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# On Cubic Equation With Four Unknowns

$$4(x^3 + y^3) + 12(s^2 - 1)(x + y)z^2 = (3s^2 + 1)w^3, (s \neq \pm 1)$$

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**Abstract:** In this paper, the cubic equation with four unknown given by  $4(x^3 + y^3) + 12(s^2 - 1)(x + y)z^2 = (3s^2 + 1)w^3, (s \neq \pm 1)$  is considered for determining its non-zero distinct integer solutions.

**Keywords:** Cubic with four unknowns, homogeneous cubic, integer solutions.

## I. INTRODUCTION

It is well-known that there are varieties of cubic equations with four unknowns to obtain integer solutions satisfying them [1-3]. In particular, different choices of cubic equations with four unknowns are presented in [4-12]. This paper has a different choice of cubic equation with four unknowns given by  $4(x^3 + y^3) + 12(s^2 - 1)(x + y)z^2 = (3s^2 + 1)w^3, (s \neq \pm 1)$  to obtain its infinitely many non-zero distinct integer solutions.

## II. METHOD OF ANALYSIS

The cubic equation with four unknowns to be solved for its non-zero distinct integer solutions is given by

$$4(x^3 + y^3) + 12(s^2 - 1)(x + y)z^2 = (3s^2 + 1)w^3 \quad (1)$$

Introduction of the linear transformations

$$x = u + v, y = u - v, w = 2u \quad (u \neq v \neq 0) \quad (2)$$

in (1) leads to

$$v^2 = s^2 u^2 - (s^2 - 1)z^2 \quad (3)$$

Again, considering the linear transformations

$$u = X + (s^2 - 1)T \quad (4)$$

$$z = X + s^2 T \quad (5)$$

in (3), it gives

$$X^2 = (s^4 - s^2)T^2 + v^2 \quad (6)$$

The fundamental solution of (6) is

$$T_0 = 2v, X_0 = (2s^2 - 1)v$$

To obtain the other solutions of (6), consider its pellian equation

$$X^2 = (s^4 - s^2)T^2 + 1$$

whose general solution  $(\tilde{T}_n, \tilde{X}_n)$  is given by

$$\tilde{X}_n = \frac{1}{2} f_n, \tilde{T}_n = \frac{1}{2\sqrt{s^4 - s^2}} g_n$$

where

$$f_n = \left( 2s^2 - 1 + 2\sqrt{s^4 - s^2} \right)^{n+1} + \left( 2s^2 - 1 - 2\sqrt{s^4 - s^2} \right)^{n+1}$$

$$g_n = \left( 2s^2 - 1 + 2\sqrt{s^4 - s^2} \right)^{n+1} - \left( 2s^2 - 1 - 2\sqrt{s^4 - s^2} \right)^{n+1}$$

Applying the lemma of Brahmagupta between the solutions  $(T_0, X_0)$  and  $(\tilde{T}_n, \tilde{X}_n)$ , the other solutions to (6) are given by

$$\left. \begin{aligned} T_{n+1} &= v f_n + \frac{(2s^2 - 1)v}{2\sqrt{s^4 - s^2}} g_n, \\ X_{n+1} &= \frac{(2s^2 - 1)v}{2} f_n + v\sqrt{s^4 - s^2} g_n, n = 0, 1, 2, \dots \end{aligned} \right\}. \quad (7)$$

Employing (4), (5) and (2), the sequence of solutions to (1) is represented as below:

$$\begin{aligned} x_{n+1} &= \frac{(4s^2 - 3)v}{2} f_n + \frac{(4s^4 - 5s^2 + 1)v}{2\sqrt{s^4 - s^2}} g_n + v, \\ y_{n+1} &= \frac{(4s^2 - 3)v}{2} f_n + \frac{(4s^4 - 5s^2 + 1)v}{2\sqrt{s^4 - s^2}} g_n - v, \\ z_{n+1} &= \frac{(4s^2 - 1)v}{2} f_n + \frac{(4s^4 - 3s^2)}{2\sqrt{s^4 - s^2}} v g_n, \\ w_{n+1} &= (4s^2 - 3)v f_n + \frac{(4s^4 - 5s^2 + 1)v}{\sqrt{s^4 - s^2}} g_n, n = -1, 0, 1, \dots \end{aligned}$$

In addition to the above solutions, there are other choices of solutions to (1) that are illustrated below:

It is worth to note that (6) can be represented as the system of double equations as presented below in Table 1:

TABLE 1  
System Of Double Equations

System	1	2	3
$X + v$	$s(s+1)T$	$s(s^2-1)T$	$s^2(s+1)T$
$X - v$	$s(s-1)T$	$sT$	$(s-1)T$

Solving each of the above systems, one obtains the values to  $X, v, T$ .

Substituting these values in (4), (5) and (2), the corresponding solutions to (1) are obtained and they are exhibited below:

Solutions to system 1:

$$x = (2s^2 + s - 1)T, y = (2s^2 - s - 1)T, z = 2s^2T, w = (4s^2 - 2)T$$

Solutions to system 2:

$$x = 2k(s^3 + s^2 - s - 1), y = 2k(s^2 + s - 1), z = k(s^3 + 2s^2), w = 2k(s^3 + 2s^2 - 2)$$

Solutions to system 3:

$$x = k(2s^3 + 4s^2 - 2), y = k(2s^2 + 2s - 4), z = k(s^3 + 3s^2 + s - 1), w = 2k(s^3 + 3s^2 + s - 3)$$

Further, (6) is written as

$$v^2 + (s^4 - s^2)T^2 = X^2 \quad (8)$$

Assume

$$X = s^2A^2 + (s^4 - s^2)s^2B^2 \quad (9)$$

write 1 as

$$1 = \frac{(s + i\sqrt{s^4 - s^2})(s - i\sqrt{s^4 - s^2})}{s^4} \quad (10)$$

Using (9) and (10) in (8) and employing the method of factorization, define

$$v + i\sqrt{s^4 - s^2}T = \left( sA + i\sqrt{s^4 - s^2} sB \right)^2 \frac{\left( s + i\sqrt{s^4 - s^2} \right)}{s^2}$$

On equating the real and imaginary parts, one obtains

$$\left. \begin{aligned} v &= s(A^2 - (s^4 - s^2)B^2) - 2AB(s^4 - s^2) \\ T &= A^2 - (s^4 - s^2)B^2 + 2ABs \end{aligned} \right\} \quad (11)$$

Substituting (9) and (11) in (4), (5) and (2), the corresponding values of  $x, y, z$  and  $w$  satisfying (1) are given by

$$\begin{aligned} x &= (2s^2 + s - 1)A^2 + (1 - s)(s^4 - s^2)B^2 + 2AB(s^3 - s - s^4 + s^2) \\ y &= (2s^2 - s - 1)A^2 + (1 + s)(s^4 - s^2)B^2 + 2AB(s^3 - s + s^4 - s^2) \\ z &= 2s^2A^2 + 2ABs^3 \\ w &= (4s^2 - 2)A^2 + 2(s^4 - s^2)B^2 + 4s(s^2 - 1)AB \end{aligned}$$

### III.CONCLUSIONS

In this paper, an attempt has been made to find non-zero distinct integer solutions to the cubic equation with four unknowns given by  $4(x^3 + y^3) + 12(s^2 - 1)(x + y)z^2 = (3s^2 + 1)w^3$ ,  $(s \neq \pm 1)$  in conclusion one may search for other sets of integer solutions to the considered cubic equation.

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