SUBDIVISION DOUBLE DOMINATION NUMBER OF A GRAPH.

In this section, we characterise the lict subdivision double domination number by giving a necessary and sufficient condition for it and also establish the conditions for a lict subdivision double dominating set

1) *Theorem 3.1:* A double dominating set D^d of the $n[S(G)]$, is a lict subdivision double dominating set of G, if and only if the following conditions hold. (i) $n[S(G)] - D^d$ has at least two vertices. (ii) For any two vertices $u, v \in V[n(G)] - D^d$, every uv path contains a vertex of D^d.

We now give a characterization of lict subdivision double dominating set of G which is minimal.

2) *Theorem 3.2:* A double dominating set D^d of the $n[S(G)]$ is minimal if and only if for every vertex $v \in D^d$ either (i) $|N(v) \cap D^d| \leq 2$ or (ii) there $\exists$ a vertex $u, \in V[n(G)] - D^d$ such that $|N(v) \cap D^d| = 2$ and $u \in N(v)$.

Proof: Let D^d be a minimal lict subdivision double dominating set of G. Suppose that there exists a vertex $v \in D^d$ for which $|N(v) \cap D^d| \geq 2$ and for every vertex $u \in V[n(S(G))] - D^d$, either $|N(v) \cap D^d| \geq 2$ or $u \notin N(v)$. Then consider$D^{d'} = D^d - \{v\}$, since v is adjacent to at least two vertices of $D^{d'}$ it follows that $D^{d'}$ is double dominating set of $n[S(G)]$, which is contradicting to the minimality of D^d. Conversely, assume that D^d is double dominating set of $n[S(G)]$ satisfying conditions (i) and (ii). For that consider the set $D^{d'} = D^d - \{v\}$ for any vertex $v \in D^d$. If condition (i) holds, then $|N(v) \cap D^{d'}| \leq 2$, which implies that $D^{d'}$ is not a double dominating set of $n[S(G)]$. If the condition (ii) holds. Then there exists a vertex $u \in V[n(S(G))] - D^{d'}$ such that $|N(u) \cap D^{d'}| = 2$ and $u \in N(v)$. But in this case that set $D^{d'}$ would not double dominating to u and hence would not be a double dominating set of $n[S(G)]$. Thus in both condition $D^{d'}$ is not a double dominating set of $n[S(G)]$. Therefore D^d is a minimal double dominating set of $n[S(G)]$.

IV. LOWER BOUNDS FOR $\gamma_{ddns}(G)$.

We establish lower bounds for $\gamma_{ddns}(G)$ in terms of elements of G.

1) *Theorem 4.1*: For any connected (p, q) graph G with $p \geq 2$, $\left\lceil \frac{p}{2} \right\rceil + 1 \leq \gamma_{ddns}(G)$. Equality hold if G is P_2.

Proof: Let $D^d = \{v_1, v_2, \dots, v_n\}$ be a double dominating set of $n[S(G)]$. By the definition of $n[S(G)]$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Let D^d be the double dominating set of $n[S(G)]$ such that any vertex $v \in V[n(S(G))] - D^d$, $|N[v] \cap D^d| \geq 2$. Then $\{V[n(S(G))] - D^d\}$ contains at least one vertex which gives $\frac{p}{2} < \left\lceil \frac{p}{2} \right\rceil < \left\lceil \frac{p}{2} \right\rceil + 1 \leq \gamma_{ddns}(G)$.

2) *Theorem 4.2:* For any connected(p, q) graph G with $p \geq 2$, $\gamma_{ddns}(G) \leq p + q - 1$.

Proof: Let G be a (p, q) graph, then $V[S(G)] = p + q$. Let $F = \{v_1, v_2, \dots, v_k\}$ be the set of vertices in $S(G)$ and $F \subseteq V[S(G)]$ such that $|F| = \alpha_0(S(G))$ similarly for $E = \{e_1, e_2, \dots, e_n\}$ be the set of edges in $S(G)$, $E \subseteq E[S(G)]$ such that $|E| = \beta_0(S(G)$ and by the definition of $n[S(G)]$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Where $E(S(G))$ is the set of edges and $C(S(G))$ is the set of cutvertices in $S(G)$. We consider a set $F \subseteq E(S(G))$. If F corresponding to such vertices $D^d = \{v_1, v_2, \dots, v_q\} \subseteq V[n(S(G))]$ such that $D^d = V[n(S(G))] - C(S(G))$ and any vertex $v \in V[n(S(G))] - D^d$ is dominated by at least two vertices of $n[S(G)]$. Thus D^d is double dominating set of $n[S(G)]$. Now consider $|D^d| \leq \alpha_0(S(G)) + \beta_0(S(G) - 1 = V[S(G)] - 1 = p + q - 1$. Hence $\gamma_{ddns}(G) \leq p + q - 1$.

3) *Theorem 4.3:* For any connected (p, q) graph$G$ with $p \geq 3$, $p + \gamma(G) \leq \gamma_{ddns}(G)$. Equality holds if $G \cong P_p$, $p \geq 3$.

Proof: Let $D = \{v_1, v_2, \dots, v_k\}$ be a minimal dominating set of G such that $|D| = \gamma(G)$. Since in $n(S(G))$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Further let $E = \{e_1, e_2, \dots, e_q\}$ be the set of all egdes which are incident to the vertices of D becomes $E(S(G)) = 2E$, in $n(S(G))$. Since $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Let $D^d = \{v_1, v_2, \dots, v_n\} \subseteq E(S(G)) \subseteq V[n(S(G))]$ be the double dominating set of $n(S(G))$ such that $|N[v] \cap D^d| \geq 2$. Then D^d form a minimal double dominating set in $n[S(G)]$. Clearly

$|E(S(G))| - |V(G)| \geq |D|$. Thus it follows that $\gamma_{ddns}(G) - p \geq \gamma(G)$. Hence $p + \gamma(G) \leq \gamma_{ddns}(G)$. For equality if $G \cong P_3$. Then in this case $|D| = 1$, further $S(P_3) = P_5, \gamma_{ddns}(G) = 4$. Hence $\gamma_{ddns}(G) = 4 = 1 + 3\,\gamma(G) + p$.

4) Theorem 4.4: For any connected (p, q) graph G with $p \geq 3$, then $diam(G) + 2 \leq \gamma_{ddns}(G)$.

Proof: Let $I = \{e_1, e_2, \dots, e_n\} \subseteq E(G)$ be the set of edges which constitutes the longest path between any two distinct vertices of G such that $|I| = diam(G)$. Let $D^d = \{v_1, v_2, \dots, v_n\}$ be the set of vertices in $V[n(S(G))]$ such that for any vertex $v \in V[n(S(G))] - D^d$ is adjacent to at least two vertices of D^d and $|N[v] \cap D^d| \geq 2$. It follows that $|D^d| \geq 2$ and the diametral path includes at least two vertices. Thus $diam \leq \gamma_{ddns}(G) - 2$ it gives $diam(G) + 2 \leq \gamma_{ddns}(G)$.

5) Theorem 4.5: For any connected (p, q) graph G with $p \geq 2$, $\frac{(2q - p(p-3))}{2} \leq \gamma_{ddns}(G)$.

Proof: *Let* $D^d = \{v_1, v_2, \dots, v_n\} \subseteq V[n(S(G))]$ be the set of vertices and every vertex $v \in V[n(S(G))] - D^d$ is adjacent to at least two vertices of D^d, thus D^d itself is a double dominating set of $n[S(G)]$. Since $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Then there exists a vertex $v \in D^d$ which is not adjacent to any vertex of $V[n(S(G))]$. This implies that $q \leq \frac{p(P-1)}{2} - (p - \gamma_{ddns}(G))$. Which is $2q \leq p^2 - p - 2p + 2\gamma_{ddns}(G)$. Hence the result.

6) Theorem 4.6: For any connected (p, q) graph G, $\left\lceil \frac{2p}{\Delta(G)+2} \right\rceil \leq \gamma_{ddns}(G)$.

Proof: Let D^d be a minimal double dominating set of $n[s(G)]$. Since in $n[S(G)]$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Let k denote the number of edges between D^d and $V[n(S(G))] - D^d$. Since for any connected graph G, there exists at least one vertex $v \in V(G)$ such that $\deg(v) = \Delta(G)$, $k \leq \Delta(G).\gamma_{ddns}(G)$. Also since each vertex v in $V[n(S(G))] - D^d$ is adjacent to at least two vertices in D^d, $k \geq 2(p - \gamma_{ddns}(G))$. From these two inequalities, $2p - 2p\gamma_{ddns}(G) \leq \Delta(G).\gamma_{ddns}(G)$. It follows that $\frac{2p}{\Delta(G)+2} \leq \left\lceil \frac{2p}{\Delta(G)+2} \right\rceil \leq \gamma_{ddns}(G)$.

7) Theorem 4.7: For any non-trivial tree T of order p with l leaves and s support vertices, $(p - l - s + 4) \leq \gamma_{ddns}(T)$.

Proof: To prove that if T is a tree of order $p \geq 2$ with l leaves and$s$ support vertices then $\gamma_{ddns}(T) \geq (p - l - s + 4)$. We use mathematical induction on p. Let $p = 2$, then $T = P_2$ and $diam(T) = 1$ it implies that $\gamma_{ddns}(T) = p - l - s + 4 = 2$. Let $p = 3$ then T is a star and $diam(G) = 2$, it implies that $\gamma_{ddns}(T) = (p - l - s + 4) = 4$. Let $p = 4$ then T may be P_4 and $diam(G) = 3$ it implies that that $\gamma_{ddns}(T) > (p - l - s + 4)$. For $p = n, \gamma_{ddns}(T) > (p - l - s + n)$. Further it is also true for $p = n + 1$. Hence we obtain the desired result.

8) Theorem 4.8: For any connected (p, q) graph G with $p \geq 2, \gamma(G) + 1 \leq \gamma_{ddns}(G)$. Equality hold if G is K_2.

Proof: Let $D = \{v_1, v_2, \dots, v_k\}$ be a minimal dominating set of G such that $|D| = \gamma(G)$. Since in $n(S(G))$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$, where $E(S(G))$ is the set of edges and $C(S(G))$ is the set of cutvertices in $S(G)$. We consider the following cases. Case 1: Suppose G is a tree. Then clearly for any tree T, $D^d = \{v_1, v_2, \dots, v_n\}$ set of vertices in $n[S(G)]$ such that $D^d = V[n(S(G))] - C(S(G))$ is dominated by at least two vertices of $n[S(G)]$. Since the number of edges of $S(G)$ are more than that of G, which gives $V(G) \subset V[n(S(G))]$. It follows that $|D^d| > |D| + 1$ which gives $\gamma_{ddns}(G) > \gamma(G) + 1$. Case 2: Suppose G is not a tree. Then there exists at least one cycle. Let D be a minimal dominating set of G, then $|D| = \gamma(G)$. Suppose there exists a cycle of length l, then in $S(G)$, the cycle length be $2l$. Let $E = \{e_1, e_2, \dots, e_k\} \subseteq E[S(G)]$, such that $|E| = \gamma'(G)$. In $n[S(G)]$, $V[n(S(G)] = E(S(G)) \cup C(S(G))$. Suppose $F \subset C[S(G)]$. Then $\{E\} \cup \{F\} \subseteq V[n(S(G))]$ such that $\forall v_i \in V[n(S(G))] - \{E\} \cup \{F\}$ is adjacent to at least two vertices of $V[S(G))]$. Hence $|E| \cup |F|$ is a minimal double dominating set $n[S(G))]$ such that $|E| \cup |F| = \gamma_{ddns}(G)$ –set. Clearly $|D| \leq |E| \cup |F|$ which gives $\gamma(G) + 1 \leq \gamma_{ddns}(G)$. For equality, let $G = K_2$, we have $\gamma(G) = 1$. Further $S(K_2) = P_3$ and $n[S(K_2)] = C_3$ and $\gamma_{ddns}(G) = 2, \gamma(G) = 1$. Hence $\gamma_{ddns}(G) = 2 = \gamma(G) + 1$.

9) Theorem 4.9: For any connected (p, q) graph G, $\left\lceil \frac{(p(9-p)-6}{6} \right\rceil \leq \gamma_{ddns}(G)$.

Proof: Let D^d be a γ_{ddns}-set of G. Since in $n[S(G)]$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Let t_1 denote the number of edges in $n[S(G)]$ incident to the vertices of $V[n(S(G))] - D^d$ only. Also t_2 denotes the number of edges in $n[S(G)]$ incident to the vertices of D^d only. Then $\frac{[p(p-1)]}{2} \geq t_1 + t_2 \geq 4|V(G) - D^d| - 2 + |D^d - 1|$ it implies that $\frac{[p(p-1)]}{2} \geq 4p - 4|D^d| - 2 + |D^d| - 1 = 4p - 3|D^d| - 3$. It gives that $3|D^d| \geq \frac{p(9-p)-6}{2}$. Hence $\left\lceil \frac{(p(9-p)-6}{6} \right\rceil \leq \gamma_{ddns}(G)$.

10) Theorem 4.10: For any connected (p, q) graph G with $p \geq 2$, $2(p - q) \leq \gamma_{ddns}(G)$.

Proof: Let $D^d = \{ v_1, v_2, \dots, v_n \}$ be the minimal set of vertices which covers all the vertices in $V[n(S(G))]$. Suppose for every vertex of $v \in V[n(S(G))]$ is adjacent to at least two vertices of D^d, clearly D^d forms a double dominating set of $n(S(G))$. Let any vertex $v \in D^d$ which is not adjacent to any vertex of $V[n(S(G))] - D^d$. Then $2q \geq |D^d| + 2|V(G) - D^d|$ it gives that $2q \geq |D^d| + 2p - 2|D^d|$. Therefore $|D^d| \geq 2p - 2q$ this implies that $2(p-q) \leq \gamma_{ddns}(G)$.

V. SPECIFIC VALUES OF $\gamma_{ddns}(G)$.

We found some constraints for which $\gamma_{ddns}(G)$ follows the equality relation with other domination parameters of G.

1) *Theorem 5.1:* For any path P_p with $p \geq 3$, $\gamma_{ddns}(P_p) = \begin{cases} 4\alpha_0(P_p) - 2, & if\ p\ is\ even \\ 4\alpha_0(P_p), & if\ p\ is\ odd \end{cases}$.

Proof: Let P_p be the path with $p \geq 3$ vertices. Consider $V = \{ v_1, v_2, \dots, v_n \}$ be the vertices and $E = \{ (v_i, v_{i+1}) \}$, i = 1,2,3,… be the edge set of path P_p. Since in $n[S(G)]$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. We consider the following cases

a) *Case 1:* If p is even. Then by Theorem A[1], we have $\alpha_0(P_p) = \frac{p}{2}$ it implies that $p = 2\alpha_0(P_p)$. Since $\gamma_{ddns}(P_p) = 2q = 2(p-1)$, we have $\gamma_{ddns}(P_p) = 2p - 2 = 2[2\alpha_0(P_p)] - 2 = 4\alpha_0(P_p) - 2$.
b) *Case 2:* If p is an odd. Then by Theorem A[1], we have $\alpha_0(P_p) = \frac{p-1}{2}$ it implies that $p - 1 = 2\alpha_0(P_p)$. Since $\gamma_{ddns}(P_p) = 2q = 2(p-1) = 4\alpha_0(P_p)$.

2) *Theorem 5.2:* For any path P_p with $p \geq 3$, $\gamma_{ddns}(P_p) = \begin{cases} 4\alpha_1(P_p) - 2, & if\ p\ is\ even \\ 4\alpha_1(P_p) - 4, & if\ p\ is\ odd \end{cases}$.

Proof: Let P_p be the path with $p \geq 3$ vertices. Consider $V = \{ v_1, v_2, \dots, v_n \}$ be the vertices and $E = \{ (v_i, v_{i+1}) \}$, i = 1,2,3,… be the edge set of path P_p. Since in $n[S(G)]$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. We consider the following cases.

a) *Case 1:* If p is even. Then by Theorem B[1], we have $\alpha_1(P_p) = \frac{p}{2}$ it implies that $p = 2\alpha_1(P_p)$. Since $\gamma_{ddns}(P_p) = 2q = 2(p-1)$, we have $\gamma_{ddns}(P_p) = 2p - 2 = 2[2\alpha_1(P_p)] - 2 = 4\alpha_0(P_p) - 2$.
b) *Case 2:* If p is an odd. Then by Theorem B[3], we have $\alpha_1(P_p) = \frac{p+1}{2}$ it implies that $p - 1 = 2\alpha_1(P_p)$. Since $\gamma_{ddns}(P_p) = 2q = 2(p-1) = 2p - 2 = 4\alpha_1(P_p) - 4$.

VI. UPPER BOUNDS FOR $\gamma_{ddns}(G)$.

We establish upper bounds for $\gamma_{ddns}(G)$ in terms of elements of G.

1) *Theorem 6.1:* For any connected (p,q) graph G with $p \geq 2$, $\gamma_{ddns}(G) \leq 2q$.

Proof: Since $V[n(S(G))] = E(S(G)) \cup C(S(G))$, $V(S(G)) = p + q$. Let D^d be double dominating set of $n[S(G)]$. Then by definition of lict subdivision double domination $|D^d| \geq 2$. Further by definition of $n[S(G)]$, $2q - \gamma_{ddns}(G) \geq 0$. Clearly it follows that $\gamma_{ddns}(G) \leq 2q$.

2) *Theorem 6.2*: For any connected (p,q) graph G with $p \geq 2$, $\gamma_{ddns}(G) \leq p + q - \delta(G)$.

Proof: Let $D^d = \{ v_1, v_2, \dots, v_n \} \subseteq V[n(S(G))]$ be the set of vertices and every vertex $v \in V[n(S(G))] - D^d$ such that $|N[v] \cap D^d| \geq 2$. Thus it is clear that $|D^d| \geq 2$. Since for any graph G there exists at least one vertex $v \in V(G)$ such that $\deg(v) = \delta(G)$. By definition of $n[S(G)]$, $V[n(S(G))] = E(S(G)) \cup C(S(G))$. Then there exists a vertex $v \in G$ such that $\deg(v) = \delta(G)$. Thus $\delta(G) \leq p - D^d + q$, which implies that $D^d \leq p + q - \delta(G)$. Hence the result.

3) Theorem 6.3: For any connected (p,q) graph G with $n[S(G))] \neq K_p$, $\gamma_{ddns}(G) + \gamma_{ssns}(G) \leq 2q + C$. Where C is the number of cut vertices in $S(G)$.

Proof: Suppose G has $p \leq 3$ then γ_{ddns} –set does not exist. Now we consider any graph with $p \leq 4$, such that $n[S(G)] \neq K_p$. Since $\gamma_{ddns}(G) \leq 2q$ and from Theorem D[3] $\gamma_{ssns}(G) = \alpha_0[n(S(G))]$. Further $\gamma_{ddns}(G) + \gamma_{ssns}(G) \leq 2q + \alpha_0[n(S(G))] \leq V[n(S(G))] \leq E(S(G)) \cup C(S(G)) \leq 2q + C$. Hence the result.

4) *Theorem 6.4:* For any non-trivial tree T, the lict subdivision of a tree is non-eulerian.

Proof: Let T be a non-trivial tree and $n[S(G)]$ always contain a point of odd degree. Hence by Theorem C[2], the result follows.

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5) *Theorem 6.5:* For any connected (p,q) graph G with $p \geq 3$ vertices,

(i)$\gamma_{ddns}(G) + \gamma_{ddns}(\bar{G}) \leq 4q$. (ii)$\gamma_{ddns}(G).\gamma_{ddns}(\bar{G}) \leq 4q^2$.

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