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# Determination of M- Power Soft Subgroup Structures

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**Abstract:** In this paper, we introduce the order of the soft structure of the group theory of power of the soft sets related with soft group. We also characterise the m-power soft subgroups and its determination.

**Keywords:** soft set, soft group, m-power set, m-power group, determination, soft subset, union, intersection.

## I. INTRODUCTION

The theory of soft sets, introduced by Molodtsov [1], is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. Maji et al. [2] give an application of soft set theory in a decision making problem by using the rough sets and they conducted a theoretical study on soft sets in a detailed way [3]. Chen et al. [4] proposed a reasonable definition of parameterizations reduction of soft sets and compared them with the concept of attributes reduction in rough set theory. The algebraic structures of set theories which deal with uncertainties have been studied by some authors. Rosenfeld [5] proposed fuzzy groups to establish results for the algebraic structures of fuzzy sets.

Fuzzification of algebraic structures was studied by many authors [5,6,7]. Many papers on soft algebras have been published since Aktas, and Cagman[8] introduced the notion of a soft group in 2007. Recently, Jun et al. [9] studied soft ideals and idealistic soft BCK/BCI-algebras. Acar et al. [10] introduced initial concepts of soft rings. Aygunoglu and Aygun [11] introduced the concept of fuzzy soft group and, in the meantime, they studied its properties and structural characteristics. Atagun and Sezgin [12] introduced and studied the concepts of soft subrings, soft ideal of a ring, and soft sub fields of a field. In this paper, we introduce the order of the soft structure of the group theory of power of the soft sets related with soft group. We also characterise the m-power soft subgroups and its determination.

## II. PRELIMINARIES AND BASIC CONCEPTS

- 1) **Definition 2.1 [D.Molodtsov]:** A pair  $K_A$  is called a soft set over  $U$ , where  $F$  is a mapping given by  $K : A \rightarrow P(U)$ . In other words, a soft set over  $U$  is a parameterised family of subsets of the universe  $U$ .
- 2) **Definition 2.2 [Maji et.al]:** For two soft sets  $K_A$  and  $G_B$  over  $U$ ,  $K_A$  is called a soft subset of  $G_B$ , if
  - (1)  $A \leq B$  and
  - (2) For all  $e \in A$ ;  $K(e)$  and  $G(e)$  are identical approximations.

It is denoted by  $K_A \leq G_B$ .  $K_A$  is called a soft super set of  $G_B$  if  $G_B$  is a subset of  $K_A$ . It is denoted by  $K_A \geq G_B$ .

- 3) **Definition 2.3[Maji et.al]:** Union of two soft sets of  $K_A$  and  $G_B$  over  $U$  is the soft set  $H_C$ , where  $C = A \cup B$  and  $e \in C$ ,

$$H_C = \begin{cases} K(e), & \text{if } e \in A-B, \\ G(e), & \text{if } e \in B-A, \\ K(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

It is denoted by  $K_A \cup G_B = H_C$ .

- 4) **Definition 2.4[Maji et.al]:** Intersection of two soft sets of  $K_A$  and  $G_B$  over  $U$  is the soft set  $H_C$ , where  $C = A \cap B$  and  $e \in C$ ,

$$H_C = \begin{cases} K(e), & \text{if } e \in A-B, \\ G(e), & \text{if } e \in B-A, \\ K(e) \cap G(e) & \text{if } e \in A \cap B. \end{cases}$$

It is denoted by  $K_A \cap G_B = H_C$ .

- 5) **Definition 2.5 [H.Aktas et.al]:** Let  $K_A$  be a soft set over  $G$ . Then  $K_A$  is said to be a soft group over  $G$  if and only if  $K(x)$  is a subgroup of  $G$  for all  $x \in A$ .
- 6) **Definition 2.6 [H.Aktas et.al]:** One considers the following.

- (1)  $K_A$  is said to be an identity soft group over  $G$  if  $K(x) = \{e\}$  for all  $x \in A$ , where  $e$  is the identity element of  $G$ .
- (2)  $K_A$  is said to be an absolute soft group over  $G$  if  $K(x) = G$  for all  $x \in A$ .
- 7) **Definition 2.7:** Let  $K_A$  be a soft set over  $G$  and  $K(x) \in K_A$  for  $x \in A$ . Then  $K(x^m) = \{a^m / a \in K(x), m \in \mathbb{Z}\}$  is called  $m$ -power of  $K(x)$ .
- 8) **Example 2.8:** Let  $K_A$  be a soft set over  $S_3$ , where  $A = S_3$ , and let  $K_A = \{K(e) = \{e\}, K(12) = \{e, (12)\}, K(13) = \{e, (13)\}, K(23) = \{e, (23)\}, K(123) = \{e, (123), (132)\}$  be a soft set over group  $S_3$ . And the third power of  $K(123)$  is  $K(123)^3 = \{e, (123)^3, (132)^3\} = \{e, e, e\} = \{e\}$ .  
Of course, when the group is additive, the  $m^{\text{th}}$  power of  $K(x)$  will be denoted by  $mK(x) = \{ma / a \in K(x), m \in \mathbb{Z}\}$ .

### III. PROPERTIES OF M-POWER SOFT SUBGROUPS

In this section, we shall derive some important results based on the above definitions.

1) **Theorem-3.1:** Let  $K_A$  be a soft set over  $G$  and  $K(x), K(y) \in K_A$  for all  $x, y \in A$ . Then, for all  $n \in \mathbb{Z}$ ,

- $(K(x) \cap K(y))^m \subset K(x)^m \cap K(y)^m$
- $(K(x) \cup K(y))^m \subset K(x)^m \cup K(y)^m$
- $(K(x) \times K(y))^m \subset K(x)^m \times K(y)^m$

**Proof:** Let  $a^m \in (K(x) \cap K(y))^m$ , for  $m \in \mathbb{Z}$ . From Definition 2.7  $a \in (K(x) \cap K(y))$  and  $a^m \in K(x)$  and  $a^m \in K(y)$ . This means that  $a^m \in K(x)^m \cap K(y)^m$ . This completes the proof. Theorem -3.1 (2) and theorem-3.1 (3) can be proved similarly by definition 2.7.

In general, the converse of the above theorem-3.1 is not true. We illustrate an example of this situation.

2) **Example 3.2:** Let  $A = \{0, 1\}$  and let  $K: A \rightarrow Q(\mathbb{Z})$  be a function such that  $K(0) = \{3r / r \in \mathbb{Z}\}$  and  $K(1) = \{3r + 1 / r \in \mathbb{Z}\}$ . The intersection is  $K(0) \cap K(1)$  is empty, so  $(K(0) \cap K(1))^3$  is empty. On the other hand  $K(0)^3 \cap K(1)^3$  is non-empty. Consequently  $(K(x) \cap K(y))^m \neq K(x)^m \cap K(y)^m$ .

3) **Definition 3.3:** Let  $K_A$  be a soft set over  $G$  and  $K(x) \in K_A$ . If there is a positive integer  $m$  such that  $K(x)^m = \{e\}$ , then the least such positive integer  $m$  is called the order of  $K(x)$ . If no such  $m$  exists, then  $K(x)$  has infinite order. The order of  $K(x)$  is denoted by  $|K(x)|$ . If  $K_A$  is a soft group over  $G$ , then the order of  $K(x) \in K_A$  coincides with the order of  $K(x)$ , which is a subgroup of  $G$ . Of course if there is any element  $x$  in  $A$  such that  $K(x) = \{e\}$ , then the order of  $K(x)$  is 1.

4) **Example 3.4:** In Example 2.8 the order of an element  $K(123)$  is 3. Let  $K_N$  be a soft group over group of integer numbers  $\mathbb{Z}$ , where  $K$  is a mapping from natural numbers  $\mathbb{N}$  to  $Q(\mathbb{Z})$  such that  $K(m) = m\mathbb{Z}$  for all  $m \in \mathbb{N}$ . There is no any positive integer such that  $m$  such that  $K(x)^m = \{0\}$ , so  $K(m)$  has infinite order for all  $m \in \mathbb{N} - \{0\}$ .

5) **Theorem 3.5:** Let  $G$  be a finite group and  $K_A$  a soft group over  $G$ . Then, the order of elements of  $K_A$  are finite.

**Proof:** It is straightforward.

6) **Theorem 3.6:** Let  $K_A$  be a soft set over finite group  $G$  and  $K(x) \in K_A$  for  $x \in A$ . Then, the order of  $K(x)$  is the least common multiple of order of elements of  $K(x)$ . **Proof:** Let  $m$  be the order of  $K(x)$ . Then  $K(x)^m = \{e\}$ . This means that  $a^m = e$  for all  $a \in K(x)$ . we know from classical group that  $|a| \mid m$ , namely,  $a$  divides  $m$  for all  $a \in K(x)$ . Thus,  $m$  is common multiple of elements of  $K(x)$ . Let,  $m$  be another common multiple elements of  $K(x)$ . Then, by reason of  $a^p = e$  for all  $a \in K(x)$ ,  $K(x)^p = \{e\}$ . However, since  $m$  is the least number that satisfies the condition  $K(x)^m = \{e\}$ , hence  $p/m$ . This completes the proof.

7) **Theorem 3.7:** Let  $G$  be finite group,  $K_A$  soft group over  $G$ , and  $K(x)$  and  $K(y)$  the elements of  $K_A$ . Then, for all  $x, y \in A$ , one has the following:

- $|K(x) \cap K(y)| \leq \text{Greatest common divisor of } (|K(x)|, |K(y)|) \text{ for all } x, y \in A.$
- $|K(x) \cup K(y)| = \text{Least common multiple of } (|K(x)|, |K(y)|) \text{ for all } x, y \in A.$
- $|K(x) \times K(y)| = |K(x)| |K(y)| \text{ for all } x, y \in A.$

**Proof:** (1)  $K(x) \cap K(y)$  is a subgroup of  $K(x)$  and  $K(y)$ , so  $|K(x) \cap K(y)| \mid |K(x)|$  and  $|K(x) \cap K(y)| \mid |K(y)|$ . It follows  $|K(x) \cap K(y)| \leq \text{Greatest common divisor of } (|K(x)|, |K(y)|) \text{ for all } x, y \in A.$

(2) Let  $|K(x) \cup K(y)| = r$ ,  $|K(x)| = p$ , and  $|K(y)| = m$ . From theorem 3.1  $(K(x) \cap K(y))^r \subset K(x)^r \cap K(y)^r = \{e\}$ . This follows  $m \mid r$  and  $p \mid r$ . Thus  $r$  is common multiple of  $m$  and  $p$ . Consider  $(K(x) \cap K(y))^t = K(x)^t \cap K(y)^t = \{e\} \cap \{e\} = \{e\}$ . Since  $r$  is the least positive integer that satisfied the condition  $(K(x) \cap K(y))^r = \{e\}$ ,  $p$  divides  $t$ . Hence  $p$  is least common multiple order of  $K(x)$  and  $K(y)$ . This completes the proof.

(3) Since  $K(x)$  and  $K(y)$  are subgroups of  $G$ , it is seen easily.

8) **Definition 3.8:** Let  $G$  be a group and  $K_A$  soft set over  $G$ . The set  $= \{K(x)^m / x \in A, m \in \mathbb{Z}\}$  is called  $m^{\text{th}}$  power of soft set.

9) **Example 3.9:** Let  $K_A$  be a soft set over  $S_3$  defined in Example 2.8. Then, the second power of  $K_A$  is that  $K_A^3 = \{K(e)^3 = K(e), \text{ and } K(12)^3 = K(e), K(13)^3 = K(e), K(23)^3 = K(e), K(123)^3 = K(123)\}$ .

10) *Theorem 3.10:* Let  $K_A$  and  $E_B$  be two soft sets over  $G$ . Then,

- (1)  $(K_A \vee E_B)^m = K_A^m \vee E_B^m$ ,
- (2) If  $A \subset B$  and for all  $a \in A$ ,  $K(a)$  and  $E(a)$  are identical approximations, then  $(K_A \wedge E_B)^m \subset K_A^m \wedge E_B^m$ .

Proof: we consider the following

- (1) Suppose that  $K_A \vee E_B = (H, A^*B)$  and  $K_A^m \vee E_B^m = (T, A^*B)$ . Using Definition 3.8 and theorem 3.1 we have

$$\begin{aligned} (H, A^*B)^m &= \{ H(a,b)^m / (a,b) \in A^*B \} \\ &= \{ (K(a) \cup E(b))^m / (a,b) \in A^*B \} \\ &= \{ K(a)^m \cup E(b)^m / (a,b) \in A^*B \} \\ &= K_A^m \vee E_B^m. \end{aligned}$$

- (2) Suppose that  $K_A \wedge E_B = (H, A^*B)$  and  $K_A^m \wedge E_B^m = (T, A^*B)$ . Using the same argument in (1), we have

$$\begin{aligned} (H, A^*B)^m &= \{ H(a,b)^m / (a,b) \in A^*B \} \\ &= \{ (K(a) \cap E(b))^m / (a,b) \in A^*B \} \\ &= \{ K(a)^m \cap E(b)^m / (a,b) \in A^*B \} \\ &= \{ T(a,b) / (a,b) \in A^*B \} \\ &= K_A^m \wedge E_B^m. \end{aligned}$$

11) In Example- 2.8: The order of  $K_A$  is 6 and in Example 3.4 if we choose  $N = N_5 = \{0,1,2,3,4,5\}$  and  $K(m) = mZ$ , for  $m \in N_5$ , then the order of the group  $K_N^5$  is 6.

#### IV. CONCLUSION

we discuss the  $m$ -structures of couples subgroups. In this paper, we introduce the order of the soft structure of the group theory of power of the soft sets related with soft group. We also characterise the  $m$ -power soft subgroups and its determination.

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