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# The Homogeneous Bi-quadratic Equations with Five Unknowns $x^{4}-y^{4}+2\left(x^{2}-y^{2}\right)\left(w^{2}+p^{2}\right)=4\left(x^{3}+y^{3}\right) z$ 

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Abstract: In this paper the homogeneous bi-quadratic equation with five unknowns given by
$x^{4}-y^{4}+2\left(x^{2}-y^{2}\right)\left(w^{2}+p^{2}\right)=4\left(x^{3}+y^{3}\right) z$ is studied for determining its non-zero distinct integer solutions. A few
interesting relations between the solutions and special figurate numbers are obtained.
Keywords: homogeneous bi-quadratic, bi-quadratic with five unknowns, integer solutions.

## I. INTRODUCTION

It is well known that the subject of diophantine equations has aroused the interest of many mathematicians since antiquity as it offers a rich variety of fascinating problems. In particular one may refer [1-11] for various problems on bi-quadratic diophantine equations with four and five variables. In this paper the homogeneous equation of degree four with five unknowns given by $x^{4}-y^{4}+2\left(x^{2}-y^{2}\right)\left(w^{2}+p^{2}\right)=4\left(x^{3}+y^{3}\right) z$ is analysed for obtaining its non-zero distinct integer solutions.

## II. NOTATIONS

1) $S O_{n}=n\left(2 n^{2}-1\right)$ - Stella octangular number of rank n
2) $C P_{6, n}=n^{3}$ - Centered hexagonal pyramidal number of rank n
3) $P R_{n}=n(n+1)$ - Pronic number of rank $n$
4) $O H_{n}=\frac{1}{3} n\left(2 n^{2}+1\right)$ - Octahedral number of rank n
5) $t_{3, n}=\frac{n(n+1)}{2}$ - triangular number of rank n
6) $C P_{n, 3}=\frac{n^{3}+n}{2}$ - centered triangular pyramidal number of rank n
7) $P_{n}^{3}=\frac{n(n+1)(n+2)}{6}$ - Tetrahedral number of rank n
8) $P_{n}^{5}=\frac{n^{2}(n+1)}{2}$ - Pentagonal pyramidal number of rank n
9) $P_{n}^{4}=\frac{n(n+1)(2 n+1)}{6}$-square pyramidal number of rank n

## III.METHOD OF ANALYSIS

The homogeneous biquadratic equation to be solved is

$$
\begin{equation*}
x^{4}-y^{4}+2\left(x^{2}-y^{2}\right)\left(w^{2}+p^{2}\right)=4\left(x^{3}+y^{3}\right) z \tag{1}
\end{equation*}
$$

Introduction of the liner transformations
$x=u+v, y=u-v, z=v$
in (1), gives
$w^{2}+p^{2}=2 v^{2}$
we present below different methods of solving (3) and thus, different sets of non-zero distinct integer solutions to (1) are obtained.

## A. Method 1

Let $z=a^{2}+b^{2}$
write 2 as

$$
\begin{equation*}
2=(1+i)(1-i) \tag{4}
\end{equation*}
$$

Using (4) and (5) in (3) and employing the method of factorization, define
$(w+i p)=(1+i)(a+i b)^{2}$
Equating the real and imaginary parts in (6), we get
$\left.\begin{array}{l}w=a^{2}-b^{2}-2 a b \\ p=a^{2}-b^{2}+2 a b\end{array}\right\}$
Substituting the values of $v$ in (2), it is seen that
$\left.\begin{array}{l}x=u+a^{2}+b^{2} \\ y=u-a^{2}-b^{2} \\ z=a^{2}+b^{2}\end{array}\right\}$
Thus, (7) and (8) represent the distinct integer solution to (1).
Note 1:
It is observed that 2 may also be written as

$$
\begin{aligned}
& 2=\frac{(1+7 i)(1-7 i)}{25} \\
& 2=\frac{(7+i)(7-i)}{25}
\end{aligned}
$$

Following the procedure as above, the corresponding two sets of solutions to (1) are presented below:
Set 1: Solutions for (i) are given as
$x=u+25 A^{2}+25 B^{2}$
$y=u-25 A^{2}-25 B^{2}$
$z=25 A^{2}+25 B^{2}$
$w=5 A^{2}-5 B^{2}-70 A B$
$p=35 A^{2}-35 B^{2}+10 A B$
Set 2: Solutions to (ii) are given as
$x=u+25 A^{2}+25 B^{2}$
$y=u-25 A^{2}-25 B^{2}$
$z=25 A^{2}+25 B^{2}$
$w=35 A^{2}-35 B^{2}-10 A B$
$p=5 A^{2}-5 B^{2}+70 A B$
Properties:

1) $x^{3}-y^{3}-8 z^{3}=6 x y z$
2) $6\left(4 z^{2}-(w+p)^{2}\right)$ is a Nasty number.
3) $p-w=8 t_{3, b}$ when $a=(b+1)$
4) $p-w=8 P_{b}^{5}$ when $a=b(b+1)$
5) $p-w=24 P_{b}^{3}$ when $a=b(b+1)(b+2)$
6) $p-w=4 S O_{b}$ when $a=\left(2 b^{2}-1\right)$
7) When $a$ and $b$ represents the sides of the Pythagorean triangle then $3(x-y)$ is a nasty number.

## B. Method 2

Note that (3) is also written as
$w^{2}+p^{2}=2 v^{2} * 1$
write 1 as $1=\frac{(3+4 i)(3-4 i)}{25}$
Using (4), (5) and (10) in (8) and employing the method of factorization, define
$(w+i p)=\frac{1}{5}(3+4 i)(1+i)\left(a^{2}-b^{2}+2 i a b\right)$
Equating the real and imaginary parts in (11) and replacing $a$ by $5 A$ and $b$ by $5 B$, we have
$w=w(A, B)=-5 A^{2}+5 B^{2}-70 A B$
$\left.p=p(A, B)=35 A^{2}-35 B^{2}-10 A B\right\}$
In this case, the corresponding integer solutions to (1) are found to be
$x=x(A, B)=u+25 A^{2}+25 B^{2}$
$y=y(A, B)=u-25 A^{2}-25 B^{2}$
$z=z(A, B)=25 A^{2}+25 B^{2}$

## Note 2:

It is worth mentioning that two more sets of integer solutions to (1) are obtained by considering

$$
\begin{array}{ll}
2=\frac{(1+7 i)(1-7 i)}{25}, & 1=\frac{(3+4 i)(3-4 i)}{25} \\
2=\frac{(7-i)(7+i)}{25}, & 1=\frac{(3+4 i)(3-4 i)}{25}
\end{array}
$$

Remark:
Note that 1 in (10) may be considered in the general form as given below:
$1=\frac{\left[2 m n+i\left(m^{2}-n^{2}\right)\right]\left[2 m n-i\left(m^{2}-n^{2}\right)\right]}{\left(m^{2}+n^{2}\right)^{2}}$
Or
$1=\frac{\left[m^{2}-n^{2}+i 2 m n\right]\left[m^{2}-n^{2}-i 2 m n\right]}{\left(m^{2}+n^{2}\right)^{2}}$
Properties:

1) When $A$ and $B$ represents the sides of the Pythagorean triangle then $3(x-y)$ is a nasty number.
2) $p+7 w+500 P R_{a}=0$ when $B=(A+1)$
3) $p+7 w+1000 P_{A}^{5}=0$ when $B=A(A+1)$
4) $p+7 w+1500 O H_{A}=0$ when $B=A\left(2 A^{2}+1\right)$
5) $p+7 w+500 C P_{A, 6}=0$ when $B=A^{2}$
6) $p+7 w+3000 P_{A}^{3}=0$ when $B=(A+1)(A+2)$
7) $p+7 w+3000 P_{A}^{4}=0$ when $B=(2 A+1)(A+1)$

## C. Method 3

Observe that (3) is written in the form of ratio as
$\frac{(w+v)}{(v+p)}=\frac{(v-p)}{(w-v)}=\frac{\alpha}{\beta}, \beta \neq 0$
which is equivalent to the system of double equation

$$
\begin{aligned}
& \beta w-\alpha p=v(\alpha-\beta) \\
& \alpha w+\beta p=v(\alpha+\beta)
\end{aligned}
$$

Applying the method of cross multiplication, we get

$$
\begin{align*}
& w=-\alpha^{2}+\beta^{2}-2 \alpha \beta \\
& p=\alpha^{2}-\beta^{2}-2 \alpha \beta  \tag{15}\\
& v=\alpha^{2}+\beta^{2}
\end{align*}
$$

In view of (2), we have

$$
\begin{align*}
& x=u+\alpha^{2}+\beta^{2} \\
& y=u-\alpha^{2}-\beta  \tag{16}\\
& z=\alpha^{2}+\beta^{2}
\end{align*}
$$

Thus, (15) and (16) represent the non-zero distinct integer solutions to (1).

## IV.CONCLUSIONS

In this paper, we have made an attempt to find infinitely many distinct integer solutions to the homogeneous bi-quadratic equation with five unknowns given by $x^{4}-y^{4}+2\left(x^{2}-y^{2}\right)\left(w^{2}+p^{2}\right)=4\left(x^{3}+y^{3}\right) z$. To conclude, one may search for other choices of solutions to the considered bi-quadratic equation with five unknowns.

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