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Frequency Domain Analysis of Control Systems

Swarup Bhattacharjee
Assistant Professor,Department of Mathematics
Iswar Chandra Vidyasagar College, Belonia, South Tripura

Abstract: The frequency domain analysis is generally done by using a sinusoidal input signal. When a sinusoidal input signal is given to a linear time-invariant system and the output response consists of transient and ready-state parts whereas when transient part dies down as $T\rightarrow\infty$, only steady-state part remains. The term frequency response means the steady-state response of a system to a sinusoidal input. Industrial control systems are often designed using frequency response methods. Many techniques are available in the frequency response methods for the analysis and design of control systems. Control systems exist in many systems of engineering, sciences, and in human body. Some type of control systems affects most aspects of our day-to-day activities.
Keywords: frequency domain, sinusoidal, laplace transform, steady state, LTI system

I. INTRODUCTION

In frequency response method, the frequency of the input signal is variedover certainrange and then, the resulting response is studied . The frequency response approach involves simple tests and can be made accurately by use of readily available sinusoidal generators and precise measurements equipment. The transfer functions of complicated components can be determined experimentally by frequency response tests.

The frequency response can be represented in any of the form

A. Bode or logarithmic plot
B. Nyquist or Polar plot.
C. Log magnitude versus phase plot(Nichols plots)

II. FREQUENCY RESPONSE ANALYSIS

Consider a system with sinusoidal input $r(t) = A\sin\omega t$. The steady-state output may be written as, $c(t) = B\sin(\omega t + \phi)$. The magnitude and the phase relationship between the sinusoidal input and the steady-state output of a system is called *frequency response*. The frequency response test is performed by keeping the amplitude *A* fixed and determining *B* and *Φ* for a suitable range of frequencies. Whenever it is not possible to obtain the transfer function of a system through analytical techniques, frequency response test can be used to compute its transfer function. The design and adjustment of open-loop transfer function of a system for specified closed-loop performance is carried out more easily in frequency domain. Further, the effects of noise and parameter variations are relatively easy to visualize and assess through frequency response. The Nyquist criteria is used to extract information about the stability and the relative stability of a system in frequency domain.

The transfer function G(s) can be written as ratio of two polynomials in s(or Laplace)

$$\mathrm{G(s)}=\frac{p(s)}{q(s)}=$$

$$\frac{p(s)}{(s+s_1)(s+s_2)\ldots(s+s_n)}$$

Then the output Y(s) can be written as Y(s)=G(s)X(s)= $\frac{p(s)}{q(s)}$ X(s), X(s) is the input function.

After steady state conditions are reached, the frequency response can be obtained by replacing s with jw, G(jw)=$Me^{j\varphi}$,M is the amplitude ratio of the output and input sinusoids. φ is the phase shift between the input sinusoid and output sinusoid.

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If

$$X(s)=\frac{\omega X}{s^2+\omega^2}$$

Then, the frequency response of the system is

$$Y(s)=G(s)X(s)=G(s)\frac{\omega X}{s^2+\omega^2}$$

$$Y(s)=G(s)\frac{As+B\omega}{s^2+\omega^2}$$

$$=G(s)\frac{As+B\omega}{(s+j\omega)(s-j\omega)}$$

$$=\frac{K_1}{s+j\omega}+\frac{K_2}{s-j\omega}+\text{partial fraction terms from G(s)}$$

$$K_1=\frac{As+B\omega}{s-j\omega}G(s)\Big|_{s\to -j\omega}=\frac{1}{2}(A+jB)G(-j\omega)=\frac{1}{2}M_ie^{-j\phi}M_Ge$$

$$=\frac{M_iM_G}{2}e^{-j(\phi_i+\phi_G)}$$

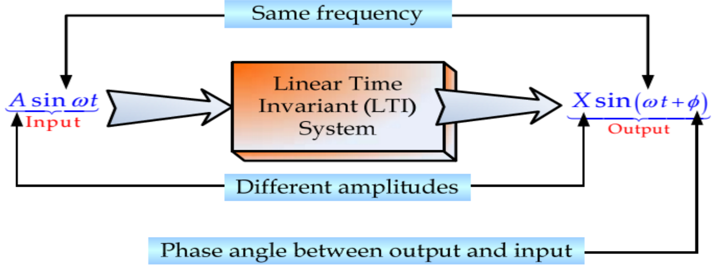
$$K_2=\frac{As+B\omega}{s+j\omega}G(s)\Big|_{s\to +j\omega}=\frac{1}{2}(A-jB)G(j\omega)=\frac{1}{2}M_ie^{j\phi_i}M_Ge^{j\phi_G}$$

$$=\frac{M_iM_G}{2}e^{j(\phi_i+\phi_G)}=K_1^*$$

where K^*_1 is the complex conjugate of K_1

III. SINUSOIDAL TRANSFER FUNCTION (STF)

When a sinusoidal input is applied to a LTI system, the system will tend to vibrate at its own natural frequency, as well as follow the frequency of the input. In the presence of damping, that portion of motion sustained by the sinusoidal input will gradually die out. As a result, the response at steady-state is sinusoidal at the same frequency as the input. The steady-state output differs from the input only in the amplitude and the phase angle. See Figure below



Thus, the output-input amplitude ratio and the phase angle between the output and input sinusoids are the only parameters needed to predict the steady state output of LTI systems when the input is a sinusoid.

$$\frac{X}{A}:\text{output-input amplitude ratio}$$

$$\phi\ :\text{phase angle between output and input}$$

IV. FORCED VIBRATION WITHOUT DAMPING

Mass system in which the mass is subjected to a sinusoidal input force $p(t)=P_0\sin\omega t$. Let us find the response of the system when it is initially at rest. The equation of motion is

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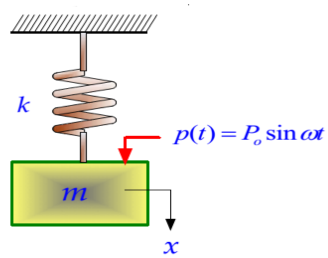
$$m\ddot{x} + kx = P_\circ \sin \omega t$$



Spring-mass system.

where x is the output, P is the amplitude of the excitation and ω is the forcing (excitation) frequency. The above equation can be written in the form

$$\ddot{x} + \frac{k}{m}x = \frac{P_\circ}{m}\sin \omega t \qquad \text{-----------(1)}$$

where $(k/m)^{1/2} = \omega_n$ is known as the natural frequency of the system. The solution of above equation consists of the vibration at its natural frequency (the complementary solution) and that at the forcing frequency (the particular solution) as shown in Figure . Thus,

$x(t)$ = complementary solution + particular solution.

Let us obtain the solution under the condition that the system is at rest. Take Laplace Transformation of both sides of quation (1) for zero initial conditions, i.e.,

$$x(0) = \dot{x}(0) = 0$$

$$\left(s^2 + \frac{k}{m}\right)X(s) = \frac{P_\circ}{m}\frac{\omega}{s^2 + \omega^2}$$

Where X(s)=L[x(t)], Substituting $k/m = \omega_n^2$ and Solving for $X(s)$ yields

$$X(s) = \frac{P_\circ}{m}\frac{\omega}{s^2 + \omega^2}\frac{1}{s^2 + \omega_n^2}$$

The above equation can be written in partial fraction as:

$$X(s) = \frac{P_\circ}{m}\frac{\omega}{s^2 + \omega^2}\frac{1}{s^2 + \omega_n^2} = \frac{A_1 s + B_1}{s^2 + \omega^2} + \frac{A_2 s + B_2}{s^2 + \omega_n^2}$$

where $A1$, $A2$, $B1$ and $B2$ can be calculated.

The expression for $X(s)$ is therefore

$$X(s) = -\frac{P_\circ \omega}{k - m\omega^2}\left(\frac{1}{\omega_n}\right)\frac{\omega_n}{s^2 + \omega_n^2} + \frac{P_\circ}{k - m\omega^2}\frac{\omega}{s^2 + \omega^2}$$

The inverse Laplace Transform of the above equation is given by

$[x(t)]=L^{-1}[X(s)]=$

$$\underbrace{-\frac{P_\circ(\omega/\omega_n)}{k-m\omega^2}}_{A}\sin\omega_n t+\underbrace{\frac{P_\circ}{k-m\omega^2}}_{B}\sin\omega t$$

$$\underbrace{A\sin\omega_n t}_{\text{Complementary Solution}}+\underbrace{B\sin\omega t}_{\text{Particular Solution}}$$

V. CONCLUSION

The performance of a control system is generally judged by its time response to test signals. The analysis and design of a control system is also carried out based on its frequency response.

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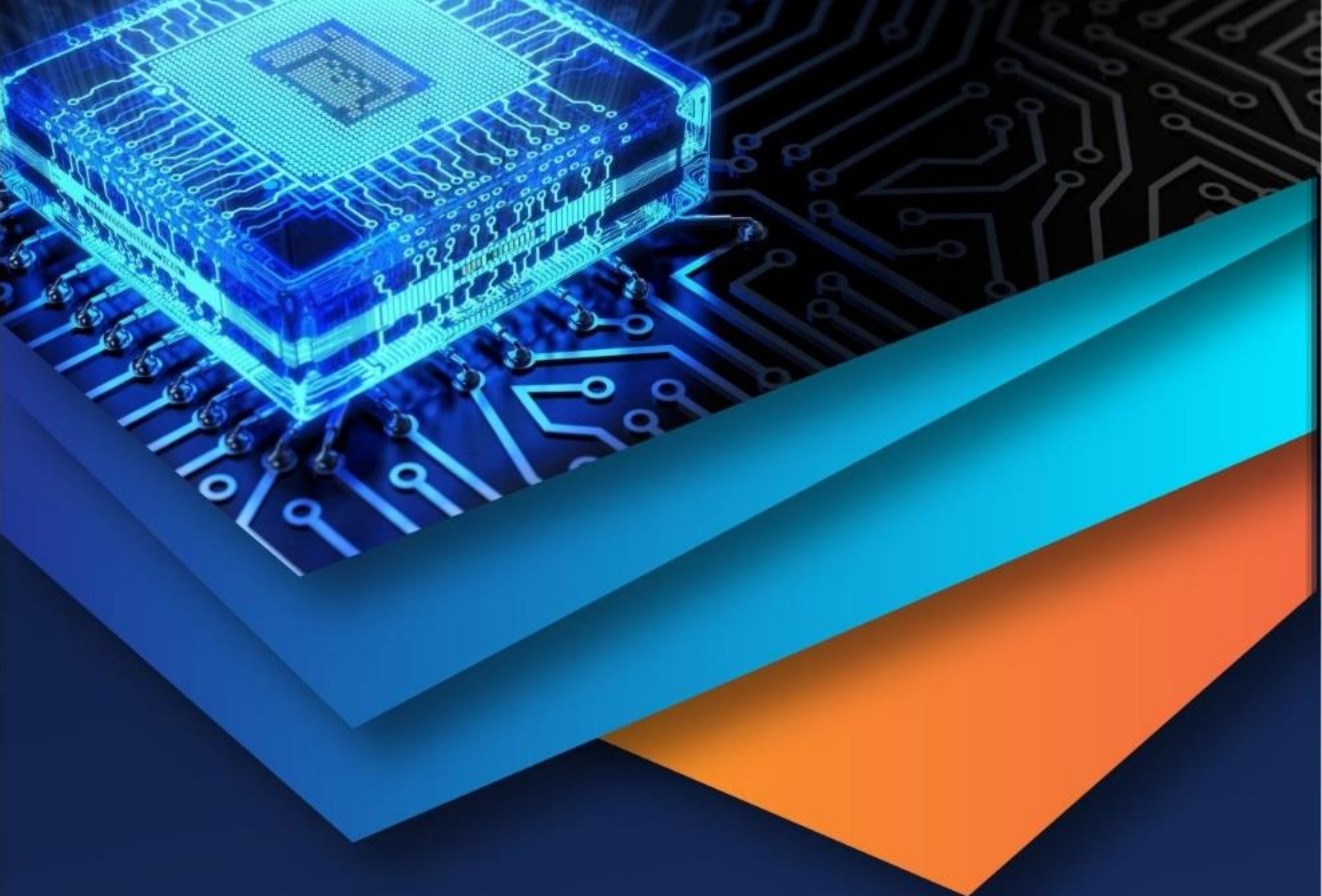

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