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The Homogeneous Bi-quadratic Equations with Five Unknowns $x^4 - y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$

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Abstract: ***In this paper the homogeneous bi-quadratic equation with five unknowns given by*** $x^4 - y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$ ***is studied for determining its non-zero distinct integer solutions. A few interesting relations between the solutions and special figurate numbers are obtained.***

Keywords: homogeneous bi-quadratic, bi-quadratic with five unknowns, integer solutions.

I. INTRODUCTION

It is well known that the subject of diophantine equations has aroused the interest of many mathematicians since antiquity as it offers a rich variety of fascinating problems. In particular one may refer [1-11] for various problems on bi-quadratic diophantine equations with four and five variables. In this paper the homogeneous equation of degree four with five unknowns given by $x^4 - y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z$ is analysed for obtaining its non-zero distinct integer solutions.

II. NOTATIONS

1) $SO_n = n(2n^2 - 1)$ - Stella octangular number of rank n
2) $CP_{6,n} = n^3$ - Centered hexagonal pyramidal number of rank n
3) $PR_n = n(n+1)$ - Pronic number of rank n
4) $OH_n = \frac{1}{3}n(2n^2 + 1`)$ - Octahedral number of rank n
5) $t_{3,n} = \frac{n(n+1)}{2}$ - triangular number of rank n
6) $CP_{n,3} = \frac{n^3 + n}{2}$ - centered triangular pyramidal number of rank n
7) $P_n^3 = \frac{n(n+1)(n+2)}{6}$ - Tetrahedral number of rank n
8) $P_n^5 = \frac{n^2(n+1)}{2}$ - Pentagonal pyramidal number of rank n
9) $P_n^4 = \frac{n(n+1)(2n+1)}{6}$ -square pyramidal number of rank n

III.METHOD OF ANALYSIS

The homogeneous biquadratic equation to be solved is

$$x^4 - y^4 + 2(x^2 - y^2)(w^2 + p^2) = 4(x^3 + y^3)z \qquad (1)$$

Introduction of the liner transformations

$$x = u + v,\ y = u - v,\ z = v \qquad (2)$$

in (1), gives

$$w^2 + p^2 = 2v^2 \tag{3}$$

we present below different methods of solving (3) and thus, different sets of non-zero distinct integer solutions to (1) are obtained.

A. *Method 1*

Let $z = a^2 + b^2$ (4)

write 2 as

$$2 = (1+i)(1-i) \tag{5}$$

Using (4) and (5) in (3) and employing the method of factorization, define

$$(w+ip) = (1+i)(a+ib)^2 \tag{6}$$

Equating the real and imaginary parts in (6), we get

$$\left.\begin{aligned} w &= a^2 - b^2 - 2ab \\ p &= a^2 - b^2 + 2ab \end{aligned}\right\} \tag{7}$$

Substituting the values of v in (2), it is seen that

$$\left.\begin{aligned} x &= u + a^2 + b^2 \\ y &= u - a^2 - b^2 \\ z &= a^2 + b^2 \end{aligned}\right\} \tag{8}$$

Thus, (7) and (8) represent the distinct integer solution to (1).

Note 1:

It is observed that 2 may also be written as

$$2 = \frac{(1+7i)(1-7i)}{25}$$

$$2 = \frac{(7+i)(7-i)}{25}$$

Following the procedure as above, the corresponding two sets of solutions to (1) are presented below:

Set 1: Solutions for (i) are given as

$$x = u + 25A^2 + 25B^2$$
$$y = u - 25A^2 - 25B^2$$
$$z = 25A^2 + 25B^2$$
$$w = 5A^2 - 5B^2 - 70AB$$
$$p = 35A^2 - 35B^2 + 10AB$$

Set 2: Solutions to (ii) are given as

$$x = u + 25A^2 + 25B^2$$
$$y = u - 25A^2 - 25B^2$$
$$z = 25A^2 + 25B^2$$
$$w = 35A^2 - 35B^2 - 10AB$$
$$p = 5A^2 - 5B^2 + 70AB$$

Properties:

1) $x^3 - y^3 - 8z^3 = 6xyz$
2) $6\left(4z^2 - (w+p)^2\right)$ is a Nasty number.
3) $p - w = 8t_{3,b}$ when $a = (b+1)$
4) $p - w = 8P_b^5$ when $a = b(b+1)$

5) $p - w = 24P_b^3$ when $a = b(b+1)(b+2)$

6) $p - w = 4SO_b$ when $a = (2b^2 - 1)$

7) When a and b represents the sides of the Pythagorean triangle then $3(x-y)$ is a nasty number.

B. Method 2

Note that (3) is also written as

$$w^2 + p^2 = 2v^2 * 1 \tag{9}$$

write 1 as $1 = \frac{(3+4i)(3-4i)}{25}$ (10)

Using (4), (5) and (10) in (8) and employing the method of factorization, define

$$(w + ip) = \frac{1}{5}(3+4i)(1+i)(a^2 - b^2 + 2iab) \tag{11}$$

Equating the real and imaginary parts in (11) and replacing a by $5A$ and b by $5B$, we have

$$\left.\begin{aligned} w &= w(A,B) = -5A^2 + 5B^2 - 70AB \\ p &= p(A,B) = 35A^2 - 35B^2 - 10AB \end{aligned}\right\} \tag{12}$$

In this case, the corresponding integer solutions to (1) are found to be

$$\left.\begin{aligned} x &= x(A,B) = u + 25A^2 + 25B^2 \\ y &= y(A,B) = u - 25A^2 - 25B^2 \\ z &= z(A,B) = 25A^2 + 25B^2 \end{aligned}\right\} \tag{13}$$

Note 2:

It is worth mentioning that two more sets of integer solutions to (1) are obtained by considering

$$2 = \frac{(1+7i)(1-7i)}{25}, \quad 1 = \frac{(3+4i)(3-4i)}{25}$$

$$2 = \frac{(7-i)(7+i)}{25}, \quad 1 = \frac{(3+4i)(3-4i)}{25}$$

Remark:

Note that 1 in (10) may be considered in the general form as given below:

$$1 = \frac{[2mn + i(m^2 - n^2)][2mn - i(m^2 - n^2)]}{(m^2 + n^2)^2}$$

Or

$$1 = \frac{[m^2 - n^2 + i2mn][m^2 - n^2 - i2mn]}{(m^2 + n^2)^2}$$

Properties:

1) When A and B represents the sides of the Pythagorean triangle then $3(x-y)$ is a nasty number.

2) $p + 7w + 500PR_a = 0$ when $B = (A+1)$

3) $p + 7w + 1000P_A^5 = 0$ when $B = A(A+1)$

4) $p + 7w + 1500\, OH_A = 0$ when $B = A(2A^2 + 1)$

5) $p + 7w + 500CP_{A,6} = 0$ when $B = A^2$

6) $p + 7w + 3000P_A^3 = 0$ when $B = (A+1)(A+2)$

7) $p + 7w + 3000P_A^4 = 0$ when $B = (2A+1)(A+1)$

C. Method 3

Observe that (3) is written in the form of ratio as

$$\frac{(w+v)}{(v+p)}=\frac{(v-p)}{(w-v)}=\frac{\alpha}{\beta},\ \beta\neq 0 \qquad (14)$$

which is equivalent to the system of double equation

$$\beta w-\alpha p=v(\alpha-\beta)$$
$$\alpha w+\beta p=v(\alpha+\beta)$$

Applying the method of cross multiplication, we get

$$\left.\begin{aligned} w &= -\alpha^2+\beta^2-2\alpha\beta \\ p &= \alpha^2-\beta^2-2\alpha\beta \\ v &= \alpha^2+\beta^2 \end{aligned}\right\} \qquad (15)$$

In view of (2), we have

$$\left.\begin{aligned} x &= u+\alpha^2+\beta^2 \\ y &= u-\alpha^2-\beta \\ z &= \alpha^2+\beta^2 \end{aligned}\right\} \qquad (16)$$

Thus, (15) and (16) represent the non-zero distinct integer solutions to (1).

IV. CONCLUSIONS

In this paper, we have made an attempt to find infinitely many distinct integer solutions to the homogeneous bi-quadratic equation with five unknowns given by $x^4-y^4+2(x^2-y^2)(w^2+p^2)=4(x^3+y^3)z$. To conclude, one may search for other choices of solutions to the considered bi-quadratic equation with five unknowns.

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