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Fibonacci Product Cordial Labelling of some Graphs

Tesssymol Abraham[1], Shiny Jose[2]
[1]Department of Mathematics, Baselius College,Kottayam-686001
[2]St.George's College,Aruvithura,Ksottayam-686112
[1, 2]Mahatma Gandhi University, Kottayam, Kerala

Abstract— An injective function φ: V (G) → { F_1, F_2, . . . , F_n}, where F_j is the j^{th} Fibonacci number (j = 1, . . . , n), is said to be Fibonacci product cordial labeling if the induced function φ* : E (G) → {0, 1} defined by $\varphi^*(uv) = (\varphi(u)\varphi(v))\, mod2$ satisfies the condition $|e_{\varphi*}(0)-e_{\varphi*}(1)| \le 1$. A graph which admits Fibonacci product cordial labeling is called Fibonacci product cordial graph. In this paper we investigated the Fibonacci product cordial labeling of Shell graph, Comb , Crown , Star graph and Complete bipartite graphs $K_{2,n}$.

Keywords— Fibonacci Product Cordial Labeling; Fibonacci Product Cordial graph, Shell ,Comb, Crown

I. INTRODUCTION

In graph labeling vertices or edges or both assigned values subject to certain conditions. There are many types of labeling. More number of labeling techniques found their starting point from graceful labeling introduced by Rosa[7]. A Fibonacci cordial labeling have been introduce by Rokad,A.K., and G.V.Ghodasara[6]. Also Fibonacci product cordial labeling have introduced by Tessymol Abraham and Shiny Jose [8].

We begin with simple,finite,connected and undirected graph $G = (V(G), E(G))$ with p vertices and q edges. In this paper we discussed Fibonacci product cordial labeling of Shell graph, Comb, Crown , Star graph and Complete bipartite graphs $K_{2,n}$. We will provide brief summary of definitions and other information which are necessary for present investigations

Notations:

$v_\varphi(0)$: Number of vertices with label 0 under the map φ

$v_\varphi(1)$: Number of vertices with label 1 under the map φ

$e_{\varphi^*}(0)$: Number of edges with label 0 under the map φ^*

$e_{\varphi^*}(1)$: Number of edges with label 1 under the map φ^*

FPCL: Fibonacci product cordial labeling

II. PRELIMINARIES

A. Definition 2.1 Fibonacci Product Cordial Labeling [8]

An injective function $\varphi: V(G) \to \{F_1, F_2, F_3, \dots F_n\}$ where F_j is the j[th] Fibonacci number $(j = 1, \dots, n)$, is said to be Fibonacci product cordial labeling if the induced function $\varphi^*: E(G) \to \{0,1\}$ defined by $\varphi*(uv) = (\varphi(u)\,\varphi(v))\mathrm{mod}2$ satisfies the condition $|e_{\varphi*}(0) - e_{\varphi*}(1)| \le 1$.

B. Definition 2.2 Fibonacci Product Cordial Graph [8]

A graph which admits Fibonacci product cordial labeling is called Fibonacci product cordial graph

C. Definition 2.3 Comb Graph [3]

The comb is a graph obtained by joining a single pendent edge to each vertex of a path. It is denoted by $P_n AK_1$

D. Definition 2.4 Shell Graph [5]

A shell graph of size n, denoted by S_n is the graph obtained from the cycle $C_n(v_1 v_2 \dots\dots v_n)$ by adding $(n-3)$ consecutive chords incident with a common vertex. The vertex at which all the chords are concurrent is called the apex vertex. The shell is also called fan fn−1. Thus, S_n= fn−1 = Pn−1 + K1.

E. Definition 2.5 Crown Graph [3]

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A crown graph is the graph obtained from the cycle $C_n(v_1 v_2 \ldots\ldots v_n)$ with pendant edge attached at each vertex of the cycle. It is denoted by Cn ʘ K1

III. PROPERTIES OF FIBONACCI PRODUCT CORDIAL LABELING

A. *Theorem 3.1*

Shell graph S_n is a Fibonacci product cordial graph for $n \geq 3, n \in N$.

1) *Proof:* Let w_1 be the common end vertex (apex) and $w_2, w_3, \ldots, w_n$ be the other vertices of a Shell Graph S_n. Then each edge is of the form $w_1 w_j$ *where* $2 \leq j \leq n$.

We define labeling function $\varphi: V(G) \to \{F_1, F_2, F_3, \ldots F_n\}$ as follows. ,

$\varphi(w_i) = F_i \quad if \;\; 1 \leq i \leq n$

Define $\varphi^*: E(G) \to \{0,1\}$ by $\varphi^*(w_1 w_j) = \left(\varphi(w_1)\varphi(w_j)\right) mod\ 2 \;\; for\ 2 \leq j \leq n$

Then we have $|e_{\varphi*}(0) - e_{\varphi*}(1)| \leq 1$. Hence Shell graph admits Fibonacci product cordial labeling and hence it is a Fibonacci product cordial graph.

2) *Illustration 3.1:*

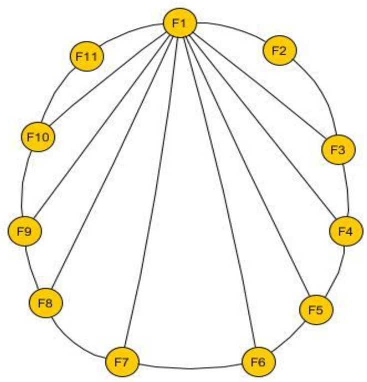

Figure 3.1. ***FPCL Of Shell Graph*** S_{11}

B. *Theorem 3.2*

The Comb is Fibonacci product cordial graph for all $n \in N$

1) *Proof:* Let G be a comb graph $P_n AK_1$ with 2n vertices and 2n-1 edges.Let $V(G) = \{v_1, v_2, \ldots, v_n\ w_1, w_2, \ldots, w_n\}$ where $v_1, v_2, \ldots, v_n$ are the vertices of the path P_n and $w_1, w_2, \ldots, w_n$ are the pendant vertices attaching by the path P_n.Then $E(G) = \{e_i, e_{ij}\}$ where $e_i = (v_i, w_i)$ and $e_{ij} = (v_i, v_j)$

Case 1: $n \equiv 0,1 mod\ 3$

Define the function $\varphi: V(G) \to \{F_1, F_2, F_3, \ldots F_{2n}\}$ as

$$\varphi(v_i) = \begin{cases} F_{2i-2} & if\ i \equiv 0\ mod\ 3 \\ F_{2i} & if\ i \equiv 1\ mod\ 3 \\ F_{2i+2} & if\ i \equiv 2\ mod\ 3 \end{cases}$$

$$\varphi(w_i) = F_{2i-1} \quad \text{if} \quad 1 \leq i \leq n$$

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Then $\varphi^*: E\ (G) \rightarrow \{0,1\}$ by $\varphi^*(e_i) = (\varphi(v_i)\varphi(w_i)) mod\ 2$ and $\varphi^*(e_{ij}) = \left(\varphi(v_i)\varphi(v_j)\right) mod\ 2$ admits Fibonacci product cordial labeling. Hence Combs are Fibonacci product cordial graphs.

2) Illustration 3.2:

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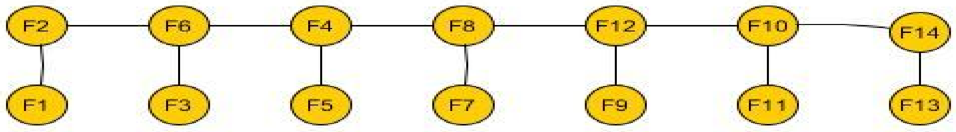

Figure 3.2. ***FPCL Of Comb*** P_7AK_1

Case 2: $n \equiv 2\ mod\ 3$

Define the function $\varphi: V\ (G) \rightarrow \{\ F_1,\ F_{2,}\ F_3,\ \ldots F_{2n}\}$ as

For $1 \leq i \leq n-1$

$$\varphi(v_i) = \begin{cases} F_{2i-2} & if\ i\ \equiv 0\ mod\ 3 \\ F_{2i} & if\ i\ \equiv 1\ mod\ 3 \\ F_{2i+2} & if\ i\ \equiv 2\ mod\ 3 \end{cases}$$

$$\varphi(v_i) = F_{2i} \qquad \text{if } i = n$$

$$\varphi(w_i) = F_{2i-1} \quad \text{if} \quad 1 \leq i \leq n$$

Then $\varphi^*: E\ (G) \rightarrow \{0,1\}$ by $\varphi^*(e_i) = (\varphi(v_i)\varphi(w_i)) mod\ 2$ and $\varphi^*(e_{ij}) = \left(\varphi(v_i)\varphi(v_j)\right) mod\ 2$ admits Fibonacci product cordial labeling. Hence Combs are Fibonacci product cordial graphs.

3) Illustration 3.3:

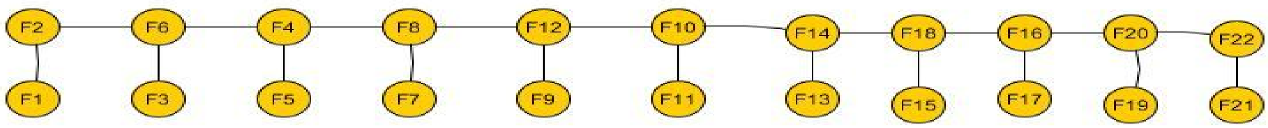

Figure 3.3. ***FPCL Of Path***$P_{11}AK_1$

C. Theorem 3.3

The Crown $C_n \odot K1$ is a Fibonacci product cordial graph for all $n \geq 3$, n ∈ N

1) Proof: Let $C_n \odot K1$ be a Crown graph with 2n vertices and 2n edges. Let $V(G) = \{v_1,\ v_2, \ldots,\ v_n\ w_1,\ w_2, \ldots,\ w_n\}$ where $v_1,\ v_2, \ldots,\ v_n$ are the vertices of the cycle C_n and $w_1,\ w_2, \ldots,\ w_n$ are the pendant vertices attaching by the cycle C_n.Then $E(G) = \{e_i, e_{ij}\}$ where $e_i = (v_i, w_i)$ and $e_{ij} = (v_i, v_j)$

.

Case 1:$n \equiv 0\ mod\ 3$

Define the function $\varphi: V\ (Cn \odot K1\ ) \rightarrow \{\ F_1,\ F_{2,}\ F_3,\ \ldots F_{2n}\}$ as

$$\varphi(v_i) = \begin{cases} F_{2i-2} & if\ i\ \equiv 0\ mod\ 3 \\ F_{2i} & if\ i\ \equiv 1\ mod\ 3 \\ F_{2i+2} & if\ i\ \equiv 2\ mod\ 3 \end{cases}$$

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$$\varphi(w_i) = F_{2i-1} \quad \text{if} \quad 1 \le i \le n$$

Then $\varphi^*: E\ (Cn \odot K1) \to \{0,1\}$ by $\varphi^*(e_i) = (\varphi(v_i)\varphi(w_i)) mod\ 2$ and $\varphi^*(e_{ij}) = \left(\varphi(v_i)\varphi(v_j)\right) mod\ 2$ admits Fibonacci product cordial labeling. Hence Crowns are Fibonacci product cordial graphs.

2) *Illustration 3.4 :*

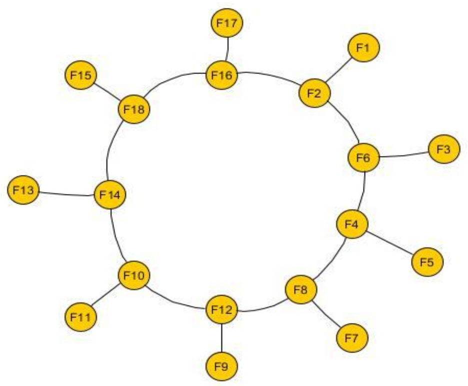

Figure 3.4. ***FPCL Of Crown*** $C_9 \odot K_1$

Case 2: $n \equiv 1, 2\ mod\ 3$

Define the function $\varphi: V\ (Cn \odot K1) \to \{F_1, F_2, F_3, \dots F_{2n}\}$ as

For $1 \le i \le n-5$

$$\varphi(v_i) = \begin{cases} F_{2i-2} & if\ i \equiv 0\ mod\ 3 \\ F_{2i} & if\ i \equiv 1\ mod\ 3 \\ F_{2i+2} & if\ i \equiv 2\ mod\ 3 \end{cases}$$

For $n-4 \le i \le n$

$$\varphi(v_i) = F_{2i}$$

$$\varphi(w_i) = F_{2i-1} \quad \text{if} \quad 1 \le i \le n$$

Then $\varphi^*: E\ (Cn \odot K1) \to \{0,1\}$ by $\varphi^*(e_i) = (\varphi(v_i)\varphi(w_i)) mod\ 2$ and $\varphi^*(e_{ij}) = \left(\varphi(v_i)\varphi(v_j)\right) mod\ 2$ admits Fibonacci product cordial labeling. Hence Crowns are Fibonacci product cordial graphs.

3) *Illustration 3.5:*

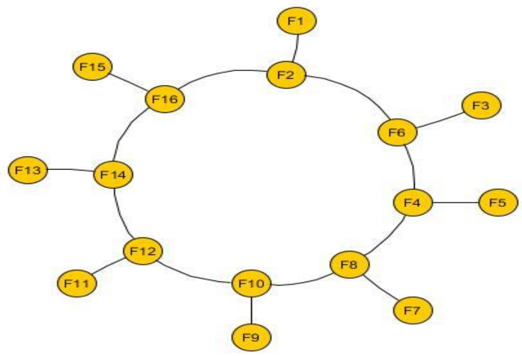

Figure 3.5. ***FPCL Of Crown*** $C_8 \odot K_1$

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D. Theorem 3.4:

The Star graphs $K_{1,n}$ are Fibonacci product cordial graphs for $n = 2,3,5$

1) Proof:

Let u be the apex vertex and $\{v_1, v_2, \dots, v_n\}$ be the pendant vertices of the star graph $K_{1,n}$ .Then $E(K_{1,n}) = \{e_i\}$ where $e_i = (u, v_i)$

Define : $V(K_{1,n}) \to \{F_1, F_2, F_3, \dots F_n\}$ as

$\varphi(u) = F_1$ and $\varphi(v_i) = F_{i+1}$ $for\ 1 \le i \le n$

Also define $\varphi^*: E(K_{1,n}) \to \{0,1\}$ by $\varphi^*(e_i) = (\varphi(u)\varphi(v_i)) mod\ 2$ .Then we have $|e_{\varphi*}(0) - e_{\varphi*}(1)| \le 1$. Hence Star graphs $K_{1,n}$,$1 \le n \le 3$ are Fibonacci product cordial graphs.

2) Illustration 3.6:

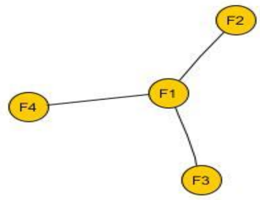

Figure 3.6.*FPCL Of Star Graph* $K_{1,3}$

E. Theorem 3.5

The Complete bipartite graphs $K_{2,n}$,$1 \le n \le 4$ are Fibonacci product cordial graphs

1) Proof: Let $X = \{u_1, u_2\}$ and $Y = \{v_1, v_2, \dots, v_n\}$ be the bipartition of the vertices of the complete bipartite graphs $K_{2,n}$

Then $E(G) = \{e_{ij}\}$ where $e_{ij} = (u_i, v_j)$ $for\ i = 1,2$, $and\ 1 \le j \le n$

Case 1: $n \equiv 0\ mod\ 2$

Define : $V(K_{2,n}) \to \{F_1, F_2, F_3, \dots F_n\}$ as

$\varphi(u_i) = F_i$ $for\ 1 \le i \le 2$

$\varphi(v_i) = F_{2+i}$ $for\ 1 \le i \le n$

Also define $\varphi^*: E(K_{2,n}) \to \{0,1\}$ by $\varphi^*(e_i) = (\varphi(u)\varphi(v_i)) mod\ 2$ .Then we have $|e_{\varphi*}(0) - e_{\varphi*}(1)| \le 1$. Hence Star graphs $K_{1,n}$,$1 \le n \le 3$ are Fibonacci product cordial graphs.

Case 1: $n \equiv 1\ mod\ 2$

Define : $V(K_{2,n}) \to \{F_1, F_2, F_3, \dots F_n\}$ as

$\varphi(u_i) = F_{2i-1}$ $for\ 1 \le i \le 2$

$\varphi(v_i) = F_{2i}$ $for\ 1 \le i \le n$

Then $\varphi^*: E(K_{2,n}) \to \{0,1\}$ by $\varphi^*(e_i) = (\varphi(v_i)\varphi(w_i)) mod\ 2$ and $\varphi^*(e_{ij}) = (\varphi(v_i)\varphi(v_j)) mod\ 2$ admits Fibonacci product cordial labeling. Hence Complete bipartite graphs $K_{2,n}$ are Fibonacci product cordial graphs .

2) Illustration 3.7:

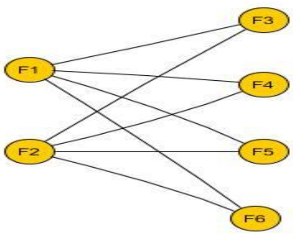

Figure 3.7. *FPCL Of Complete Graph* $K_{2,4}$

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IV. CONCLUSION

Here we proved five new theorems corresponding to Fibonacci product cordial labeling. Analogous results can be derived for other graph families as well as for different graph labeling problems..

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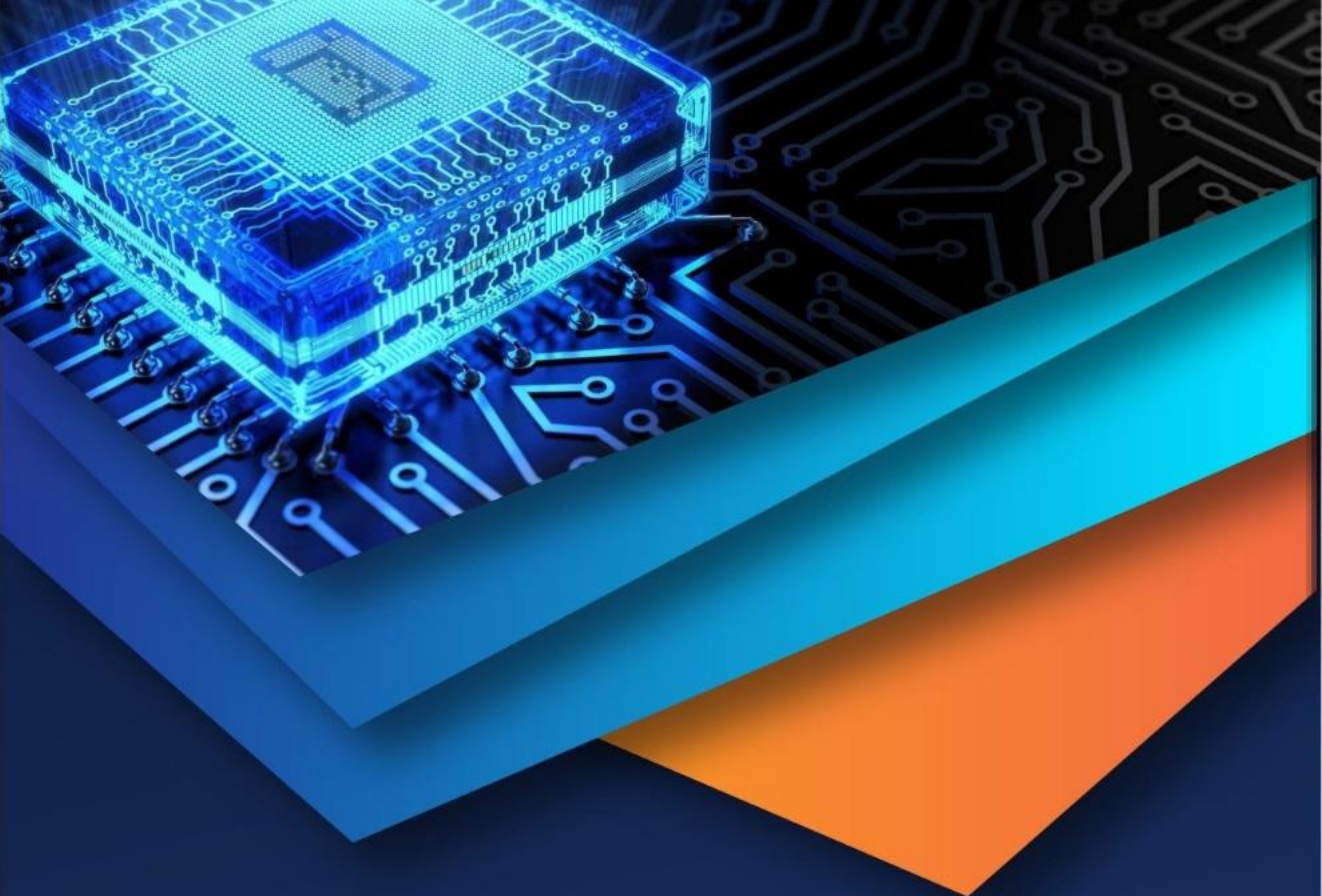

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