www.ijraset.com Volume 3 Issue III, March 2015
IC Value: 13.98 ISSN: 2321-9653
International Journal for Research in Applied Science & Engineering Technology (IJRASET)

Onset Instability of Thermosolutal Convective Flow of Viscoelastic Maxwell Fluid Thorough Porous Medium with Linear Heat Source Effect

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Abstract- The Instability mechanism of Thermosolutal convection in a horizontal layer of viscoelastic Maxwell fluid through porous medium with internal linear heating is presented in this manuscript. The flow is also affected with temperature and concentration gradient in their medium. The Darcy model is adopted in the momentum equation. The onset of instabilities of the viscolastic Maxwell fluid layer is determined between free-free boundaries. The main emphasis is given to the internal heating which is linear in nature. The entire result section is presented in form critical heat source of intensity with respect to other governing physical parameter.

Keywords – Viscoelasic Maxwell fluid, Porous media, Rayleigh number, Internal heat source.

I. INTRODUCTION

The study of the onset of thermosolutal or double – diffusive convection in fluid saturated porous layer has been an active area of research interest for many years. These phenomena of combined heat and mass transfers where both temperature and solute fields contribute to the buoyancy of the fluid have many applications in the behaviour of fluids in the crust of the earth, geophysics, metallurgy, material science and petroleum engineering. For instance, in geo- logical processes thermosolutal convection in porous media may be important in dolomitisation of carbonate platforms (Kaufman)[1], soil salinisation (Gilman and Bear)[2] .Comprehensive reviews of the literature on double – diffusive natural convection in porous media and its applications can be found in Nield and Bejan [3]

There are large number of practical situations in which convection is driven by internal heat source in the porous media. The wide applications of such convections occur in nuclear reactions, nuclear heat cores, nuclear energy, nuclear waste disposals, oil extractions, and crystal growth. The study concerning internal heat source in porous media is provided by Tveitereid [4], who obtained the steady solution in the form of hexagons and two dimensional rolls for convection in a horizontal porous layer with internal heat source. Bejan [5] studied analytically the buoyancy induced convection with internal heat source, Parthiban and Patil [6] studied the effect of non-uniform boundaries temperatures on thermal instability in a porous medium with internal heat source and predicted that internal heat source parameter advances the onset of convection. Hill [7] performed linear and nonlinear stability analyses of double-diffusive convection in a porous layer with a concentration based internal heat source. Saravanan [8] investigated linear stability analysis for the onset of natural convection in a fluid saturated porous medium with uniform internal heat source and density maximum in an local thermal nonequilibrium model and predicted that internal heat source parameter advances the onset of convectionRecently Bhadauria group [9-12] have studied the problem of thermal instability in porous media with internal heating, considering various physical models

Straughan and Hutter [13] have investigated the double diffusive convection with Soret effect in a porous layer using Darcy–Brinkman model and derived a priori bounds. An analytical and numerical study of double diffusive parallel flow in a horizontal sparsely packed porous layer under the influence of constant heat and mass flux is performed using a Brinkman model by Amahmid et al. [11]. Mamou and Vasseur [12] have studied the double diffusive instability in a horizontal rectangular porous enclosure subject to vertical temperature and concentration

gradients Double diffusive convection in a vertical enclosure filled with anisotropic porous media has been studied numerically by Bennacer et al. [13].

II. MATHEMATICAL FORMULATIONS OF THE PROBLEM

Let us Consider an infinite horizontal layer of viscoelastic Maxwell fluid with thickness “d,” cramped in the planes around $z = 0$ and $z = d$ in a dense porous whose porosity is ε & media permeability is approximately 1 this layer is instigate by aligned gravitational vector $\mathbf{g}(0,\ 0,\ -g)$. The fluid layer is at the constant temperature and concentration at the boundaries. The temperature & concentration is taken T0 and C_0 at the boundary $z = 0$ and assumes to be T_1 and C_1 be the difference in temperature and concentration across the boundaries.

Here the symoble is representing as $\mathbf{q}(u,\ V,\ w)$, p, ρ, T, C, α, α', μ, κ, and k' , $\boldsymbol{Q_0}$ be the Darcy velocity vector, hydrostatic pressure, density, temperature, solute concentration, coefficient of thermal expansion, an analogous solvent coefficient of expansion, viscosity, thermal diffusivity, solute diffusivity, and linear heat source of fluid, respectively.

The following assumptions is made for the mathematical formulation of the physical problem

A. Assumptions

1) Thermo physical properties expect for density difference at the buoyancy are constant.
2) Darcy's model is adopted in momentum equation.
3) The medium is assumed to be isotropic and homogeneous in nature.
4) The fluid and solid matrix are taken to be in thermal equilibrium state.
5) Radiation heat transfer is neglected during the process but internal heating is taken care during the heat transfer of the fluid flow process

Under the above assumptions the governing equations are given below

B. Governing Equations

*The G*overning equations for viscoelastic Maxwell fluid through porous medium is governed in form of partial differential equation which may written as

$$\nabla . q = 0$$

$$\left(\mathbf{1} + \boldsymbol{\lambda}\frac{\partial}{\partial t}\right)(-\nabla p+\rho\left(1-\alpha(T-T_0) + \alpha'(C-C_0)g\right)) - \frac{\mu}{k_1}\boldsymbol{q} = \mathbf{0}$$

$$\boldsymbol{\sigma}\frac{\partial T}{\partial t} + q.\nabla T = k\,\boldsymbol{\nabla^2 T} + \boldsymbol{D_{TC}\nabla^2 C} + \boldsymbol{Q_0(T' - T_0)},$$

$$\boldsymbol{\varepsilon}\frac{\partial C}{\partial t} + q.\nabla C = k'\boldsymbol{\nabla^2 C} + \boldsymbol{D_{CT}\nabla^2 T}, \qquad \textbf{(1)}$$

Where $\boldsymbol{D_{TC}}$ and $\boldsymbol{D_{CT}}$ are the dufour and Soret coefficients ; $\boldsymbol{\sigma} = (\boldsymbol{\rho c_p})_{\boldsymbol{m}} / = (\boldsymbol{\rho c_p})_{\boldsymbol{f}}$ **is the thermal capacity , $\boldsymbol{c_p}$**is specific heat, and the subscript m and f refer to porous medium and fluid, respectively.

Here the wall temperature and concentration assumed to be constant w.r.t the boundaries of the fluid layer. Therefore , the boundary condition are define as follows

$$w = 0, T = T_o\ , C = C_o \quad at\ z = 0$$

$$w = 0, T = T_1\ , C = C_1 \quad at\ z = d \qquad (2)$$

C. Steady state and its solutions

The steady state solution can be obtained by assuming

$$u = v = w = 0, p = p(z), T = T_s(z), C = C_s(z) \qquad (3)$$

The steady state solution is given by

$$T_s = T_0 - \Delta T\left(\frac{z}{d}\right),$$

$$C_s = C_0 - \Delta C\left(\frac{z}{d}\right) \qquad (4)$$

$$p_s = p_0 - p_0 g(z + \alpha\frac{\Delta T}{2d}z^2 + \alpha'\frac{\Delta C}{2d}z^2)$$

Where subscript 0 shows the value of the variable at boundary z = 0

D. Disturbance in flow

Volume 3 Issue III, March 2015
ISSN: 2321-9653
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In order to investigate the stability of the flow dynamic, it necessary to give imposed infinitesimal perturbations on the basic state which is well documented in the book of Chandershakra rao (1992), The perturbation on the base flow is defined as .

$$q = 0 + q' , T = 0 + T', C = C_s + C' , p = p_s + p' \quad (5)$$

where the parameters q' , T', C' , p' is known as the perturbed quantities of the mean flow dynamics. Substituting (5) into (1) and neglecting higher order terms of the perturbed quantities, then we get

$$\nabla . q' = 0$$

$$\left(\mathbf{1} + \boldsymbol{\lambda}\frac{\partial}{\partial t}\right)(-\nabla p' + \rho_0(\alpha T' + \alpha' C')g) = \frac{\mu}{k_1}\boldsymbol{q}'$$

$$\sigma\frac{\partial T'}{\partial t} - \omega'\frac{\Delta T}{d} = \kappa(\nabla^2 + \boldsymbol{Q})\boldsymbol{T}' + \boldsymbol{D}_{TC}\nabla^2\boldsymbol{C}'$$

$$\boldsymbol{\varepsilon}\frac{\partial C'}{\partial t} - \omega'\frac{\Delta C}{d} = \kappa'(\nabla^2)\boldsymbol{C}' + \boldsymbol{D}_{CT}\nabla^2\boldsymbol{T}' \quad \textbf{(6)}$$

the dimensionless parameters are defines as follows.

$$(x^{"}, y^{"}, z^{"}) = \frac{1}{d}(x', y', z'), \quad (u^{"}, v^{"}, w^{"}) = \frac{d}{k}(u', v', w'), \quad t^{"} = \frac{k}{\sigma d^2}t', \quad T^{"} = \frac{T\prime}{\Delta T\prime}, \quad C^{"} = \frac{C\prime}{\Delta c\prime}, \quad p^{"} = \frac{k_1 d^2}{\mu k}p',$$

$$\boldsymbol{Q} = \frac{\boldsymbol{h^2 Q_0}}{\boldsymbol{\alpha_m(\rho c_p)_f}}, \quad (7)$$

Remove asterisk for the simplicity

$$\nabla . q = 0$$

$$\left(1 + F\frac{\partial}{\partial t}\right)(-\nabla p + RaT + RaC) - q = 0$$

$$\frac{\partial T}{\partial t} - w == (\nabla^2 + \boldsymbol{Q})\boldsymbol{T} + \boldsymbol{D_f}\nabla^2\boldsymbol{C},$$

$$\frac{\varepsilon}{\sigma}\frac{\partial C}{\partial t} - w = \frac{1}{Le}\nabla^2\boldsymbol{C} + \boldsymbol{S_r}\nabla^2\boldsymbol{T} \quad \textbf{(8)}$$

The different non-dimensional parameters are defined as follows.

, $Ra = \frac{\alpha\, dk_1\, \Delta T g \rho_0}{\mu K}$ is the thermal Rayleigh number , $Rs = \frac{\alpha'\, dk_1\, \Delta C g \rho_0}{\mu k'}$ is the solutal Rayleigh number, $Le = \frac{k}{k\prime}$ is the Lewis number, $F = (\frac{k}{\sigma d^2})\lambda$ is the stress relaxation parameter and Q is the rate of heat addition per unit mass by internal sources $\boldsymbol{Q} = \frac{\boldsymbol{h^2 Q_0}}{\boldsymbol{\alpha_m(\rho c_p)_f}}$. $D_f = \frac{D_{TC}}{k}\frac{\Delta C}{\Delta T}$, s the Dufour parameter, and $Sr = \frac{D_{CT}}{k}\frac{\Delta T}{\Delta C}$is the Soret parameter.The nondimensional boundary conditions are

The non dimensional boundary conditions are

$$w = T = C = 0 \; at \; z = 0, z = 1 \quad (9)$$

III. NORMAL MODES AND STABILITY ANALYSIS

The disturbances of the mean flow are taken into account in term of normal modes analysis which is well documented in the book of Drazin (1987).

$$[w, T, C] = [W(z), \Theta(z), \Gamma(z)] \exp(ik_x x + ik_y y + nt) \quad (10)$$

Where the parameters k_x , k_y are called as wave numbers along with different coordinate axis x and y respectively, and n is defined as growth rate of disturbances. By using eq (10) the system of eq (8) becomes

$$(D^2 - a^2)W + (1 + Fn)(a^2 Ra\Theta + a^2 Rs\Gamma) = 0$$

$$W + (D^2 - a^2 - n - Q)\,\Theta + D_f(D^2 - a^2)\Gamma) = 0$$

$$W + S_r(D^2 - a^2)\Theta + \left(\frac{1}{Le}(D^2 - a^2) - \frac{\varepsilon}{\sigma}n\right)\Gamma = 0$$

Where $D = \frac{d}{dz}$ *and* $a = \sqrt{k_x^2 + k_y^2}$ is define as dimensionless wave number.
The corresponding free – free boundary conditions are
$W = 0, D^2W = 0, \Theta = 0, \Gamma = 0, at\ z = 0$
$W = 0, D^2W = 0, \Theta = 0, \Gamma = 0, at\ z = 1$ (12)

We assume the solution to $W,\ \Theta, and\ \Gamma$ is of the form
$W = W_0 \sin \pi z,\ \Theta = \Theta_0 \sin \pi z, \Gamma = \Gamma_0 \sin \pi z,$ (13)

Those satisfy the boundary conditions (12).

Substituting solution (13) in (11), integrating each equation from $z = 0$ to $z = 1$ by parts, we obtain the following matrix equation as

$$\begin{bmatrix} J & -a^2(1+Fn)Ra & -a^2(1+Fn)Rs \\ -1 & (J+n+Q) & D_fJ \\ -1 & S_rJ & \left(\frac{J}{Le}+\frac{\varepsilon n}{\sigma}\right) \end{bmatrix} \begin{bmatrix} W_0 \\ \Theta_0 \\ \Gamma_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

Where $J = \pi^2 + a^2$
The nontrivial solution corresponding to the matrix given in eq (14)

$$Ra = \frac{(J)(J+n+Q)\left(\left(\frac{J}{Le}+\frac{\varepsilon n}{\sigma}\right)-S_rD_fJ^2\right)}{a^2(1+Fn)J\left(\left(\frac{1}{Le}\right)-D_f\right)+\left(\frac{\varepsilon n}{\sigma}\right)} + \frac{a^2(1+Fn)(S_rJ-(J+n+Q))}{J\left(\left(\frac{1}{Le}\right)-D_f\right)+\left(\frac{\varepsilon n}{\sigma}\right)}Rs \quad (15)$$

For neutral instability $n = i\omega$, (where ω is real and dimensionless frequency of oscillation) and equating real and imaginary parts of (15), we have

$$J\left(\left(\frac{J^2}{Le} - \frac{\varepsilon\omega^2}{\sigma} + \frac{QJ}{Le}\right) - S_rD_fJ^2\right) + a^2Ra\left(J\left(D_f - \frac{1}{Le}\right) + \frac{\varepsilon\omega^2F}{\sigma} - a^2Rs(J + Q - S_rJ - \omega^2F\right) = 0$$

$$J^2\left(\frac{1}{Le} + \frac{\varepsilon}{\sigma}\right) + a^2Ra\left(FJ\left(D_f - \frac{1}{Le}\right) - \frac{\varepsilon}{\sigma}\right) - a^2Rs(JF - S_rJF + FQ) = 0 \quad (16)$$

For stationary convection ω=0 (n=0), we have

$$Ra = \frac{(J)(S_rD_fJ^2 - J - Q)\left(\frac{J}{Le}\right)}{a^2\left(D_fJ - \frac{J}{Le}\right)} + \frac{a^2[J+Q-S_r]}{D_fJ - \left(\frac{J}{Le}\right)a^2}Rs \quad (17)$$

Here the onset instability is measured in form of stationary convection. The different parameter is defined as, the Rayleigh number Ra is a function of dimensionless wave number a, Dufour parameter D_f, Soret parameter S_r, Lewis number Le and solutal Rayleigh number Rs, and internal heat source Q. Thus for stationary convection of viscoelastic Maxwell fluid is work as an ordinary Newtonian fluid.
The critical cell size at the onset of instability is calculated from.
$\left(\frac{\partial Ra}{\partial a}\right)_{a=a_c} = 0$ Which gives $a_c = \pi$
The corresponding critical Rayleigh number Ra_c for the steady onset is

$$Ra_c = 4\pi^2\left(\left(\frac{S_rD_fLe-1}{D_fLe-1}\right) + \frac{(S_r-1)Le}{1-D_fLe}Rs \quad (18)$$

If $S_r = D_f = Rs = 0$ then $Ra_c = 4\pi^2$ (19)

IV. RESULT AND DISCUSSION

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The onset of double diffusive convection in a horizontal layer of Maxwell viscoelastic fluid in the presence of internal generation effect along with Soret and Dufour in a porous medium is investigated analytically. The stationary convection through critical Rayleigh numbers is characterize the stability of the flow dynamics. All the fundamental solution is obtained analytically. The normal mode analysis is taken in to account to find the stability equations. During the study it is found that the stationary critical Rayleigh number independent from the viscoelastic parameter F; in such case the Maxwell viscoelastic binary fluid behaves like ordinary Newtonian binary fluid. The basic flow of the above study shows that the critical Rayleigh number and critical wave number are independent from viscoelastic parameter.

The vide range of different controlling parameter is defined below the solutal Rayleigh number Rs, Soret Parameter S_r ,Dufour Parameter D_f and Lewis Number Le are in the range of $10^2 \leq Rs \leq 10^3$ (solutal Rayleigh number), $0 \leq S_r \leq 1$ (Soret parameter), , $0 \leq D_f \leq 1$ (Dufour parameter), and $10^{-2} \leq Le \leq 1$ respectively. Wherever the thermal Rayleigh number is taken to be in between $10^2 \leq Ra \leq 10^5$. The of Dufour effect Df and Lewis number Le in taken in such a way that $D_f Le \neq 1$.

The stability boundary curve for the different values of internal heat generation is shown below. The internal heat source Q is taken to be 0.5, 1 , 5, 10 respectively. There are four graph has been plotted for the variety of parameters to explain the stability of the flow dynamics. In each figure the attempt has been taken care to explain various parameter in a single figure. Keeping in view the above statement.

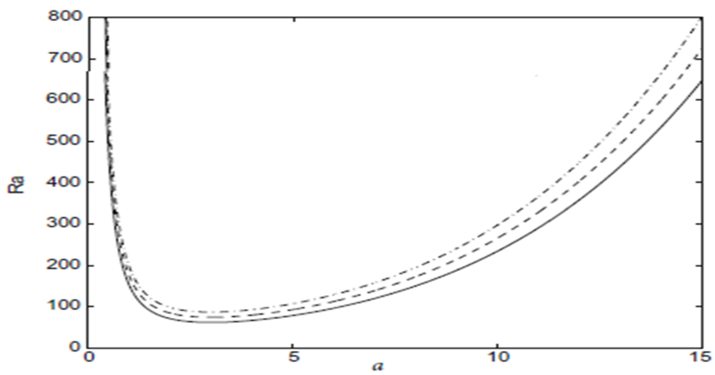

Figure 1 Variation of Critical Rayleigh Number Ra and Wave number for a Different value of Dufour Effect at Q=0.5

Figure 1 is plotted in Ra Critical and wave number a plane. Which shows the variation of stationary Rayleigh number with wave number for different values of Dufour parameter while fixing the value if Le=1 , Q =0.5 , Sr = 0.7 and $Rs = 5 \times 10^2$ respectively. There are three types of line is plotted Dash dot (--….-…) , dahsesd (- - - - - - -) and solid line (------------) which shows the different value of Dufour parameter 0.1, 0.5 and 0.9 respectively. In this plot it has been found that the as we increase the dufour effect the slop of the critical Ra is being decreases which is at 102, 99 and 84 respectively which also shifted in case of wave number. It means that the critical boundary is not stagnant in this case. This also pointed out from the plot that the between $0 < a < 5$ the Ra Criticaln decreases rapidly then increases smoothly. Concluding, it is found that the Critical Rayleigh number first increases then decreases and finally increases with increase in the value of Dufour parameter; thus for stationary convection Dufour parameter shows both the stabilizing and destabilizing effects depending upon different condition conditions. But for large value of a it shows the stabilizing character.

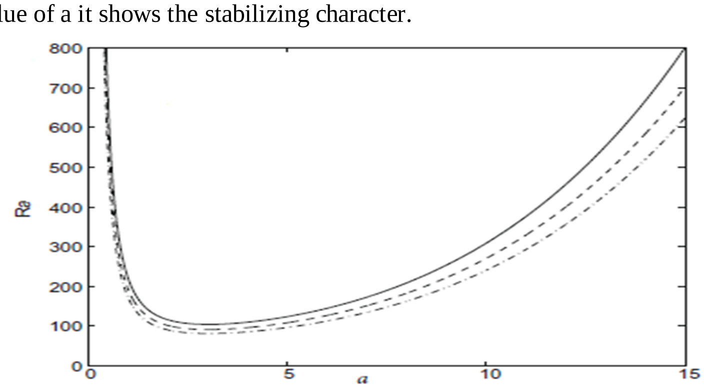

Figure 2 Variation of Critical Rayleigh number Ra and Wave number for a different value of soret parameter Sr at Q=1

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Figure 2 is plotted between Ra Critical and wave number a plane for the different range of soret parameter. Around this the objective is identify the role of thermo-diffusion at partially high value of internal heat source. This phenomenon observed in mixtures of porous particles during fluid flow where the different particle types exhibit different responses to the force of a thermal gradient. Thermodiffusion is labeled "positive" when particles move from a hot to cold region and "negative" when the reverse is true. Here we fix the internal heat source parameter is at 5 , which shows the 5 times effect from the previous. Here we observed that there is not much different is recorded in term of stability curve which but the characteristics profile is reverse. There are three types of line is plotted Dash dot (--….-…) , dahsesd (- - - - - - -) and solid line (------------) which shows the different value of soret parameter 0.1, 0.5 and 0.9 respectively. In this plot it has been found that the as we increase the dufour effect the slop of the critical Ra is being decreases which is at 101.8 , 99.3 and 84.7 respectively. Concluding, it is found that the Critical Rayleigh number first increases then decreases and finally increases with increase in the value of Dufour parameter; thus for stationary convection Dufour parameter shows both the destabilizing and stabilizing effects depending upon different condition conditions. But for large value of a it shows the stabilizing character as we seen in Figure1..

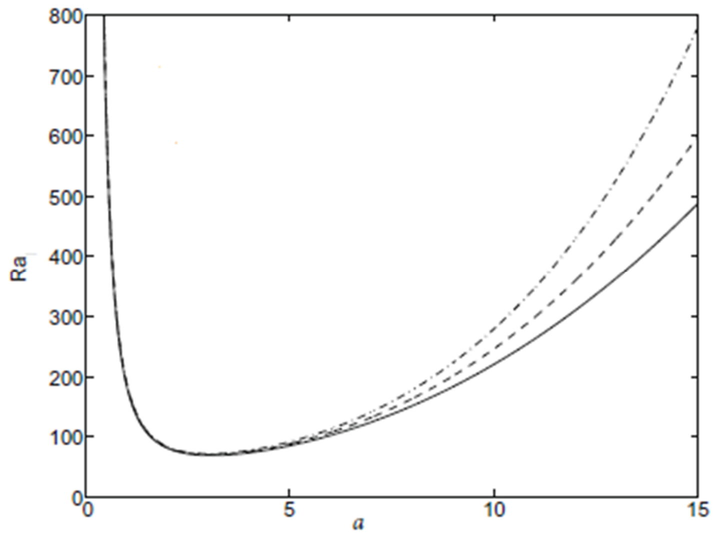

Figure 3 Varaition of Critical Rayliegh number Ra and wave number for a different value of Lewis number Le at Q =5

Figure 3 shows the variation of stationary Rayleigh number with wave number for different values of Lewis number and it is found that the Rayleigh number first increases then decreases and finally increases with increase in the value of Lewis number; thus for stationary convection Lewis number has both the stabilizing and destabilizing effects depending upon certain conditions.

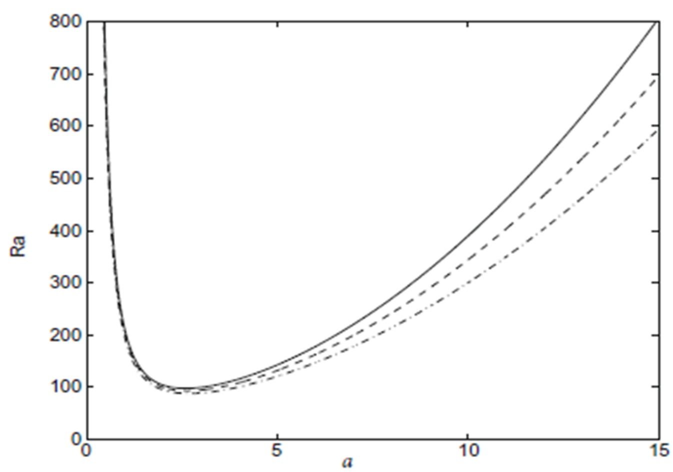

Figure 4 Variation of Rayliegh number Ra and Wave number number a for different value of Solutal Reyleigh number Rs at Q=10

Figure 4 shows the variation of Rayleigh number with wave number for different value of the solutal Rayleigh number Rs and it is found that the Rayleigh number decreases with increase in the value of solutal Rayleigh number Rs; thus solutal Rayleigh number Rs has destabilizing effect on the.

From all the figure it is also pointed out that there no much different is recorded while changing the heat source effect qualitatively but quantitatively minor effect is found which shows the destabilizing character in small interval of wave number of the flow dynamics.

V. CONCLUSIONS

The linear stability analysis of double diffusive convection in a horizontal layer of Maxwell viscoelastic fluid in the presence of Soret and Dufour in a porous medium is performed analytically. The normal mode analysis is taken into account for stability analysis. The onset of stationary instability through critical Rayleigh numbers is analyzed ,

The following points conclude the study of this paper.

A. During stationary convection the Maxwell - viscoelastic fluid is shows the effect like a ordinary Newtonian fluid.

B. Dufour, Soret parameter, both shows the stabilizing and destabilizing character during the stationary convection.

C. Lewis parameter is also have both stabilizing and destabilizing character during stationary convection.

D. Solutal Rayleigh number is play destabilizes characteristics during the stationary convection.

E. Internal heat source destabilizes the stationary convection in the small interval of wave number.

VI. ACKNOWLEDGEMENT

One of the author Mr Neetu Kumar thanks to UGC for providing the financial support during the preparation of this manuscript.

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