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Remark on the Paper Entitled Lattice Points of a Cubic Diophantine Equation $11(x+y)^2 = 4(xy+11z^3)$

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Abstract: In this paper, new sets of solutions to the cubic equation with three unknowns given by $11(x+y)^2 = 4xy+44z^3$ are presented.
Keywords: Ternary cubic, Integer solutions

I. INTRODUCTION

When a search is made for cubic diophantine equations, the authors noticed a paper by Manju Somanath, J. Kannan, K. Raja [1] in which they have presented lattice points of the cubic diophantine equation $11(x+y)^2 = 4xy+44z^3$. However, there are other interesting sets of solutions to the above equations that are exhibited in this paper.

II. METHOD OF ANALYSIS

Consider the cubic equation with three unknowns given by

$$11(x+y)^2 = 4xy+44z^3 \tag{1}$$

To start with, the substitution

$$y = (2k-1)x \tag{2}$$

in (1) gives

$$(11k^2-2k+1)x^2 = 11z^3$$

which is satisfied by

$$x = 121(11k^2-2k+1)\alpha^3 \tag{3}$$

$$z = 11(11k^2-2k+1)\alpha^2 \tag{4}$$

Note that (2) – (4) satisfies (1)

Again, the substitution

$$y = 2kx \tag{5}$$

in (1) leads to

$$(44k^2+36k+11)x^2 = 44z^3$$

whose solutions are

$$x = 242(44k^2+36k+11)\alpha^3 \tag{6}$$

$$z = 11(44k^2+36k+11)\alpha^2 \tag{7}$$

Thus, (5)-(7) satisfy (1)

Further,

Introduction of the linear transformations

$$x = u+v,\quad y = u-v,\quad z = u \tag{8}$$

in (1) leads to

$$v^2 = u^2(11u-10) \tag{9}$$

After performing some algebra, it is noted that (9) is satisfied by the following two choices of u and v:

1) $u = 11k^2-2k+1,\ v = (11k-1)(11k^2-2k+1)$

2) $u = 11k^2+2k+1,\ v = (11k+1)(11k^2+2k+1)$

In view of (8), the corresponding two sets of values to x, y, z satisfying (1) are represented below:

a) *Set 1:* Consider choice (i). The values of x, y, z are:

$$x = 11k\left(11k^2 - 2k + 1\right)$$
$$y = \left(2 - 11k\right)\left(11k^2 - 2k + 1\right)$$
$$z = 11k^2 - 2k + 1$$

b) *Set 2:* Consider choice (ii). The values of x, y, z are:

$$x = \left(11k + 2\right)\left(11k^2 + 2k + 1\right)$$
$$y = -11k\left(11k^2 + 2k + 1\right)$$
$$z = 11k^2 + 2k + 1$$

REFERENCE

[1] Manju Somanath, J.Kannan, K.Raja, "Lattice Points Of A Cubic Diophantine Equation $11(x+y)^2 = 4(xy + 11z^3)$", IJRASET, Volume 5, Issue 5,1797-1800, 2017.